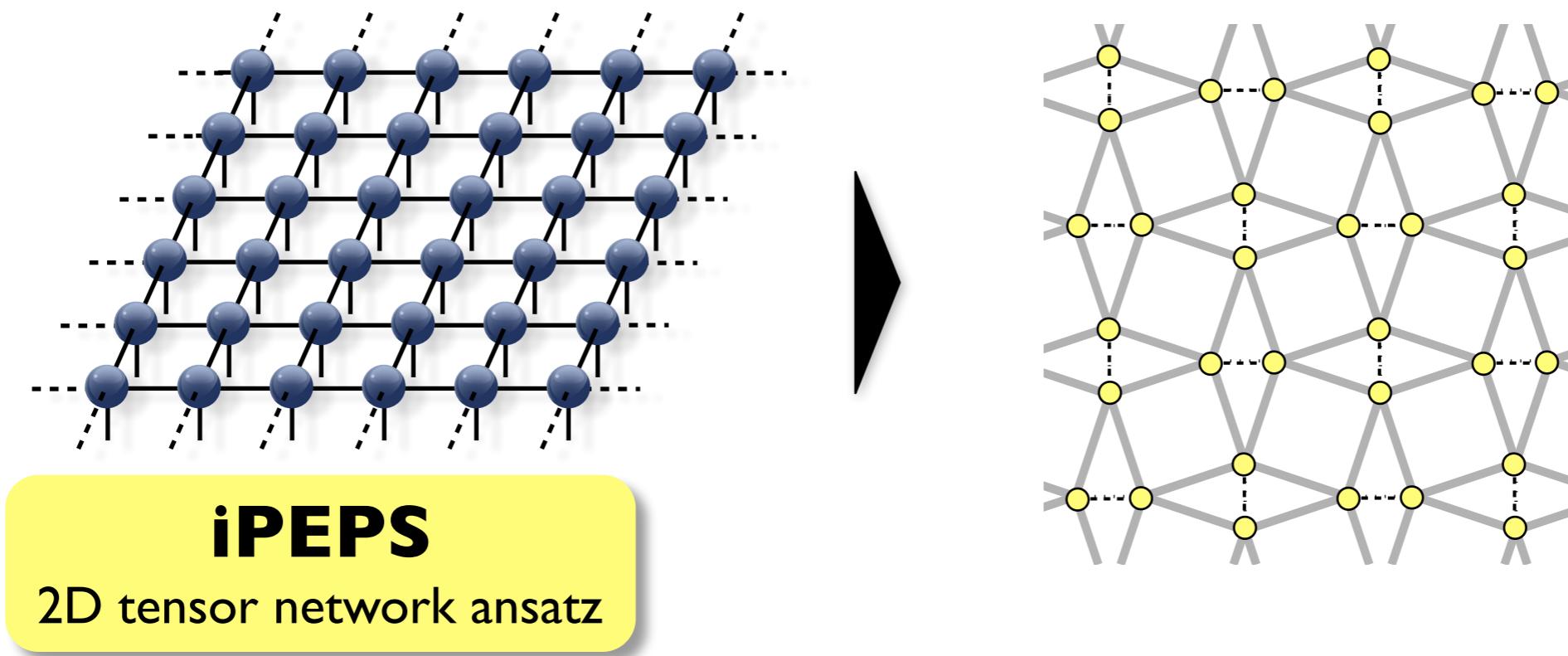


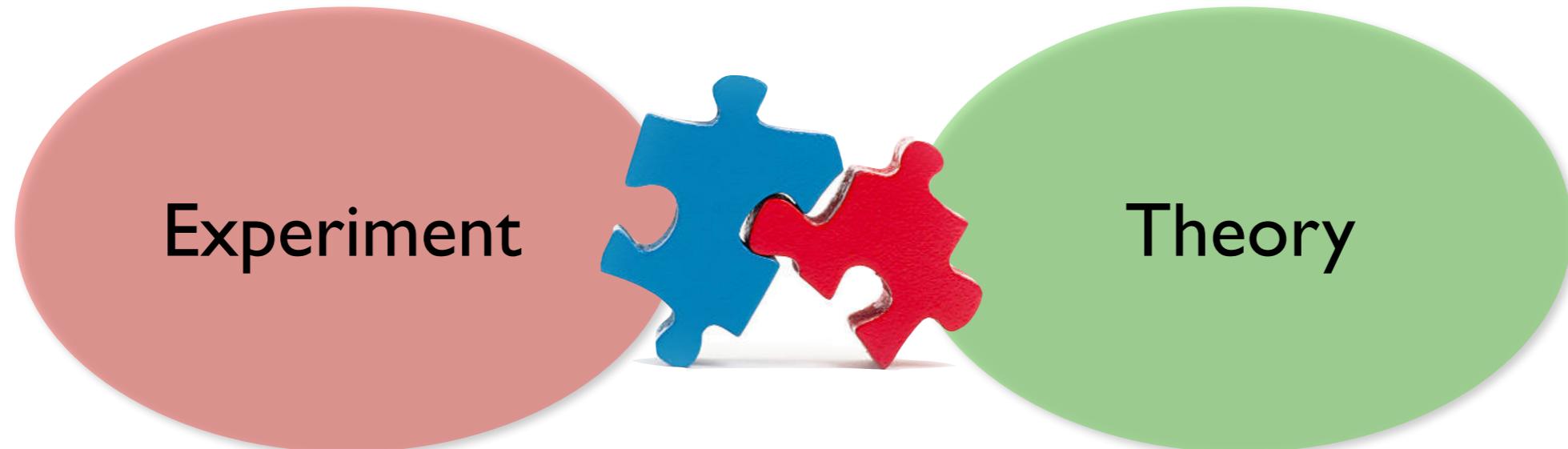
# Tensor network studies of the Shastry-Sutherland model ( $\text{SrCu}_2(\text{BO}_3)_2$ )

Philippe Corboz, Institute for Theoretical Physics, University of Amsterdam

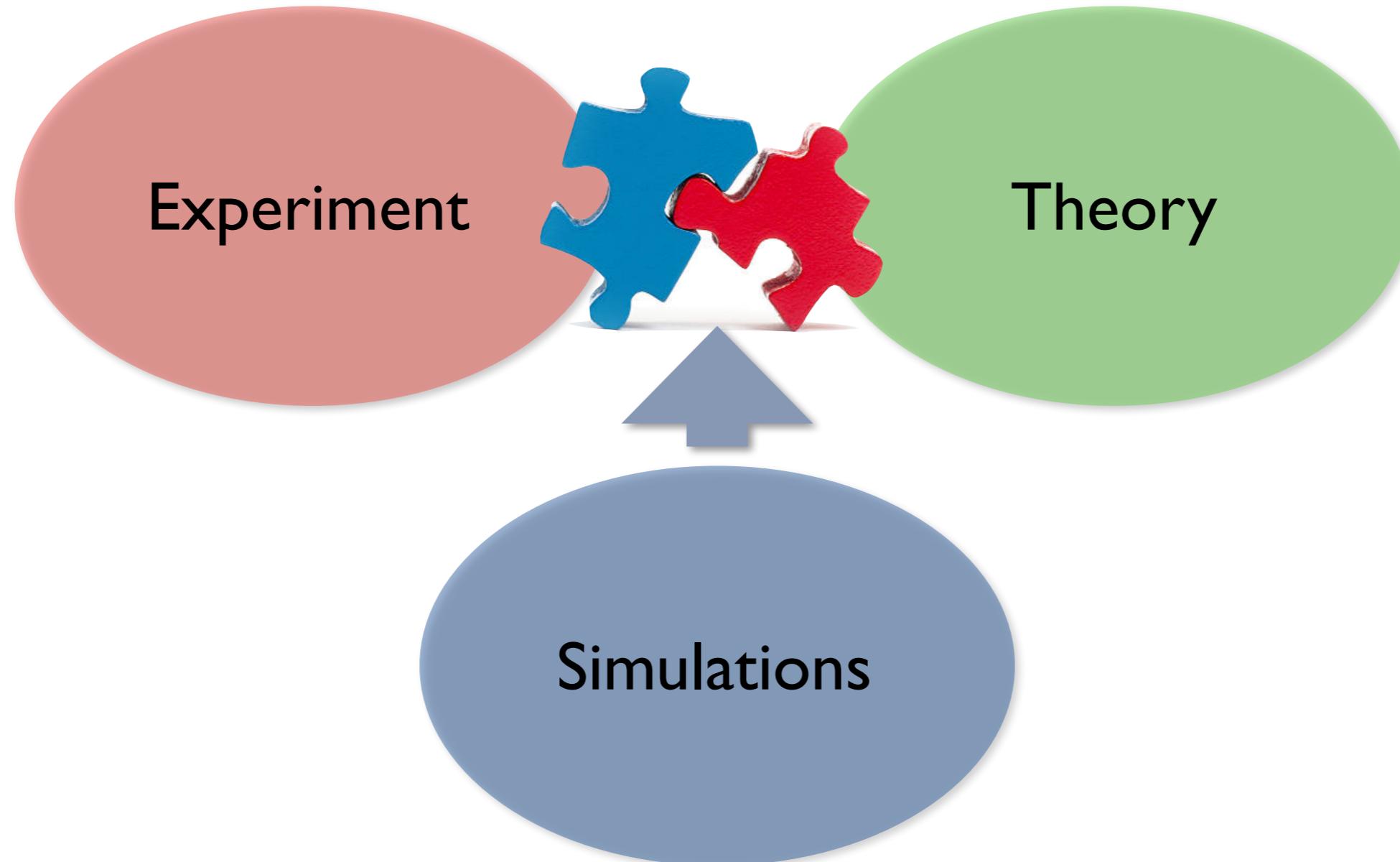


# The Shastry-Sutherland model ( $SrCu_2(BO_3)_2$ )

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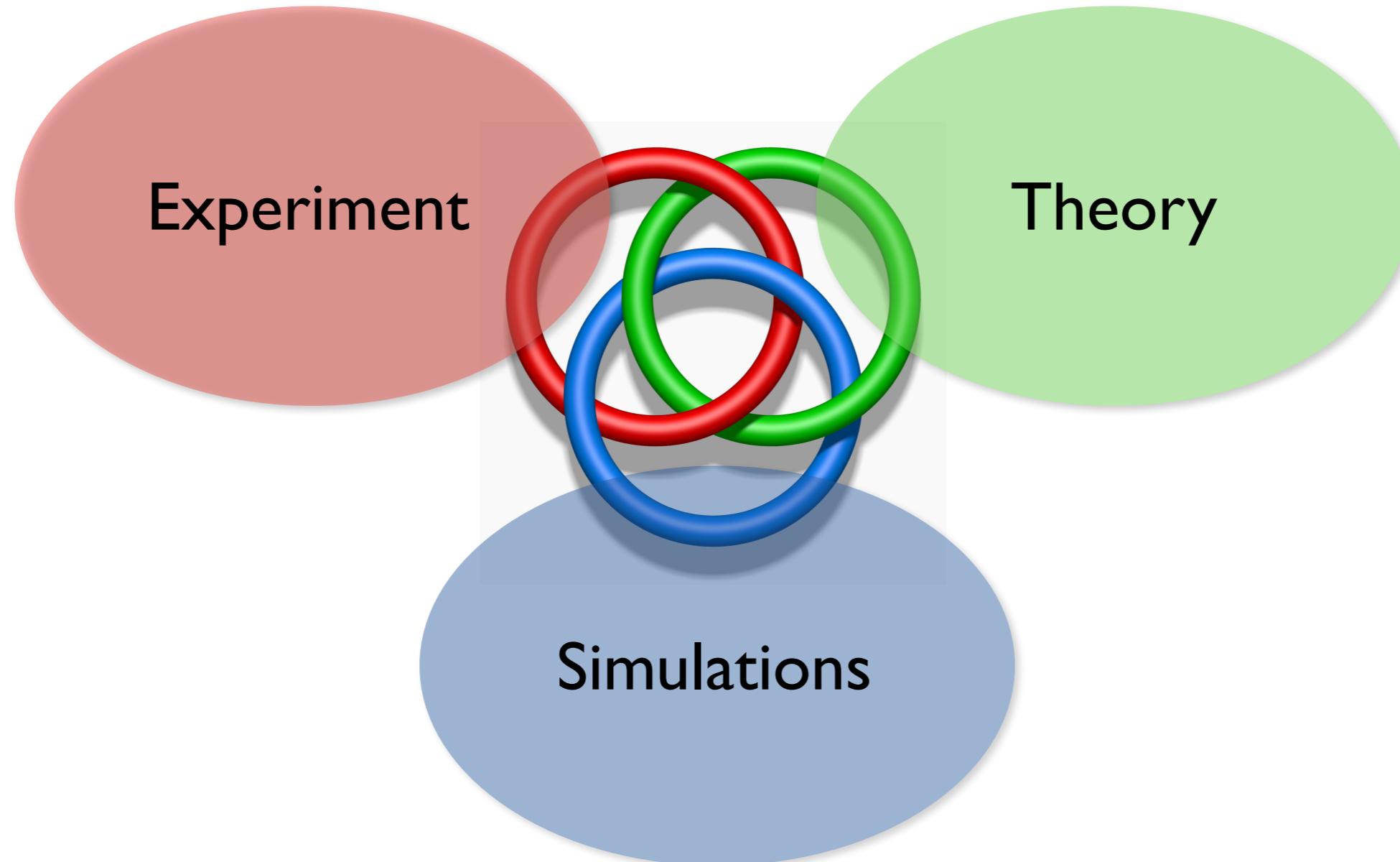


# The Shastry-Sutherland model ( $\text{SrCu}_2(\text{BO}_3)_2$ )



- ▶ *Unbiased numerical simulations* → **SURPRISE**
  - ◆ New understanding of the magnetization process in  $\text{SrCu}_2(\text{BO}_3)_2$

# The Shastry-Sutherland model ( $\text{SrCu}_2(\text{BO}_3)_2$ )



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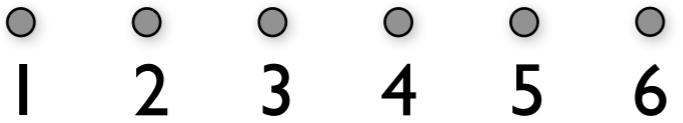
# Outline

- ▶ Introduction to iPEPS (2D tensor network ansatz)
- ▶ The Shastry-Sutherland model (SSM) in a magnetic field
  - ◆ *Magnetization plateaus = crystals made of Sz=2 bound states (not triplets!)*
  - ◆ *Supersolid phases at high magnetic fields*
- ▶ The chemically doped Shastry-Sutherland system ( $\text{SrCu}_{2-x}\text{Mg}_x(\text{BO}_3)_2$ )
  - ◆ *New anomalies in the magnetization process*
- ▶  $\text{SrCu}_2(\text{BO}_3)_2$  under pressure: extended SSM
  - ◆ *Competition between full plaquette phase vs empty plaquette phase*
- ▶ Outlook & summary

# Tensor network ansatz for a wave function

Lattice:  2 basis states per site:  $\{| \uparrow \rangle, | \downarrow \rangle\}$   
 $2^6$  basis states

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                                **2<sup>6</sup> basis states**

State:     $|\Psi\rangle = \sum_{i_1 i_2 i_3 i_4 i_5 i_6} \Psi_{i_1 i_2 i_3 i_4 i_5 i_6} |i_1 \otimes i_2 \otimes i_3 \otimes i_4 \otimes i_5 \otimes i_6\rangle$       **2<sup>6</sup> coefficients**

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**2<sup>6</sup> coefficients**

Tensor/multidimensional array

Big tensor  $\Psi_{i_1 i_2 i_3 i_4 i_5 i_6}$



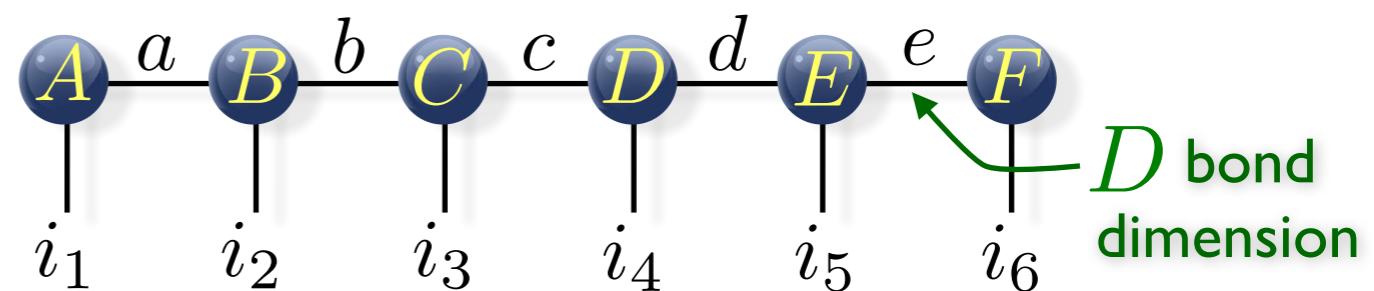
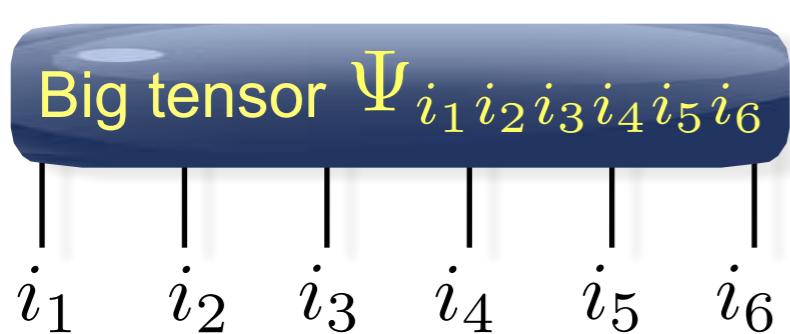
$i_1 \quad i_2 \quad i_3 \quad i_4 \quad i_5 \quad i_6$

# Tensor network ansatz for a wave function

Lattice:     2 basis states per site:  $\{| \uparrow \rangle, | \downarrow \rangle\}$   
 $i \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$     **2<sup>6</sup> basis states**

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Tensor/multidimensional array    **Tensor network:** matrix product state (**MPS**)

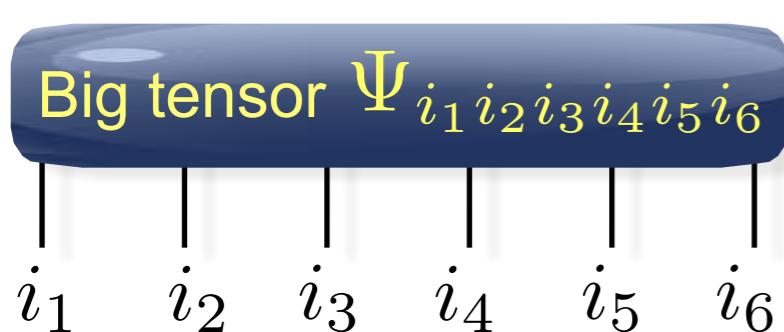


# Tensor network ansatz for a wave function

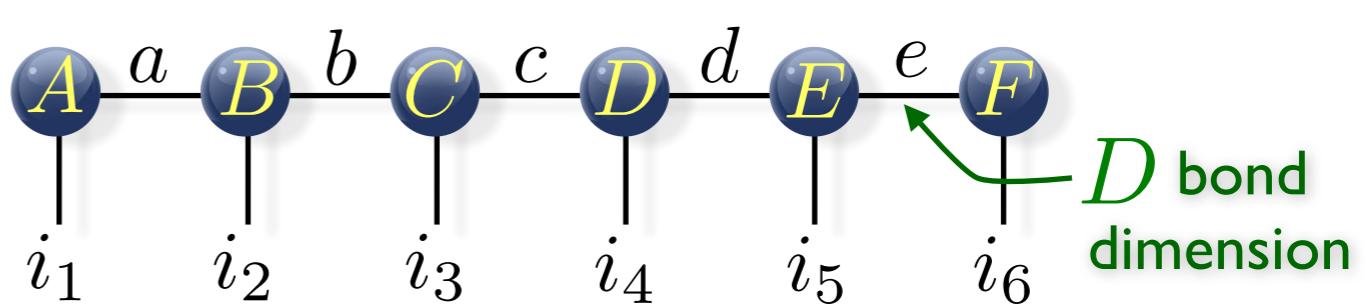
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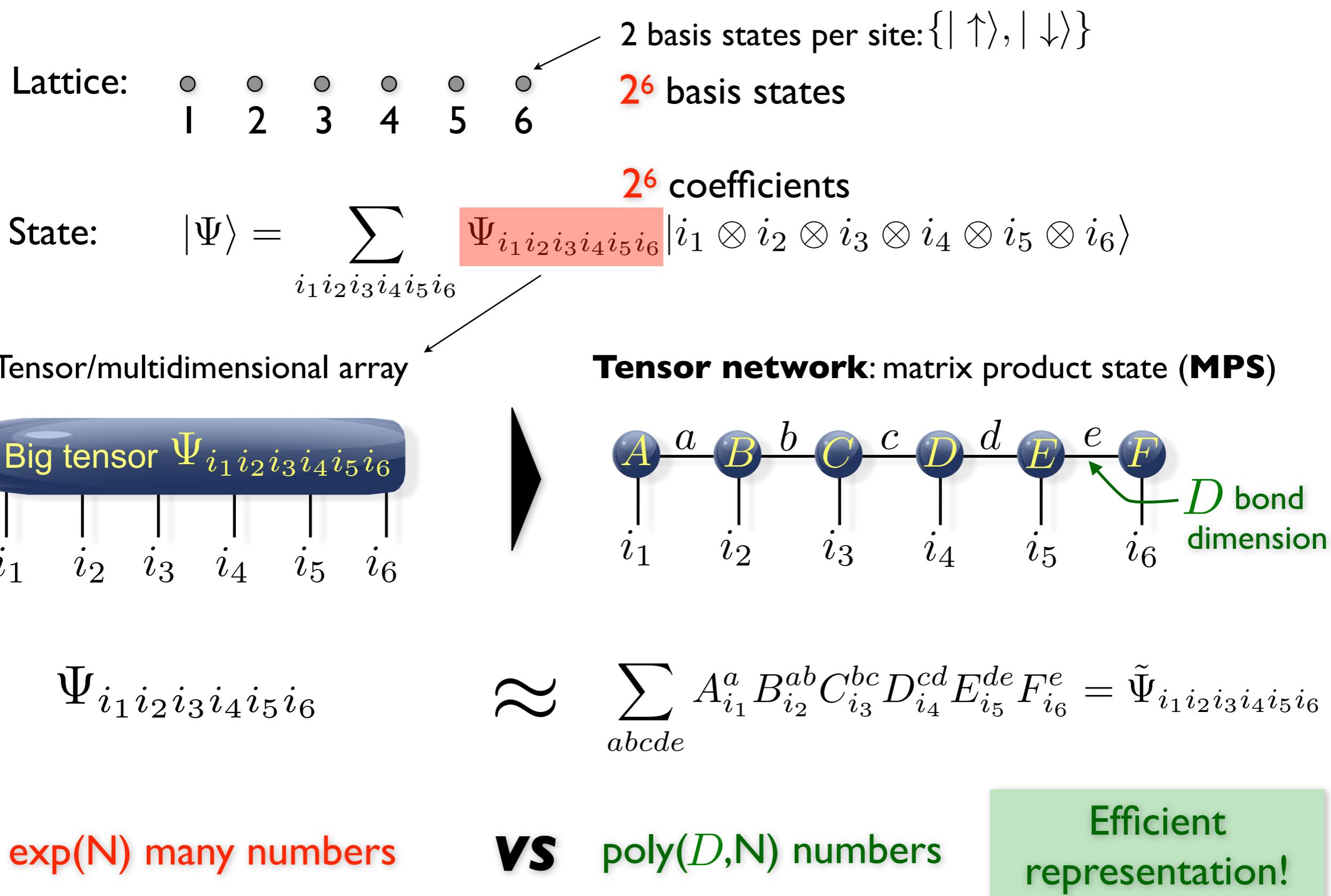


**Tensor network:** matrix product state (**MPS**)



$$\Psi_{i_1 i_2 i_3 i_4 i_5 i_6} \approx \sum_{abcde} A_{i_1}^a B_{i_2}^{ab} C_{i_3}^{bc} D_{i_4}^{cd} E_{i_5}^{de} F_{i_6}^e = \tilde{\Psi}_{i_1 i_2 i_3 i_4 i_5 i_6}$$

# Tensor network ansatz for a wave function

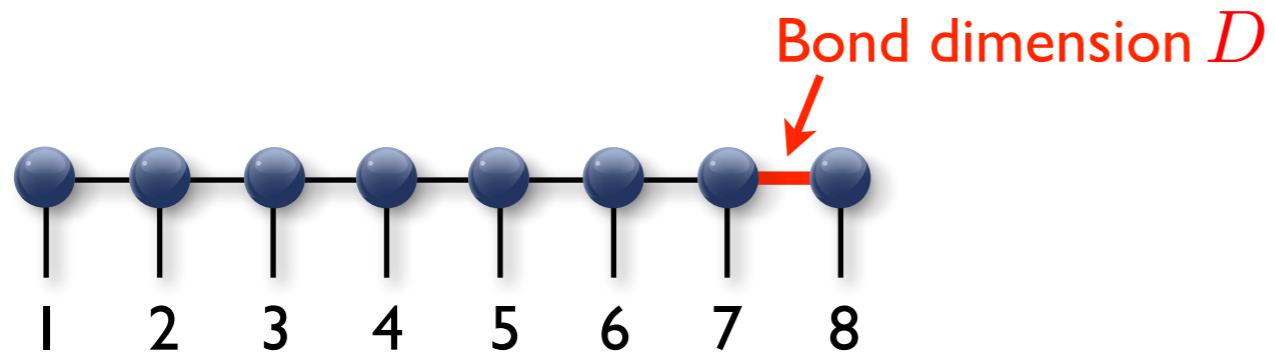


# MPS & PEPS

ID

**MPS**

Matrix-product state



Physical indices (lattice sites)

S. R. White, PRL 69, 2863 (1992)

Fannes et al., CMP 144, 443 (1992)

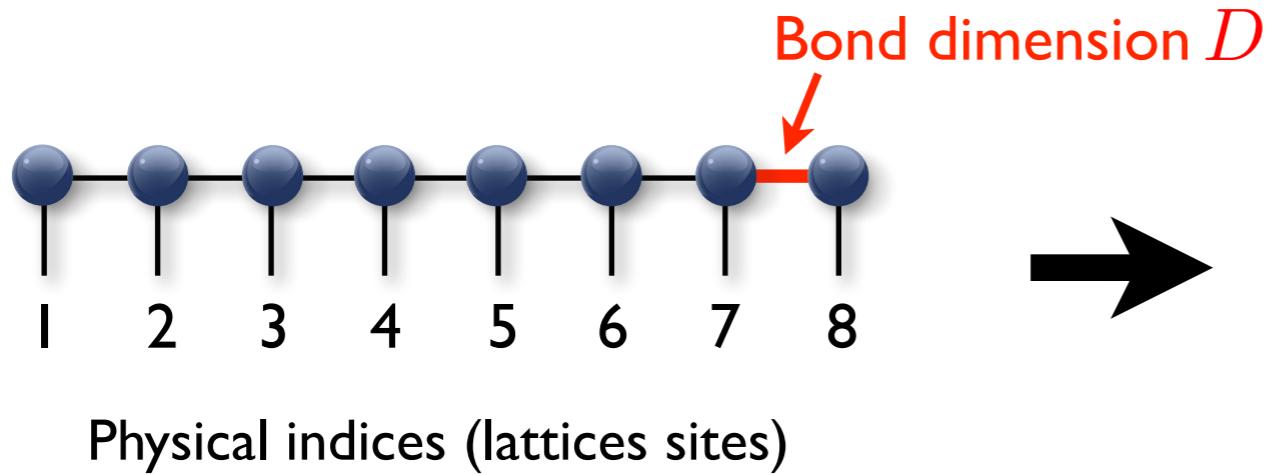
Östlund, Rommer, PRL 75, 3537 (1995)

# MPS & PEPS

**ID**

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Matrix-product state



S. R. White, PRL 69, 2863 (1992)

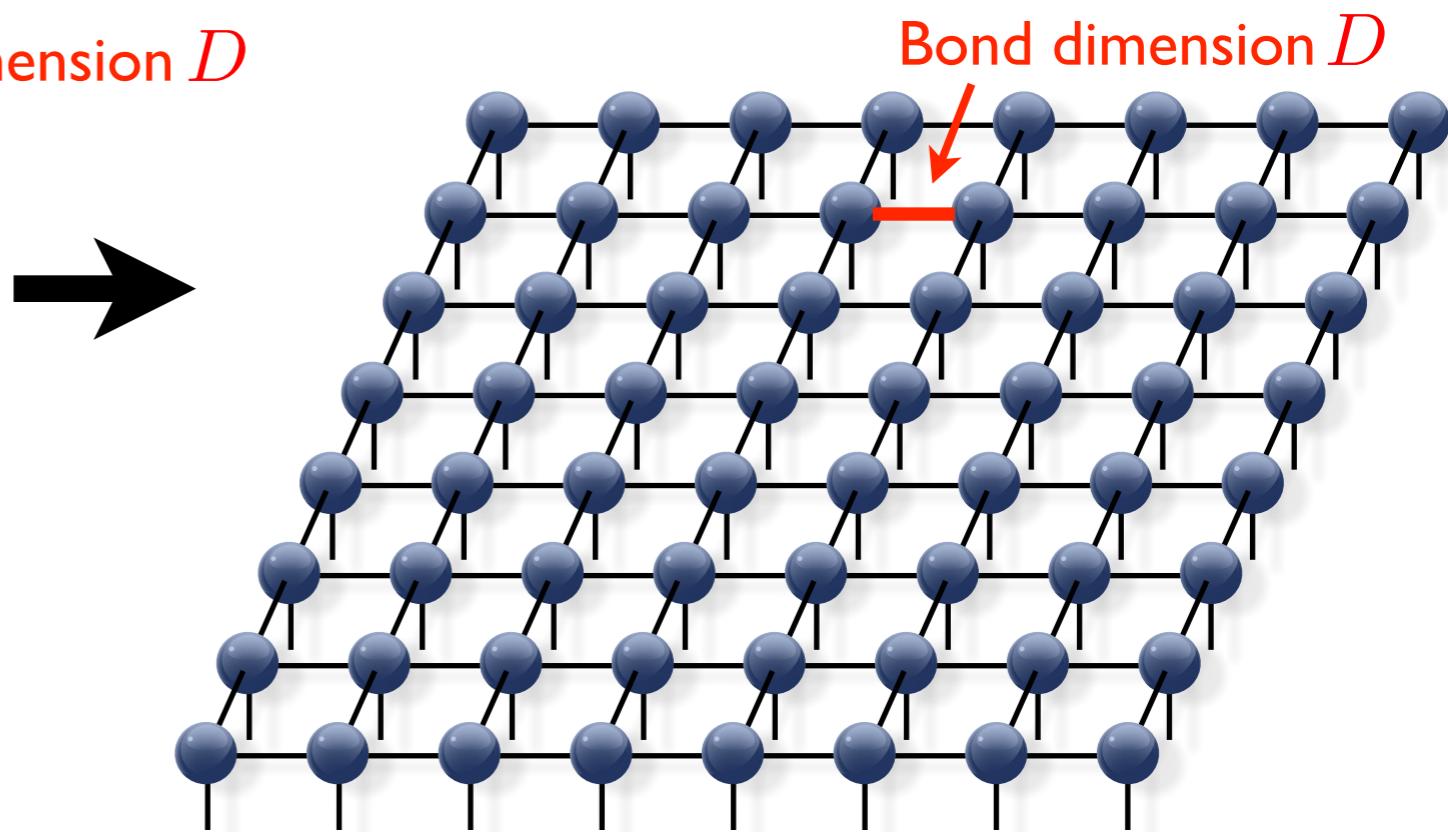
Fannes et al., CMP 144, 443 (1992)

Östlund, Rommer, PRL 75, 3537 (1995)

**2D**

**PEPS (TPS)**

projected entangled-pair state  
(tensor product state)



F. Verstraete, J. I. Cirac, cond-mat/0407066

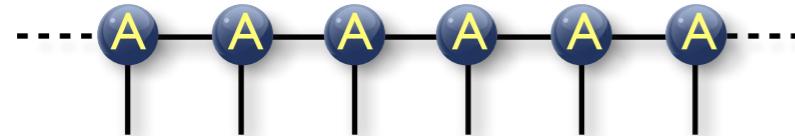
Nishio, Maeshima, Gendiar, Nishino, cond-mat/0401115

# Infinite PEPS (iPEPS)

ID

iMPS

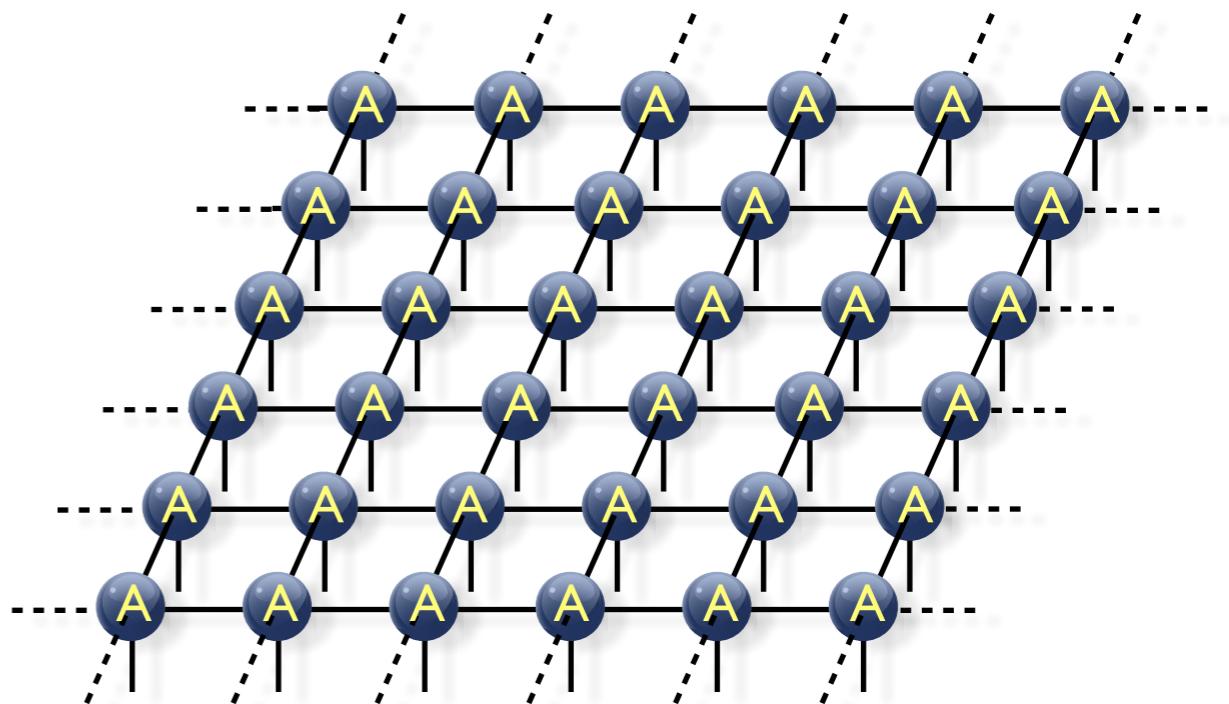
infinite matrix-product state



2D

iPEPS

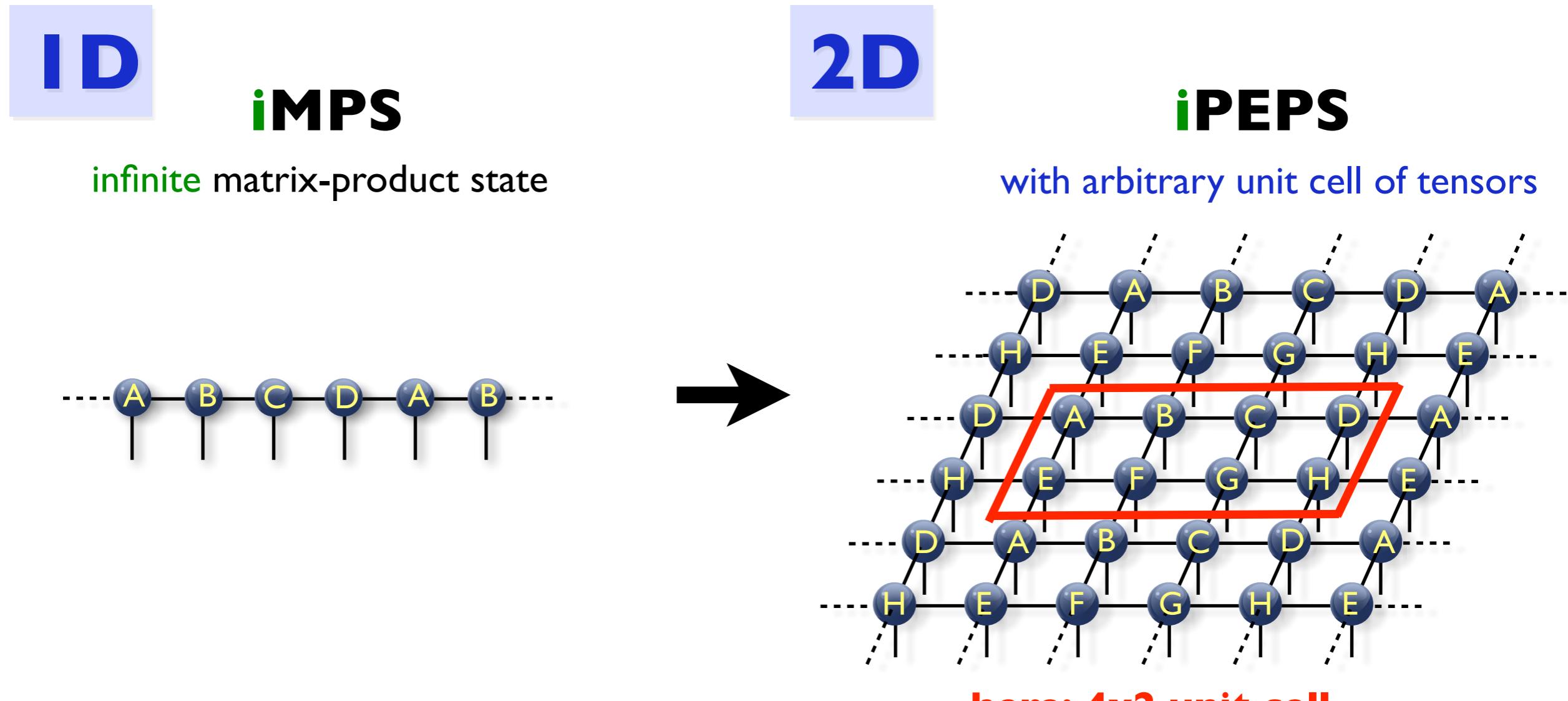
infinite projected entangled-pair state



Jordan, Orus, Vidal, Verstraete, Cirac, PRL (2008)

★ Work directly in the thermodynamic limit:  
**No finite size and boundary effects!**

# iPEPS with arbitrary unit cells



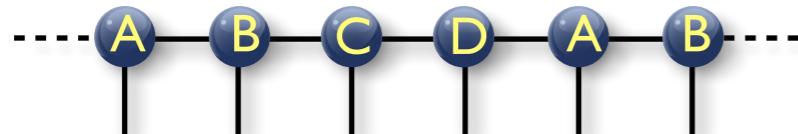
PC, White, Vidal, Troyer, PRB **84** (2011)

# iPEPS with arbitrary unit cells

**ID**

**iMPS**

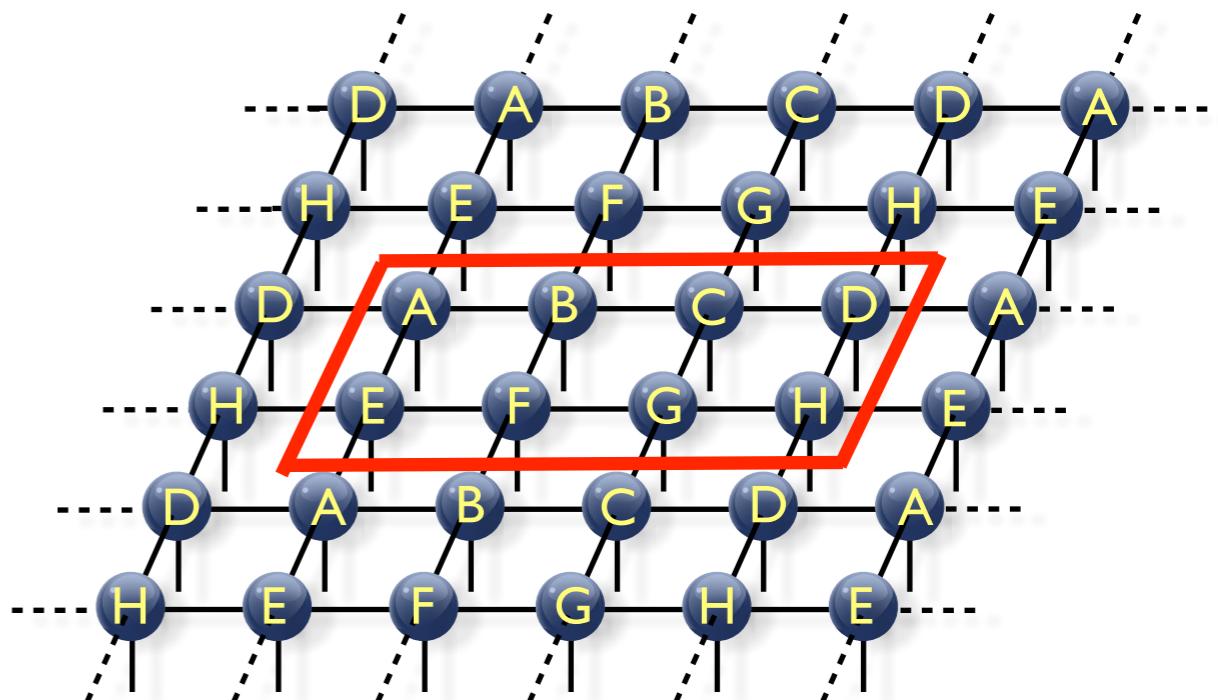
infinite matrix-product state



**2D**

**iPEPS**

with arbitrary unit cell of tensors



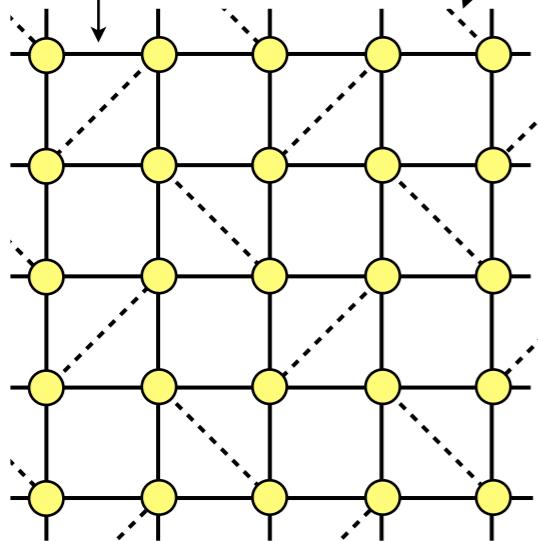
**here: 4x2 unit cell**

PC, White, Vidal, Troyer, PRB **84** (2011)

★ Run simulations with different unit cell sizes and compare variational energies

# The Shastry-Sutherland model

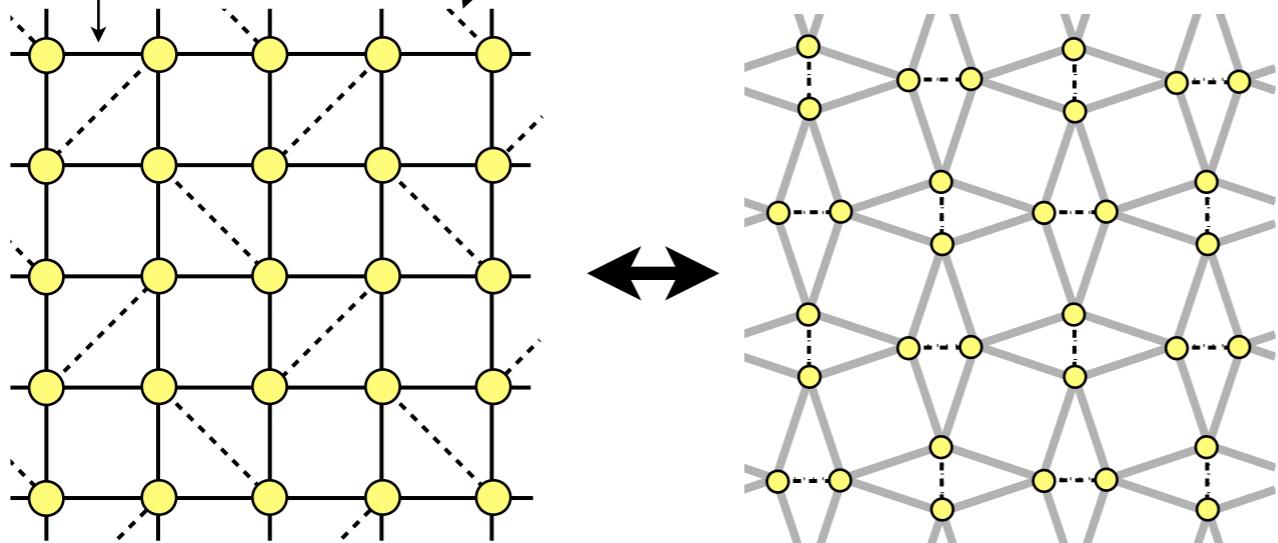
$$\hat{H} = J' \sum_{\langle i,j \rangle} S_i \cdot S_j + J \sum_{\langle\langle i,j \rangle\rangle_{\text{dimer}}} S_i \cdot S_j$$



Shastry & Sutherland, Physica B+C **108** (1981).

# The Shastry-Sutherland model

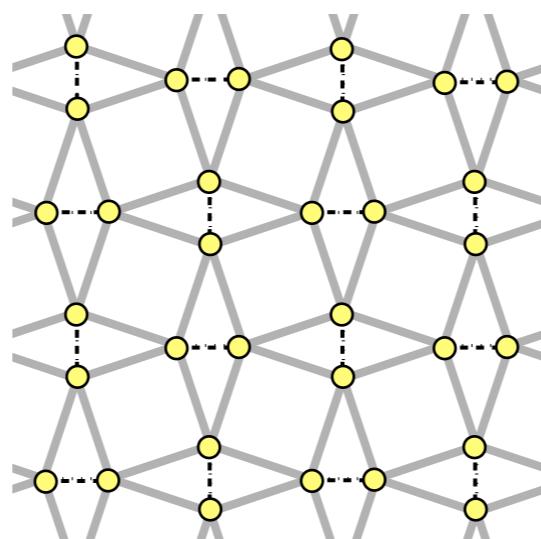
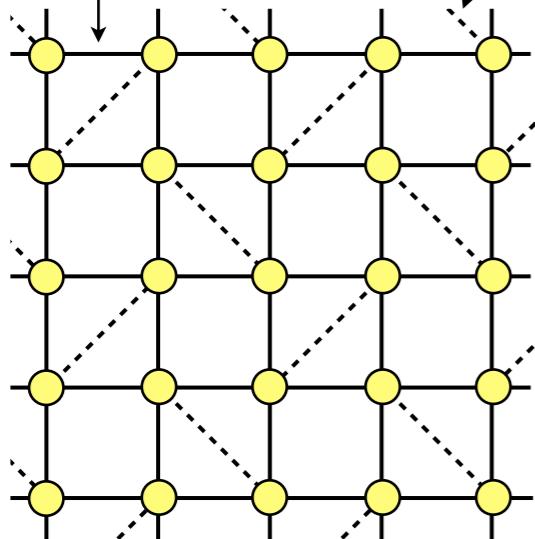
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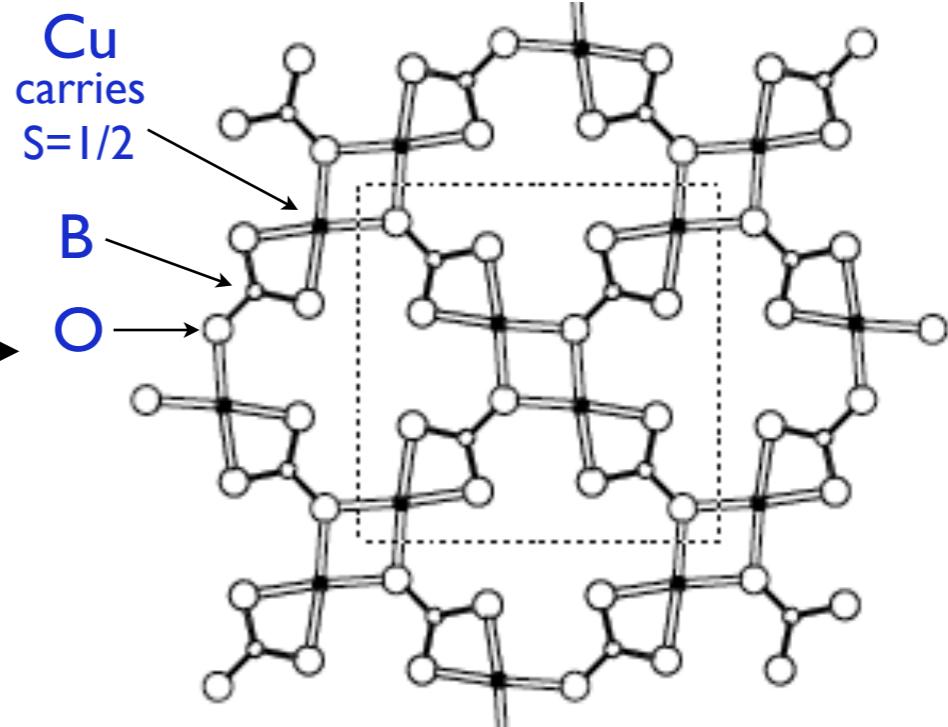
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**SrCu<sub>2</sub>(BO<sub>3</sub>)<sub>2</sub>**  
Spin-gap system (~35K)

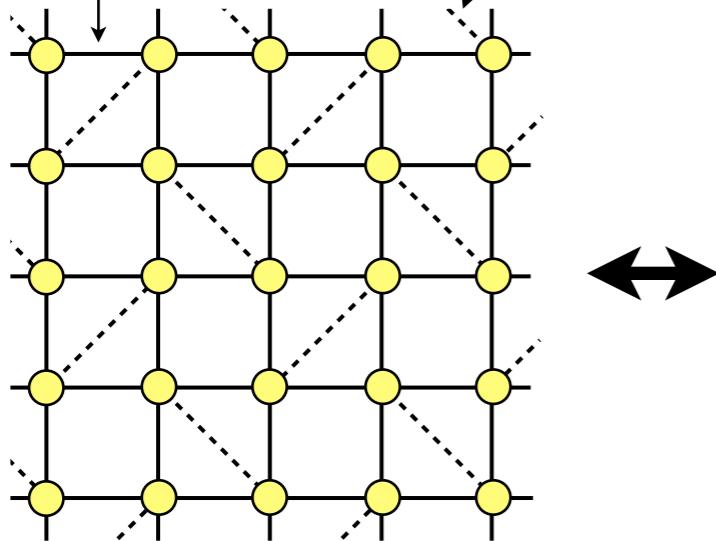


Shastry & Sutherland, Physica B+C **108** (1981).

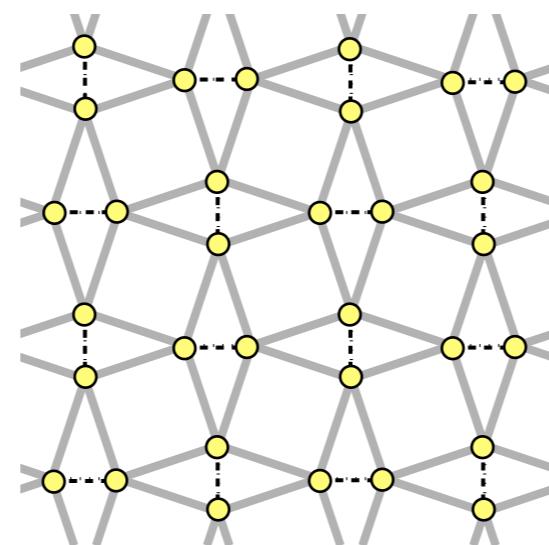
Kageyama et al. PRL **82** (1999)

# The Shastry-Sutherland model

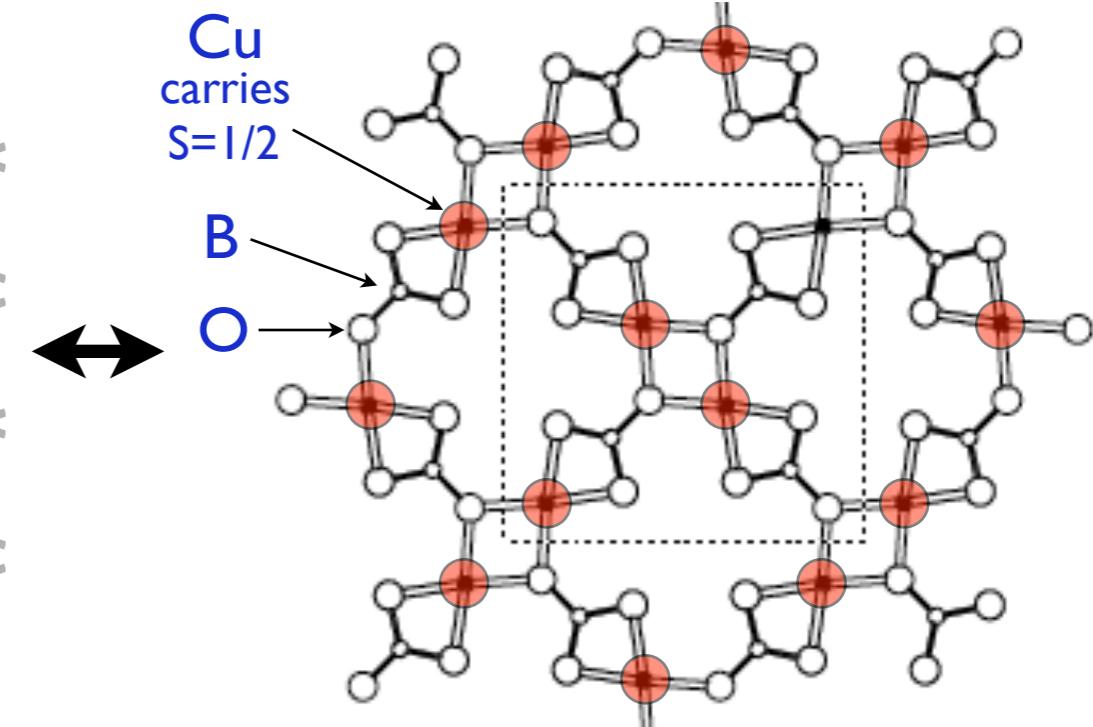
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Shastry & Sutherland, Physica B+C **108** (1981).



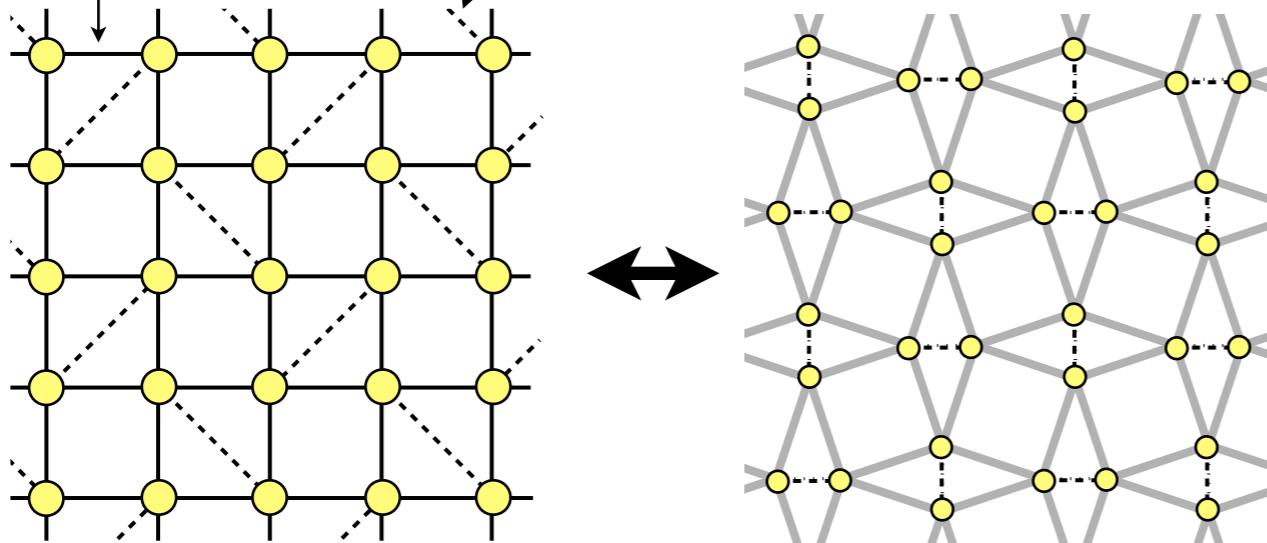
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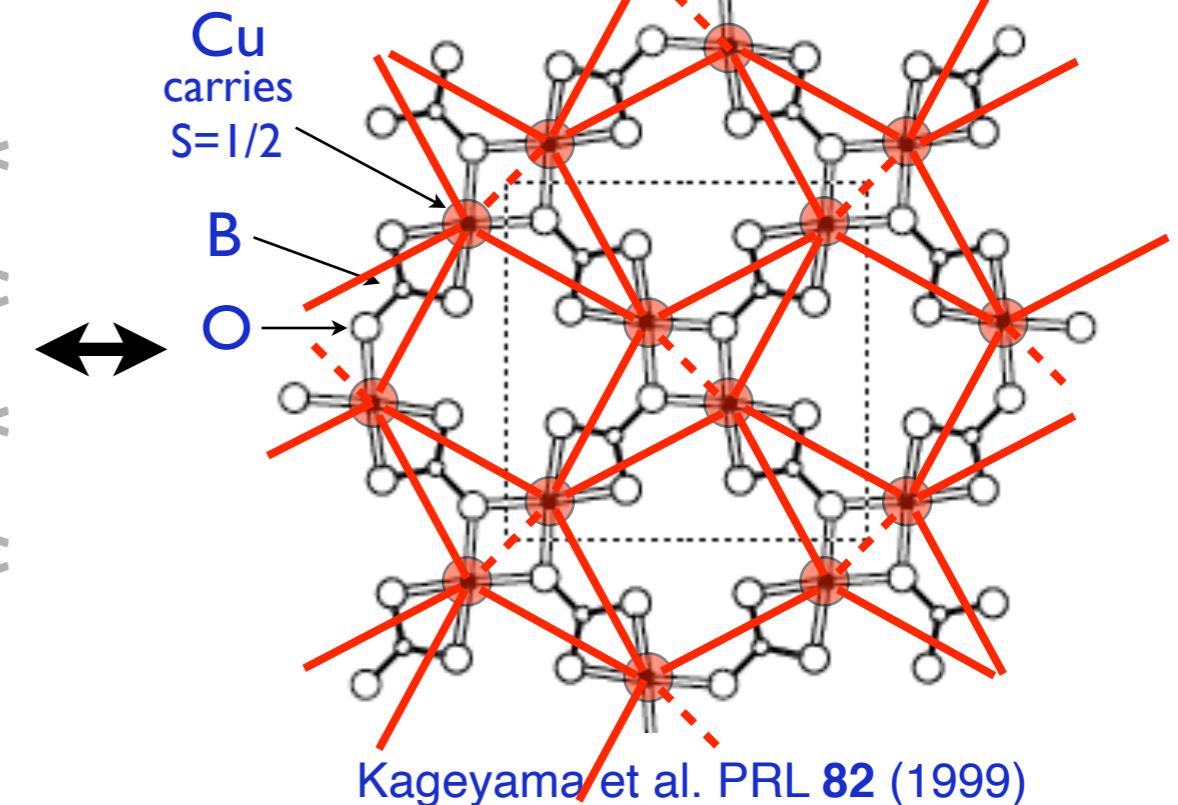
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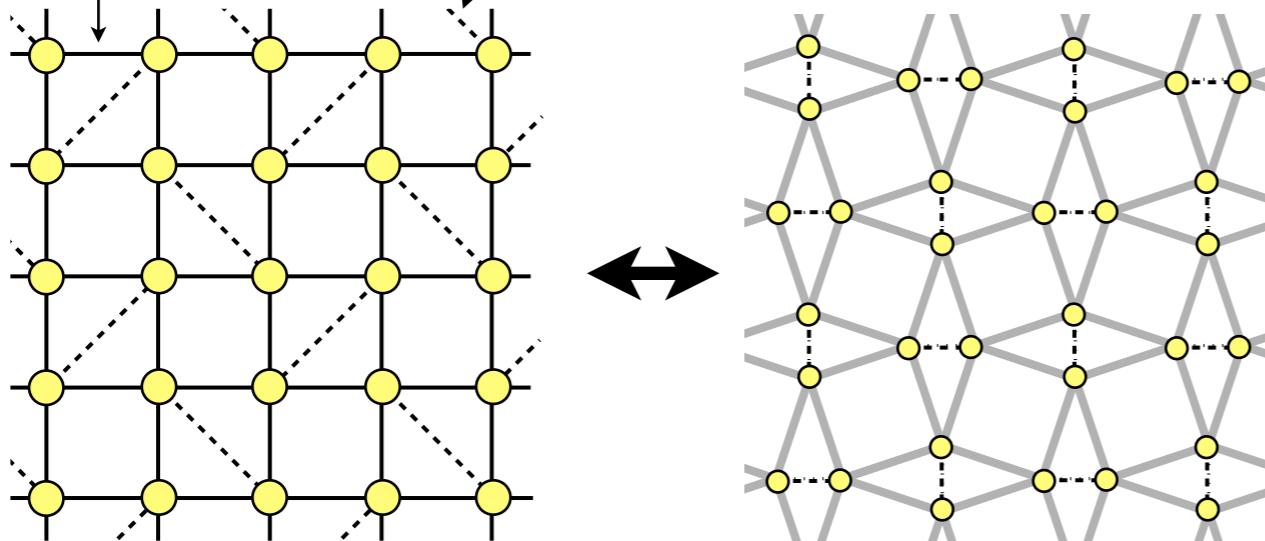
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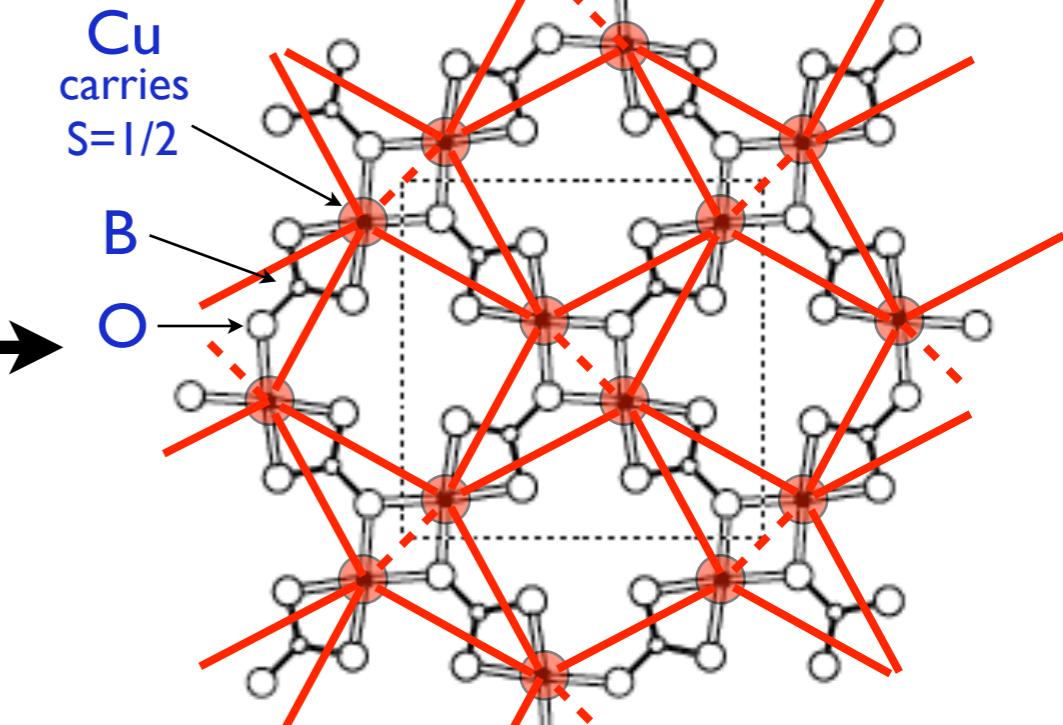
Kageyama et al. PRL **82** (1999)

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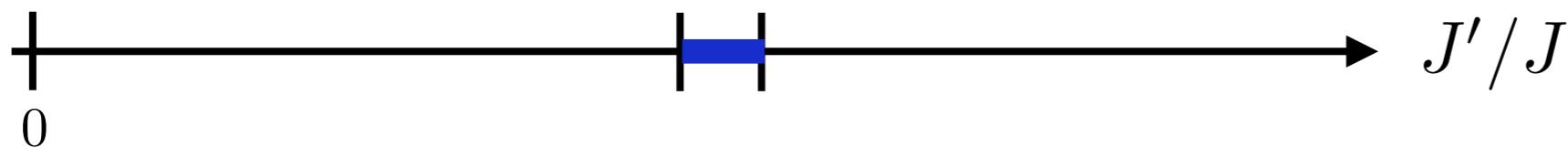
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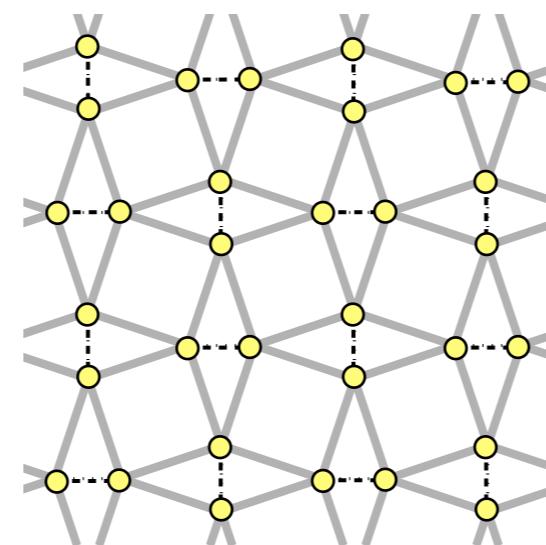
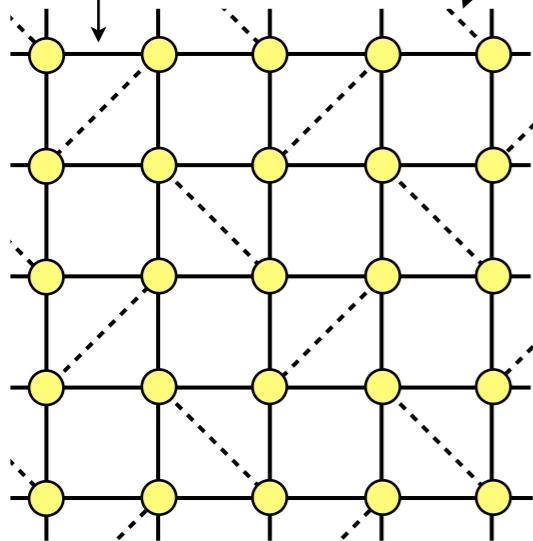


Kageyama et al. PRL 82 (1999)

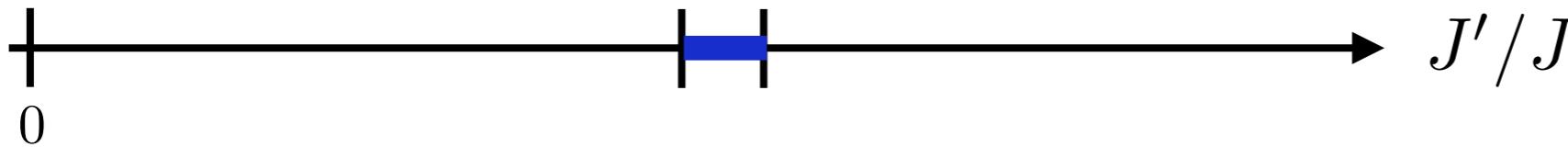
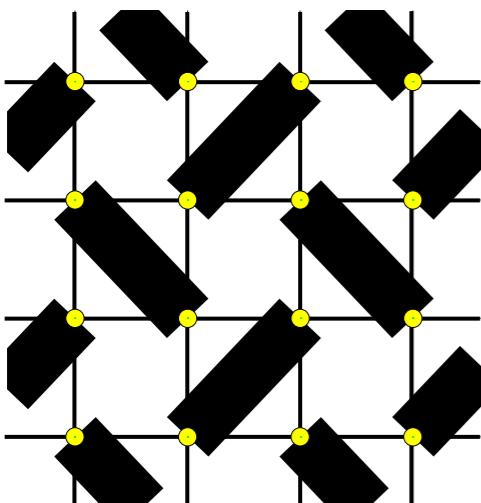


# The Shastry-Sutherland model

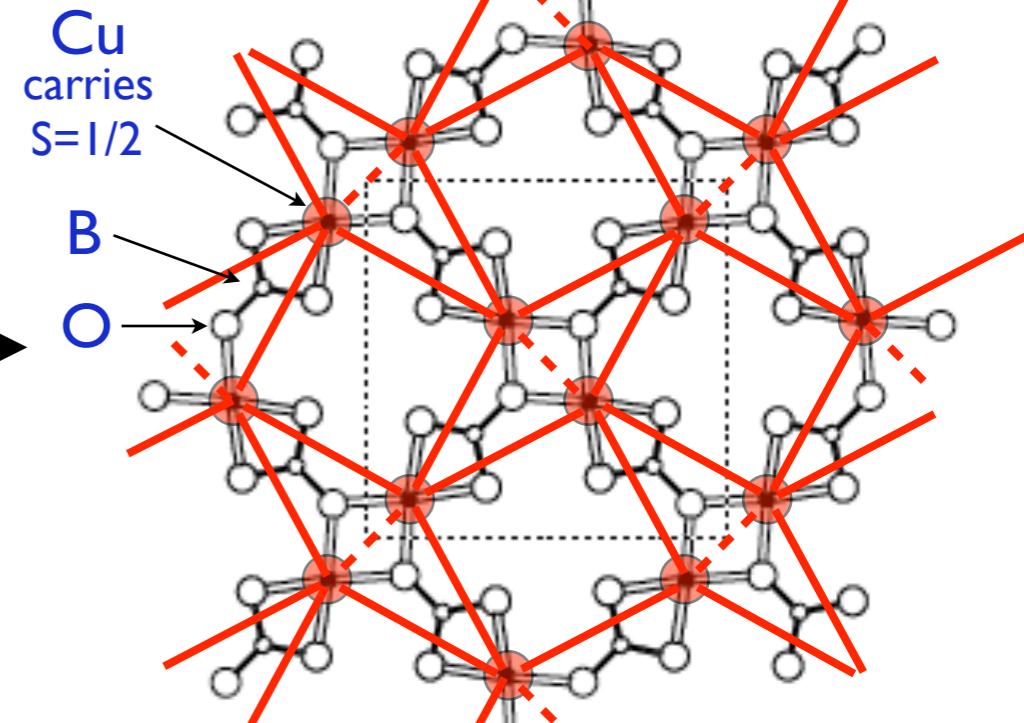
$$\hat{H} = J' \sum_{\langle i,j \rangle} S_i \cdot S_j + J \sum_{\langle\langle i,j \rangle\rangle_{\text{dimer}}} S_i \cdot S_j$$



Dimer phase



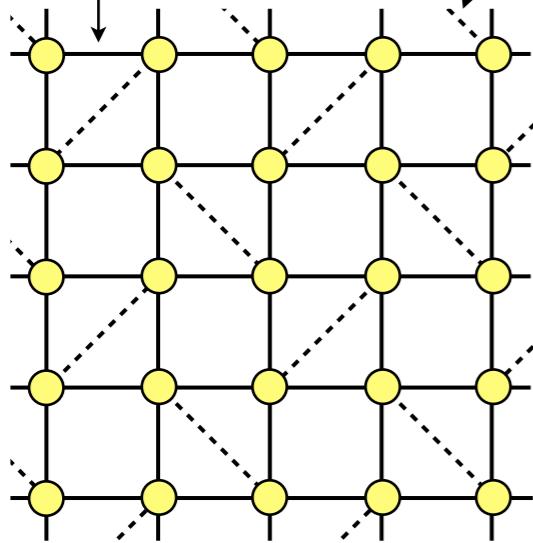
$\text{SrCu}_2(\text{BO}_3)_2$   
Spin-gap system ( $\sim 35\text{K}$ )



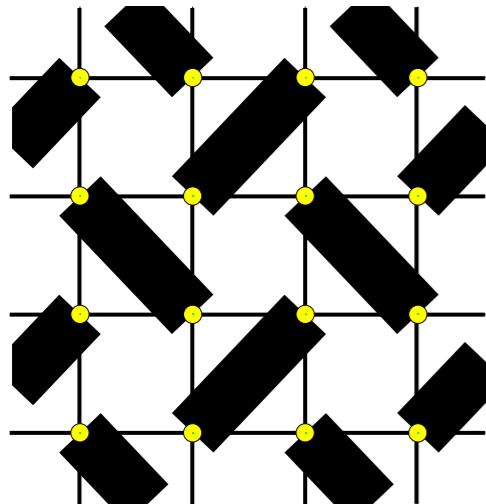
Kageyama et al. PRL 82 (1999)

# The Shastry-Sutherland model

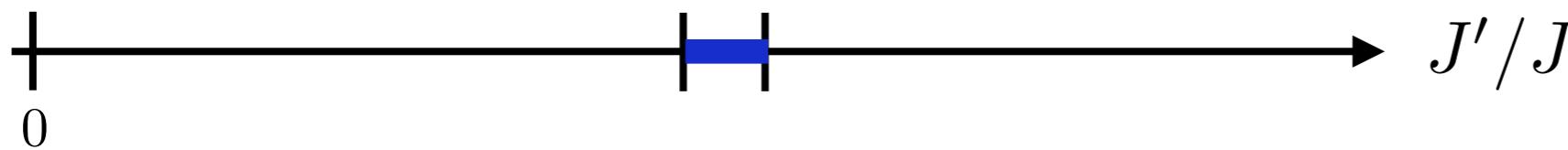
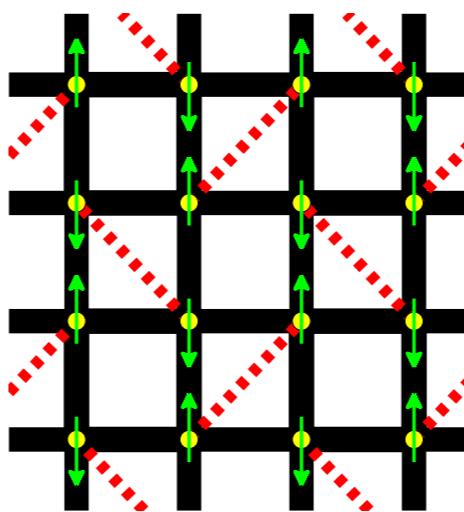
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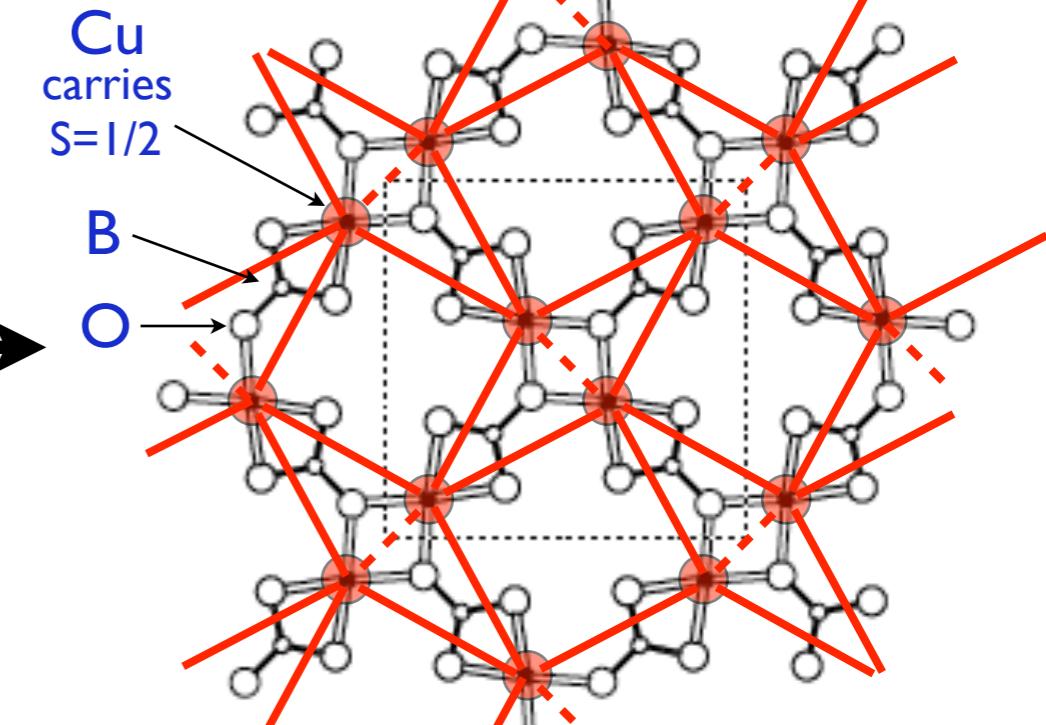
Dimer phase



Néel phase



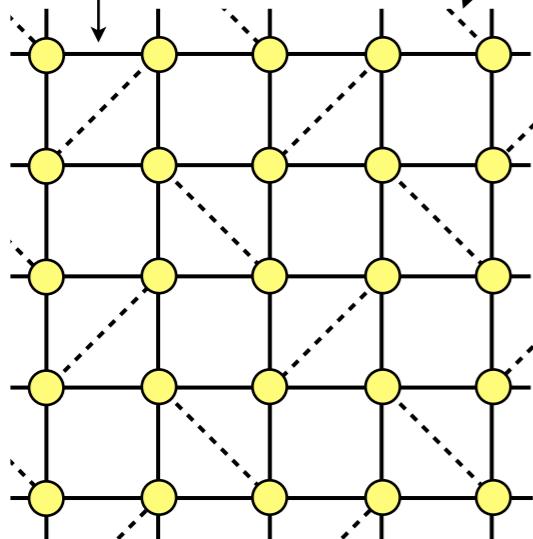
**SrCu<sub>2</sub>(BO<sub>3</sub>)<sub>2</sub>**  
Spin-gap system (~35K)



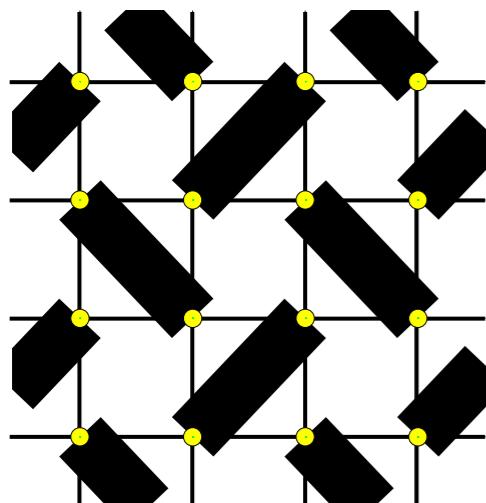
Kageyama et al. PRL 82 (1999)

# The Shastry-Sutherland model

$$\hat{H} = J' \sum_{\langle i,j \rangle} S_i \cdot S_j + J \sum_{\langle\langle i,j \rangle\rangle_{\text{dimer}}} S_i \cdot S_j$$

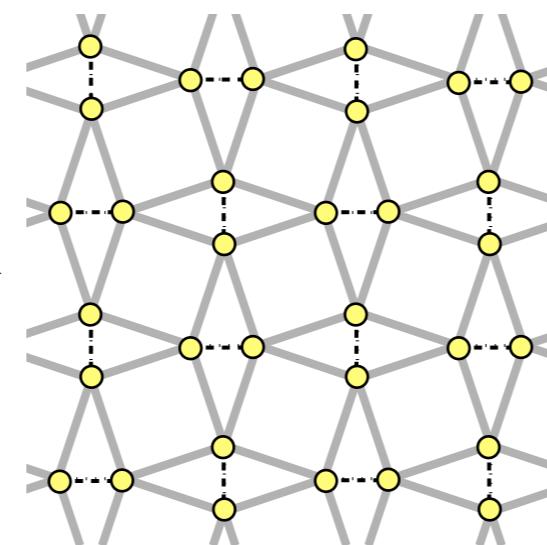
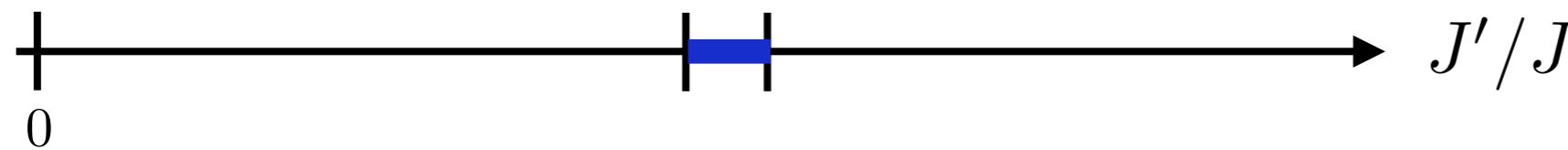


Dimer phase

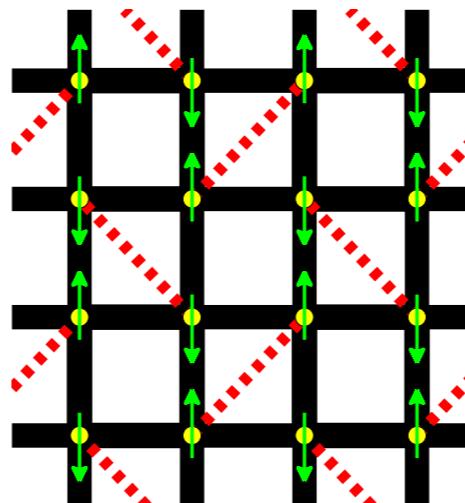


?

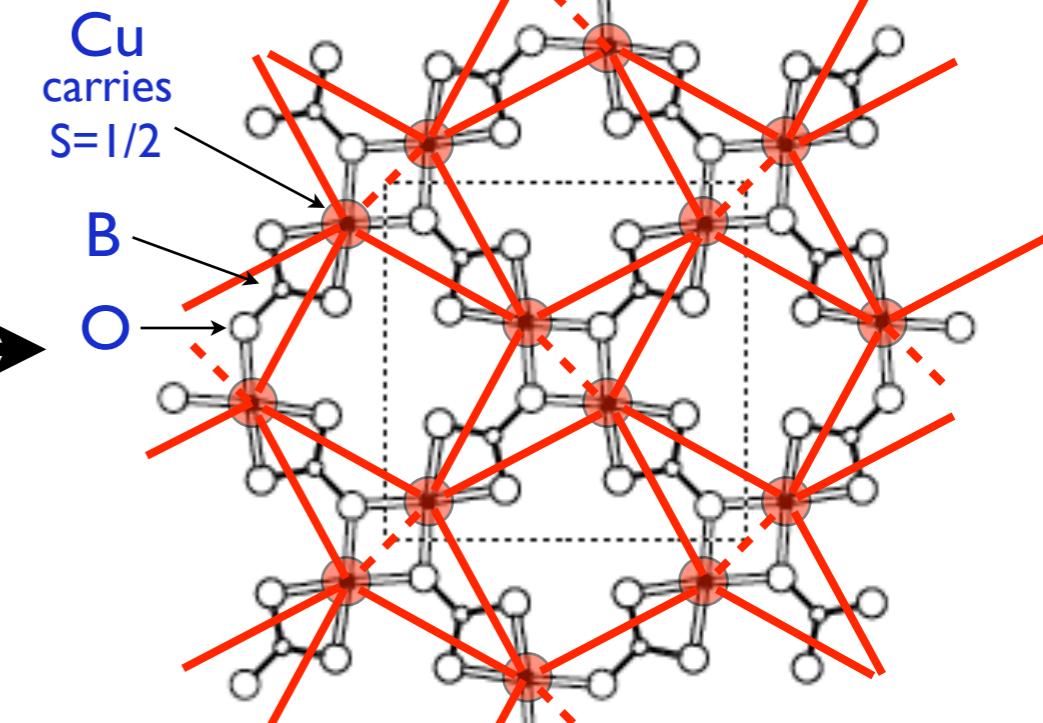
helical?  
columnar-dimer?  
plaquette?  
spin-liquid?



Néel phase



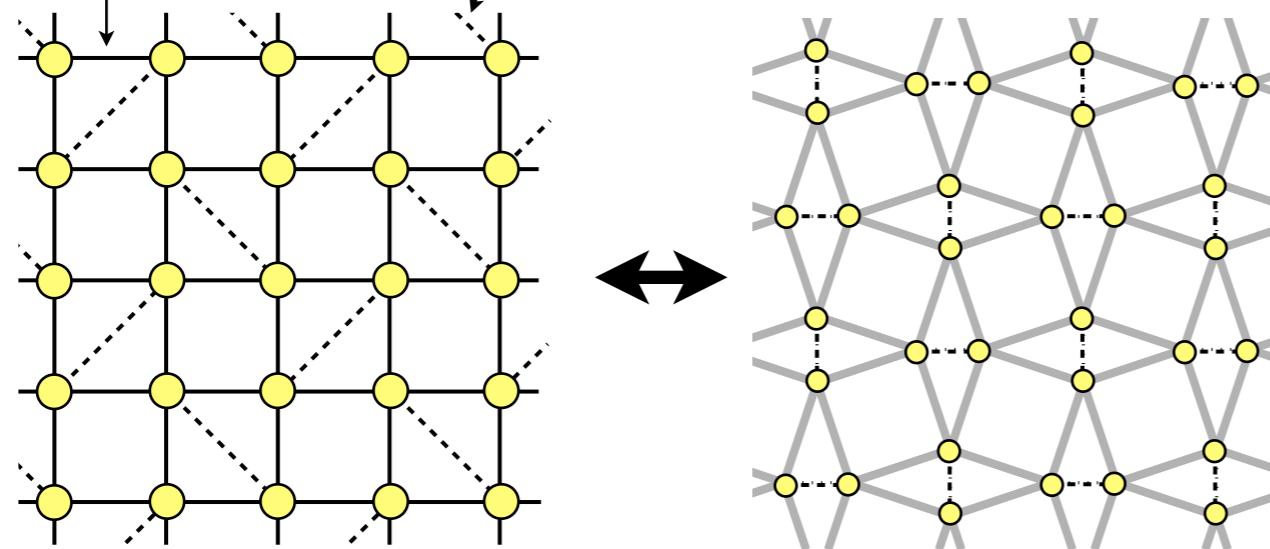
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Spin-gap system ( $\sim 35\text{K}$ )



Kageyama et al. PRL 82 (1999)

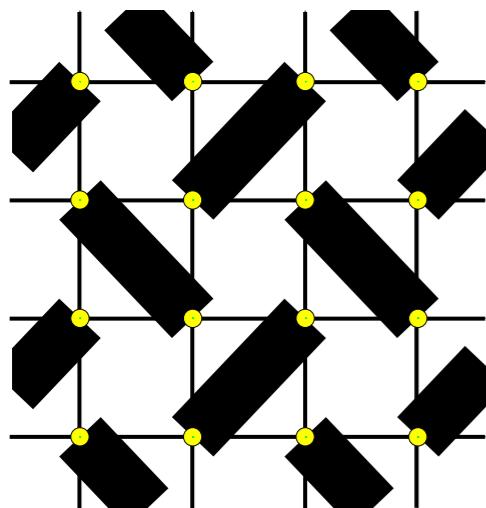
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$$\hat{H} = J' \sum_{\langle i,j \rangle} S_i \cdot S_j + J \sum_{\langle\langle i,j \rangle\rangle_{\text{dimer}}} S_i \cdot S_j$$

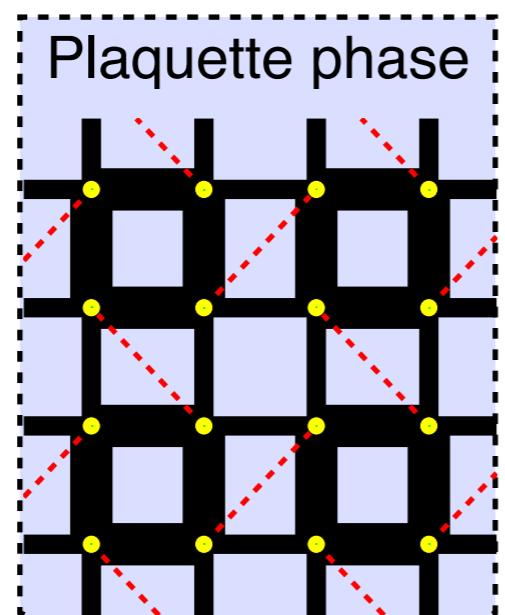


$\leftrightarrow$

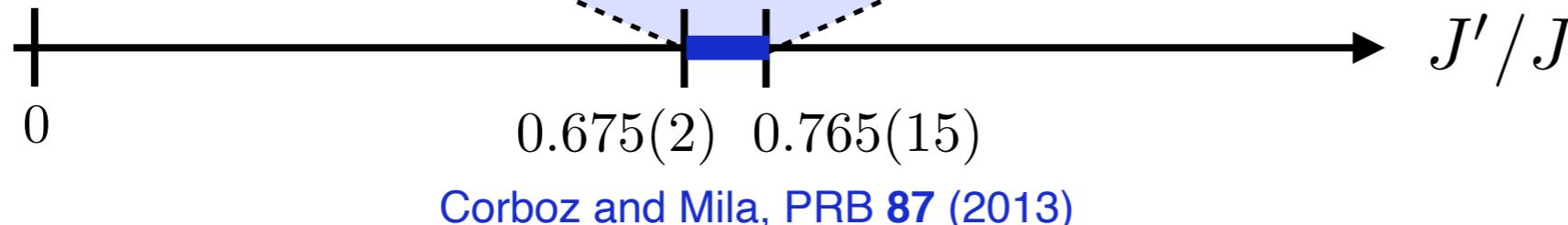
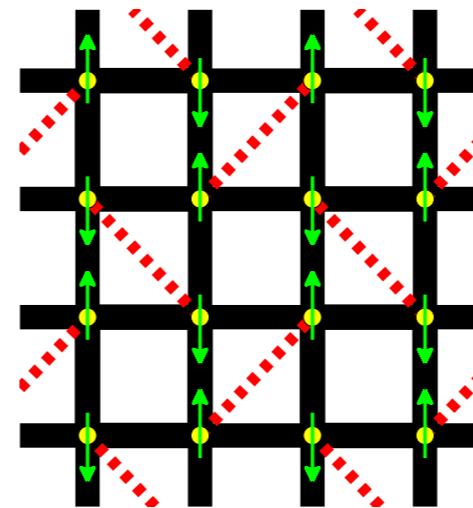
Dimer phase



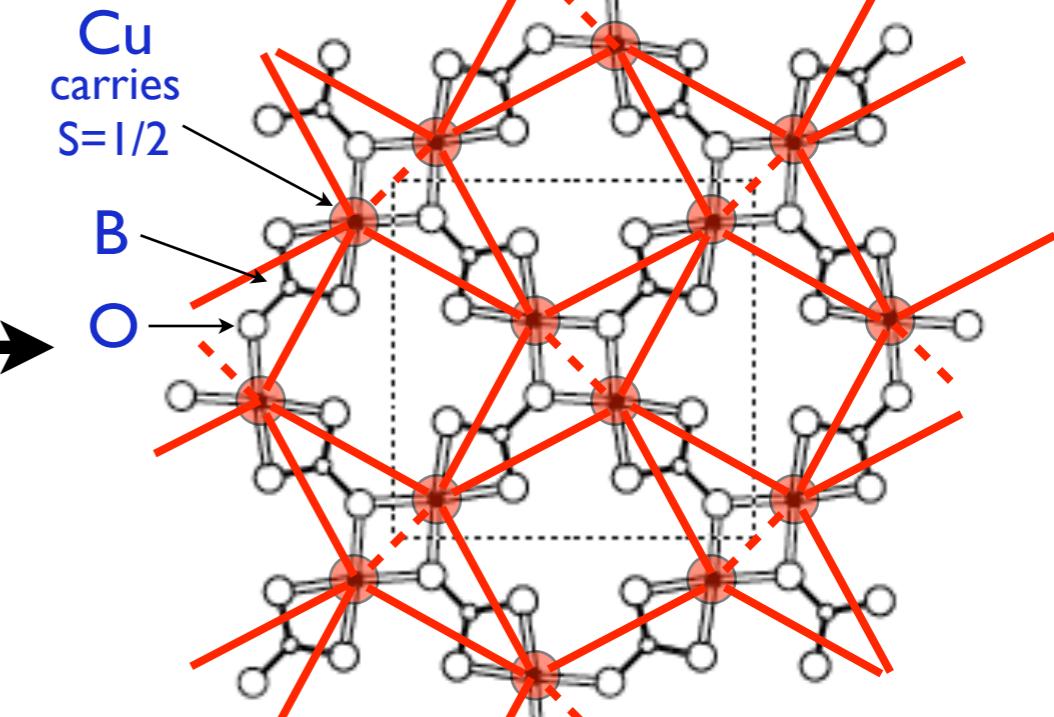
Plaquette phase



Néel phase



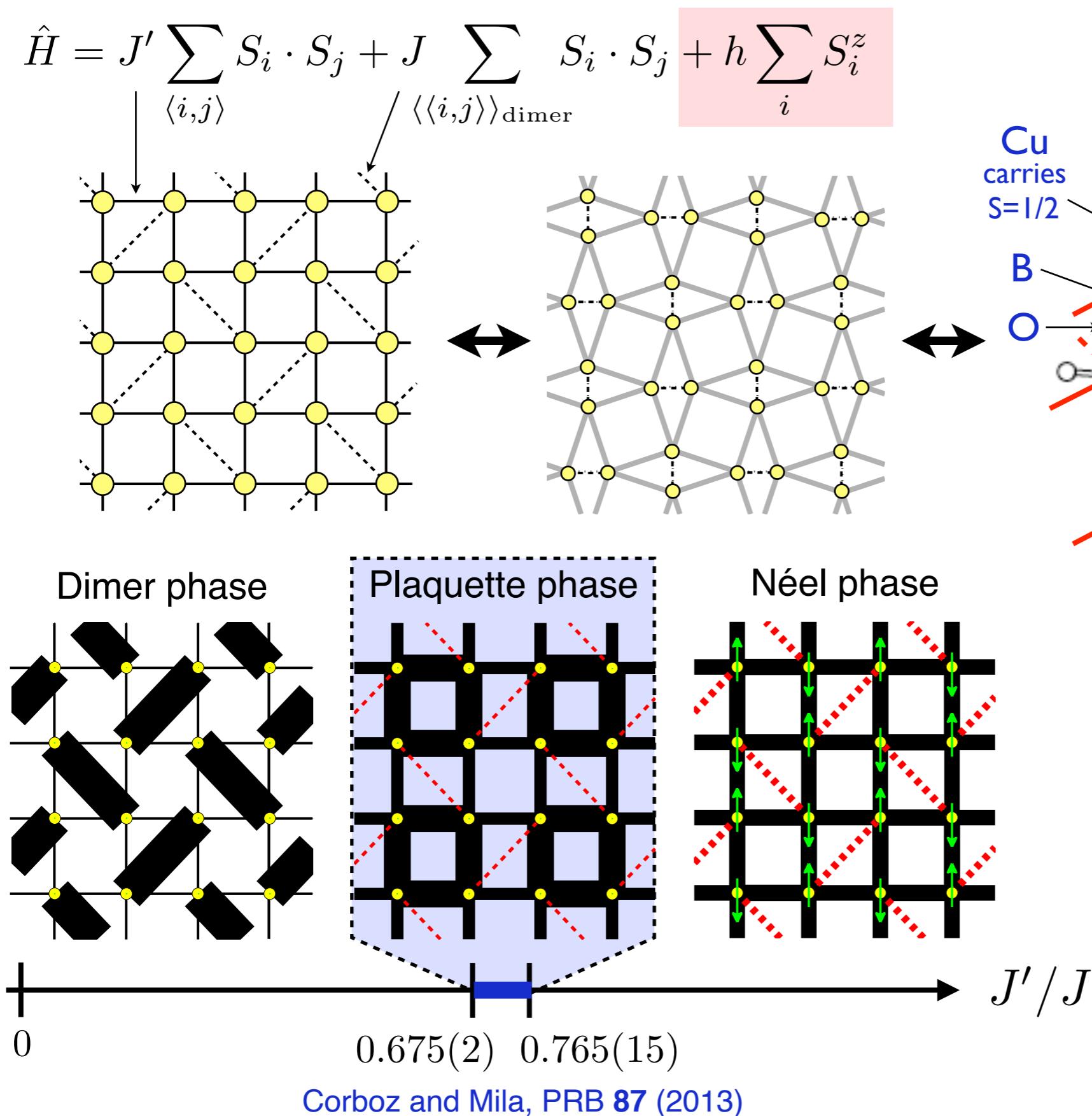
**SrCu<sub>2</sub>(BO<sub>3</sub>)<sub>2</sub>**  
Spin-gap system ( $\sim 35K$ )



previously found in:

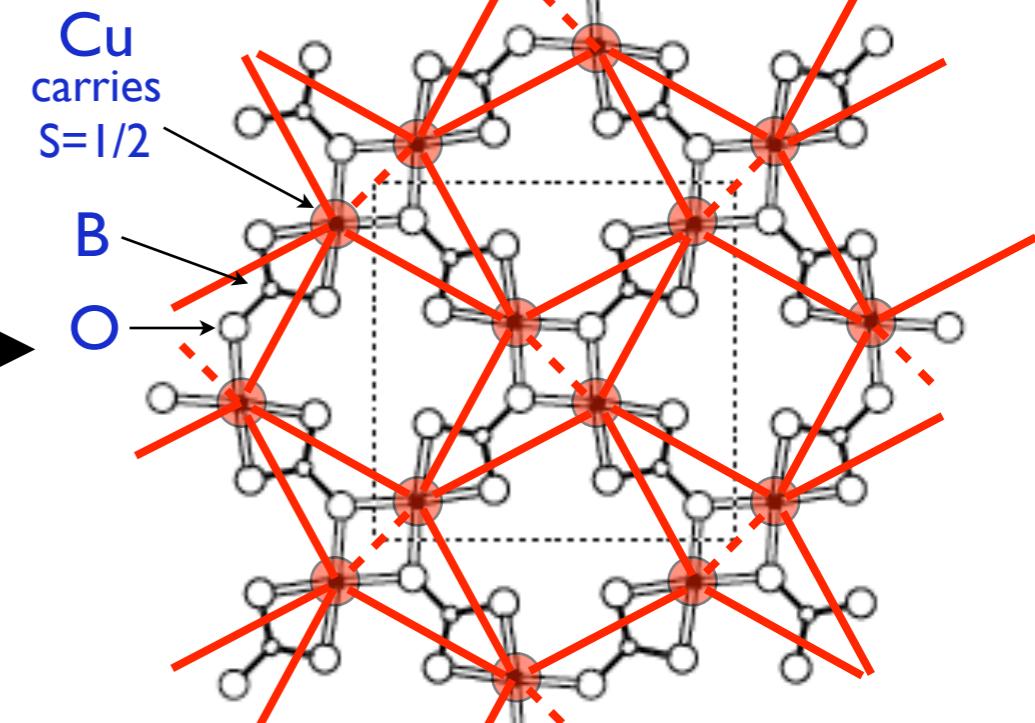
- Koga and Kawakami, PRL 84 (2000)
- Takushima et al., JPSJ 70 (2001)
- Chung et al, PRB 64 (2001)
- Läuchli et al, PRB 66 (2002)

# The Shastry-Sutherland model



**SrCu<sub>2</sub>(BO<sub>3</sub>)<sub>2</sub>**

Spin-gap system (~35K)

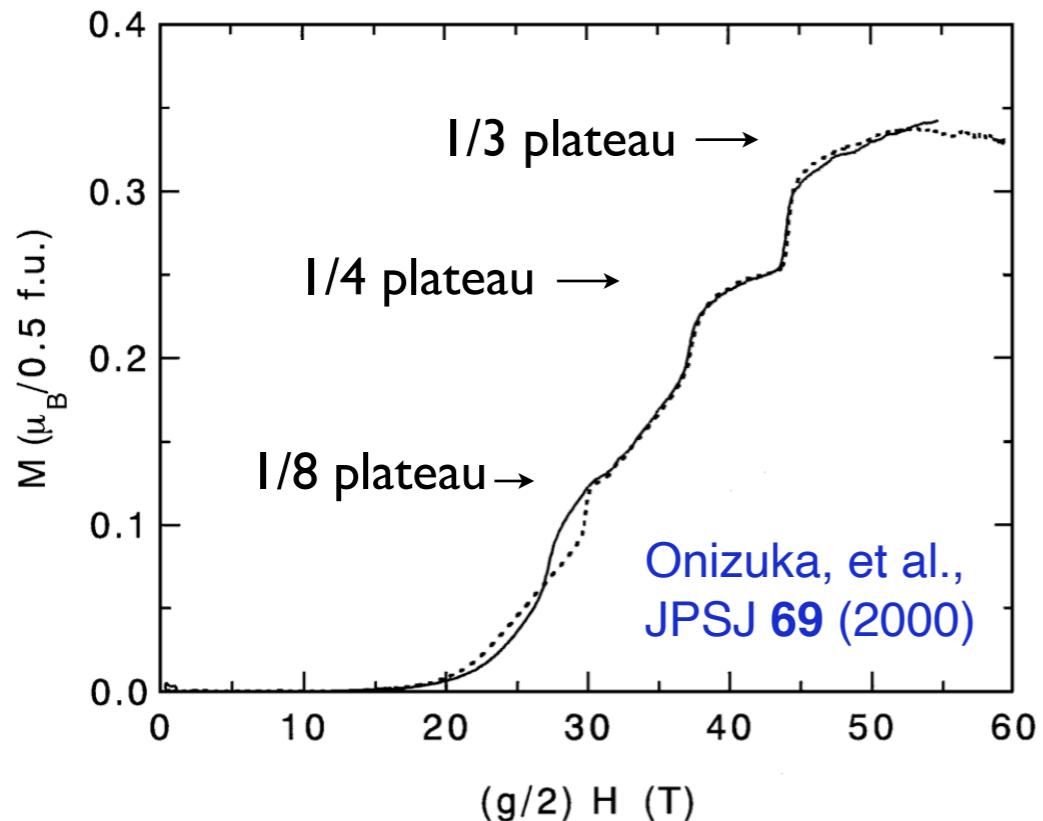


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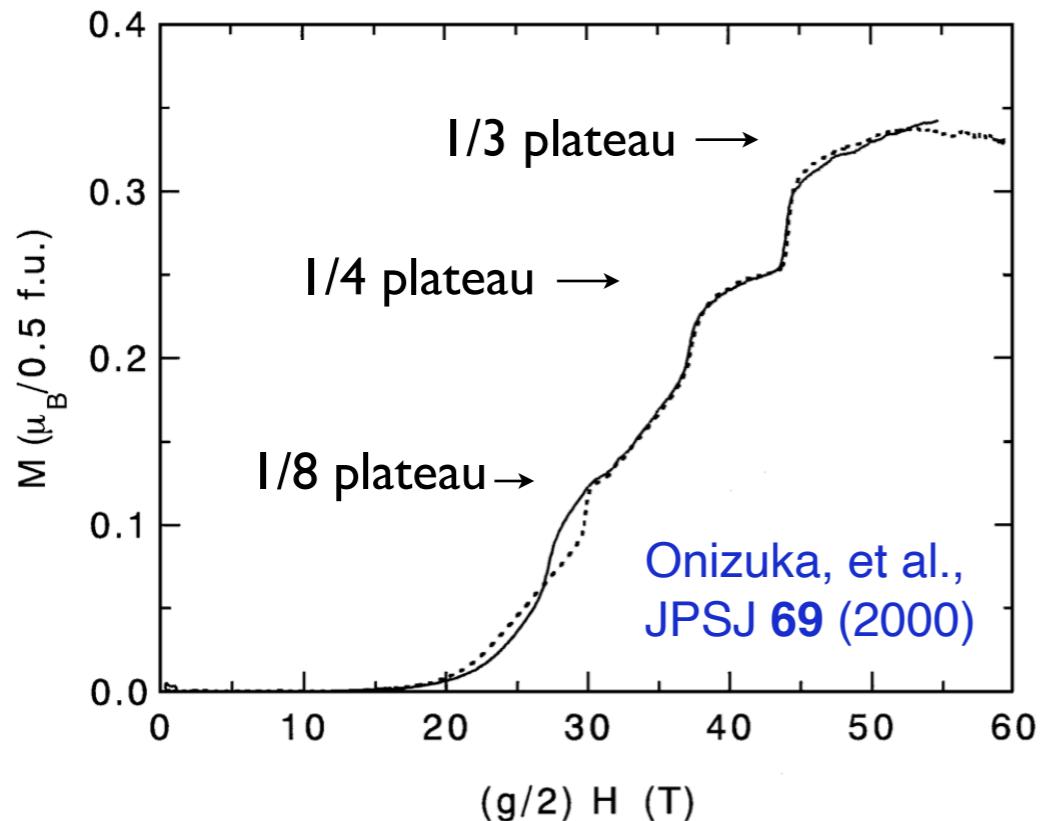
# Magnetization plateaus

$\text{SrCu}_2(\text{BO}_3)_2$  in a magnetic field exhibits several magnetization plateaus



# Magnetization plateaus

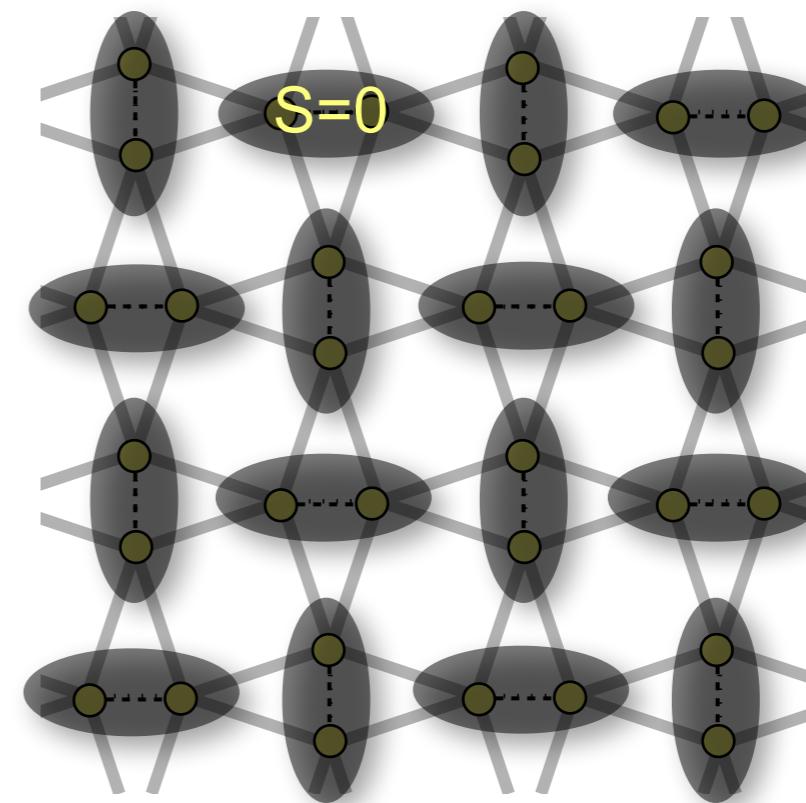
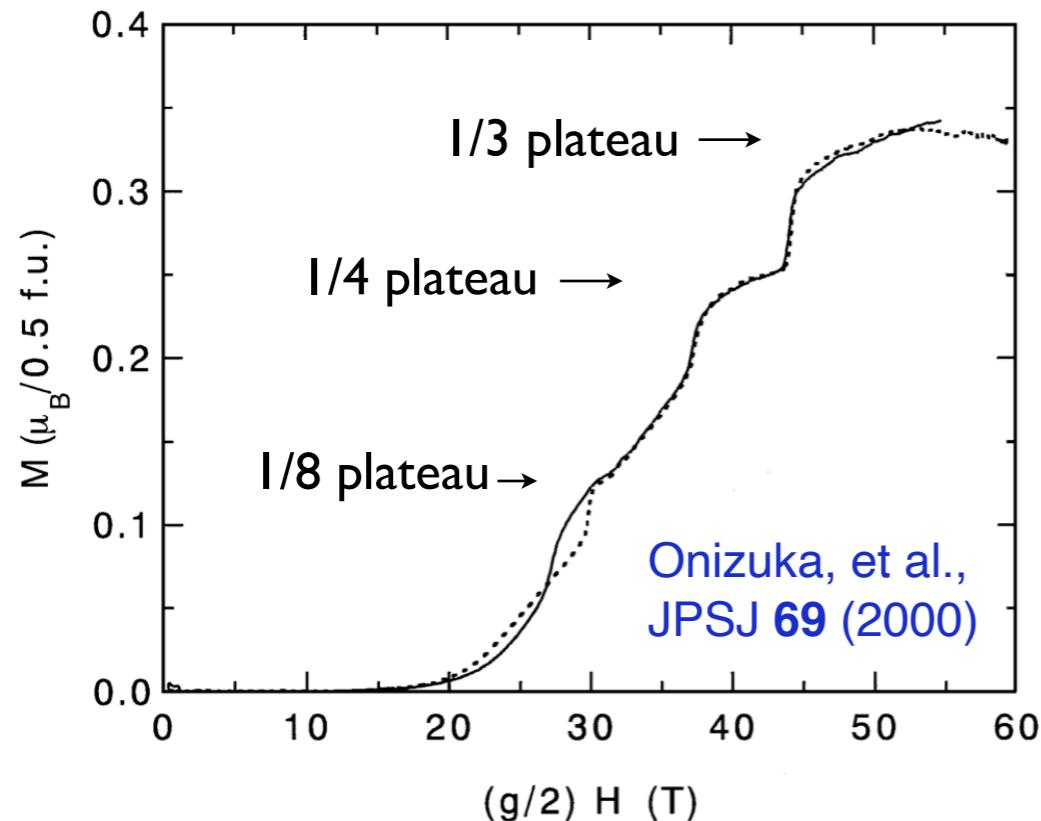
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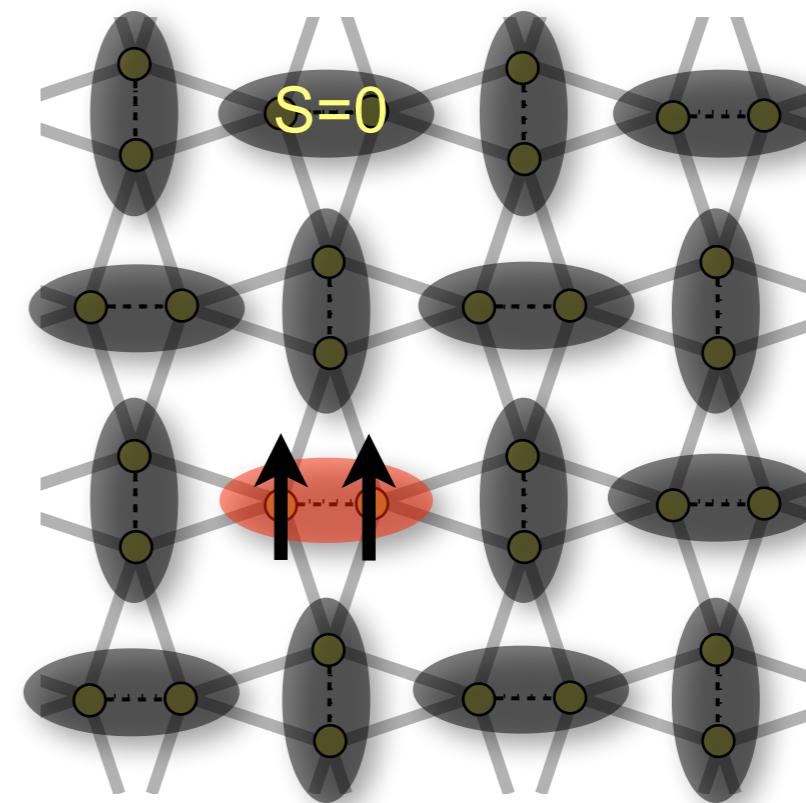
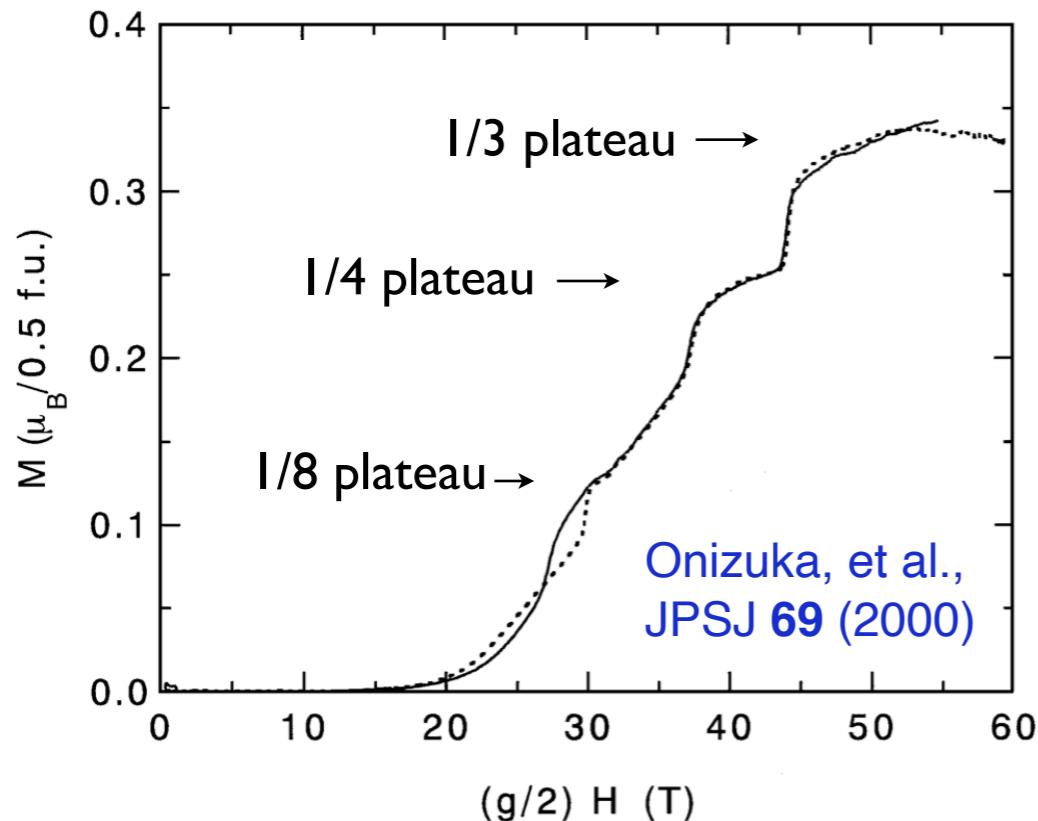
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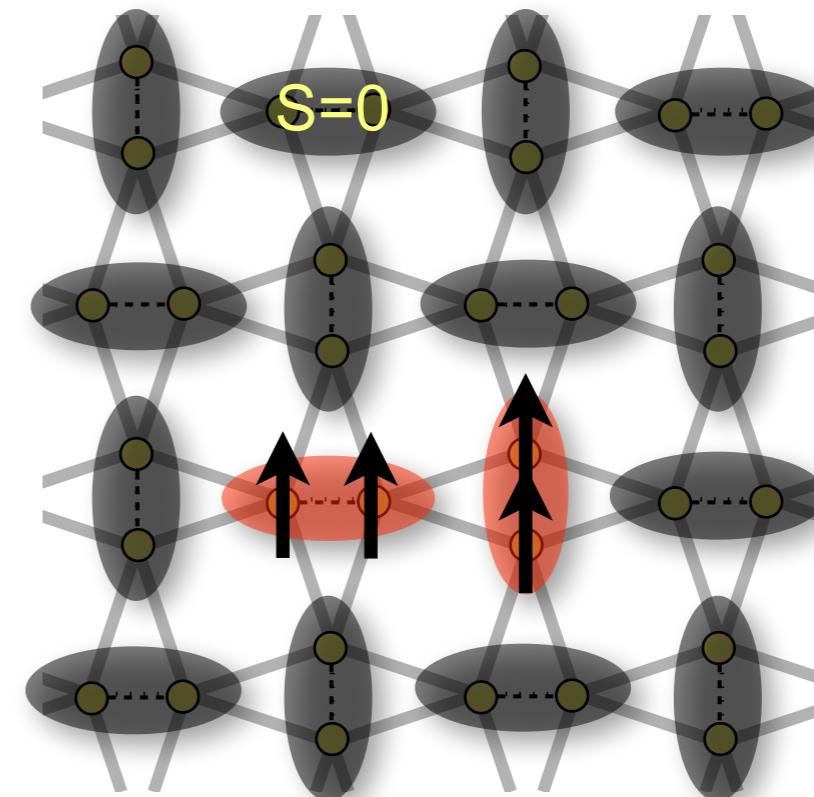
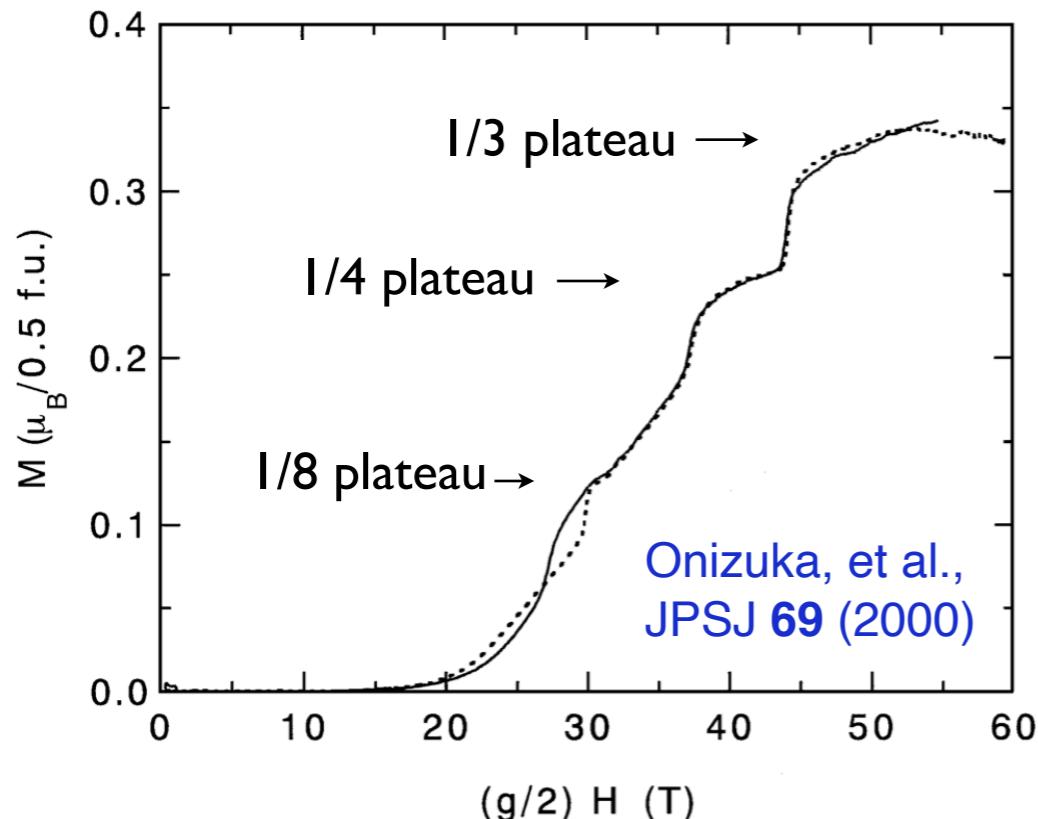
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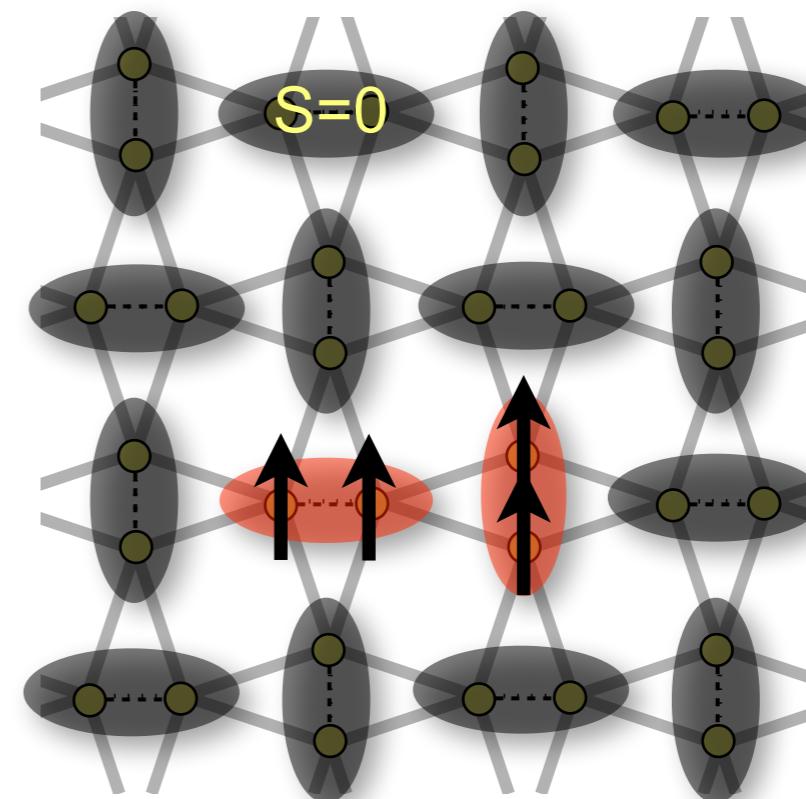
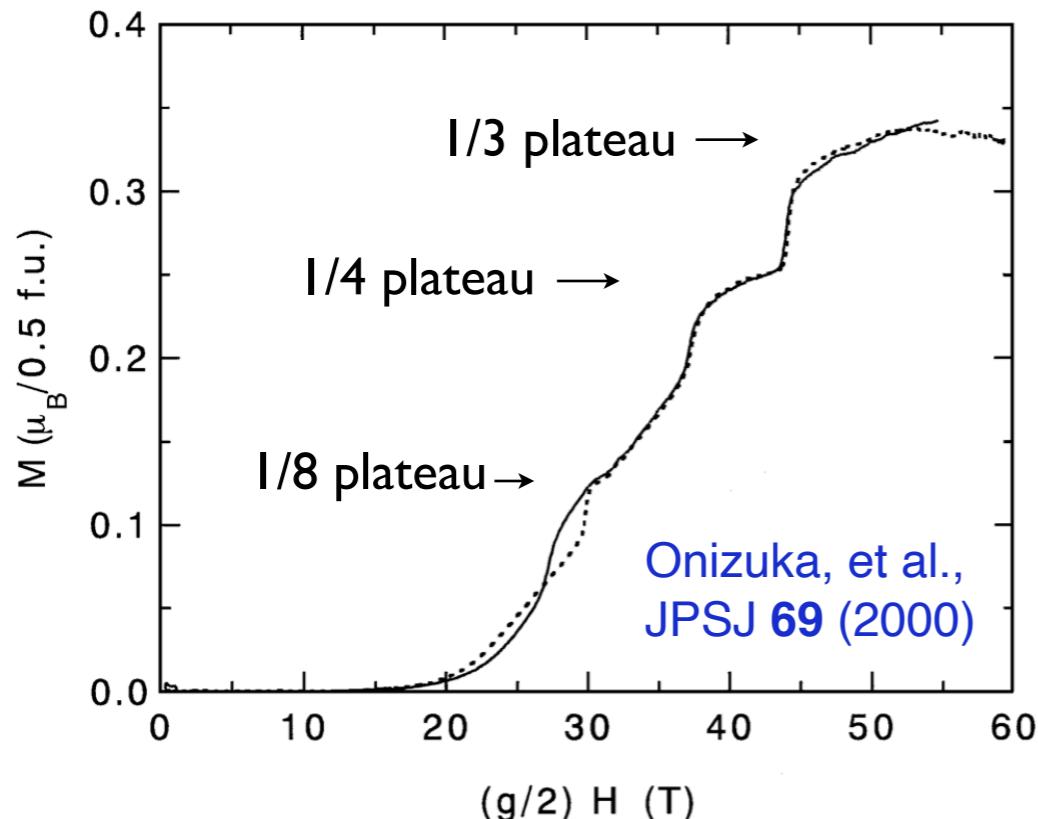


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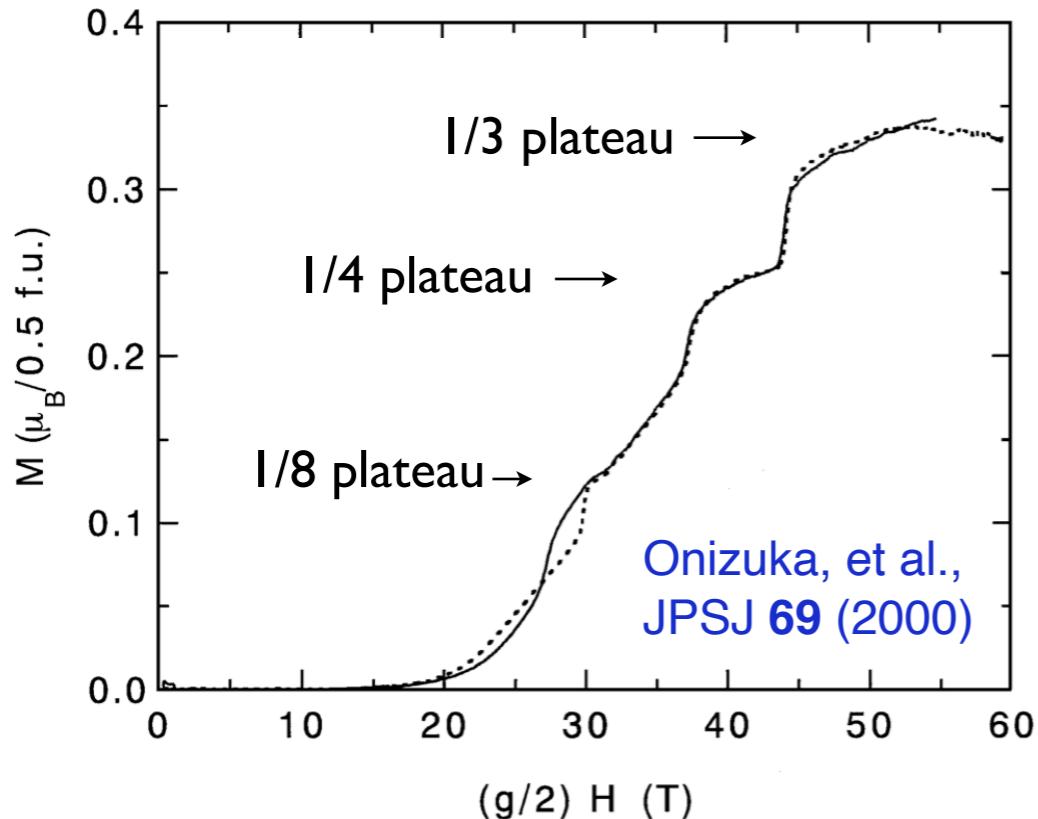
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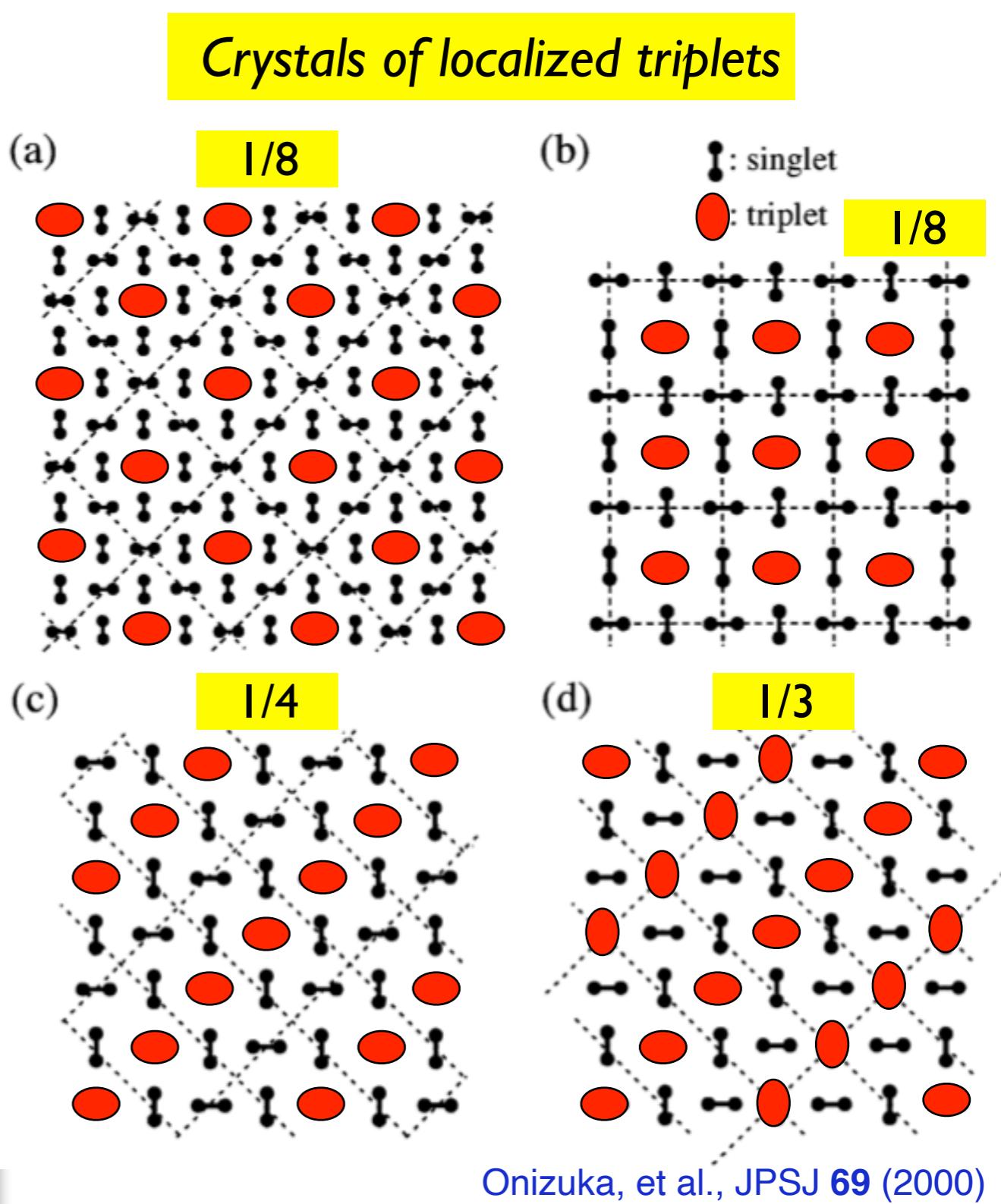
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★ Ideal problem for iPEPS: simulating large unit cell embedded in infinite system and compare variational energies of the proposed crystals



**BUT!**  
**SURPRISE!**

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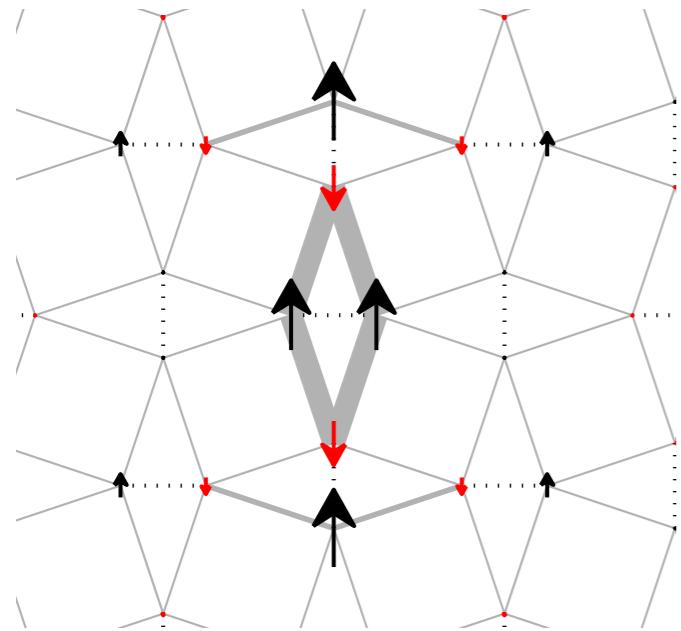
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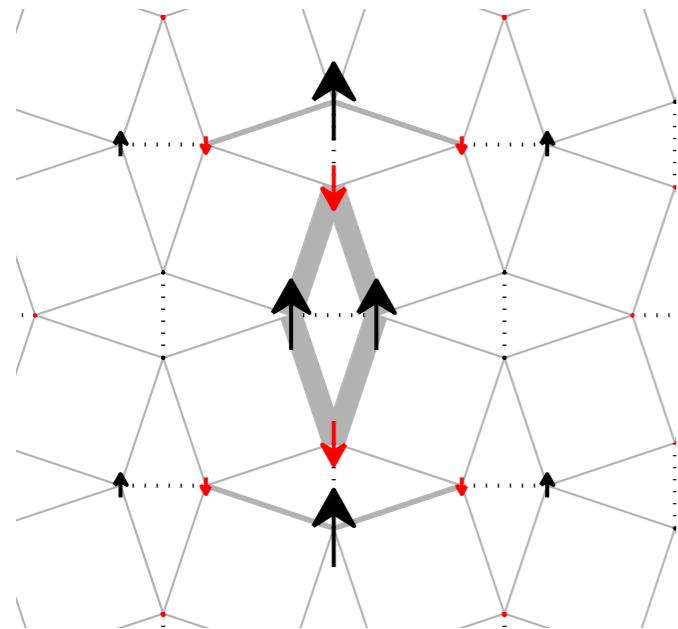


spin structure of 1  
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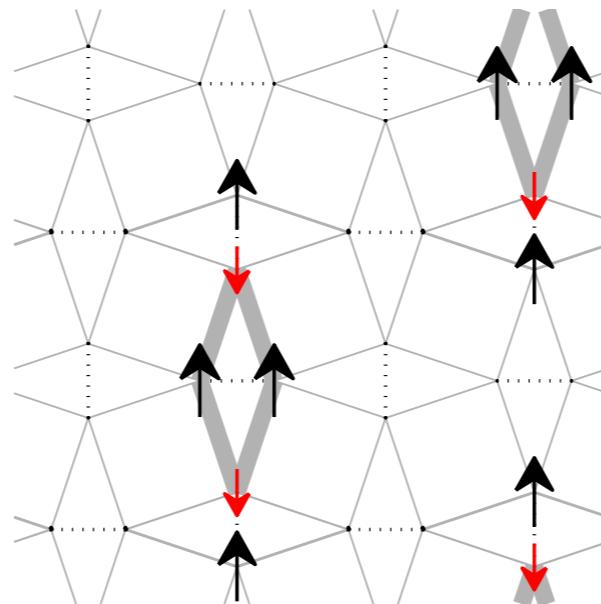
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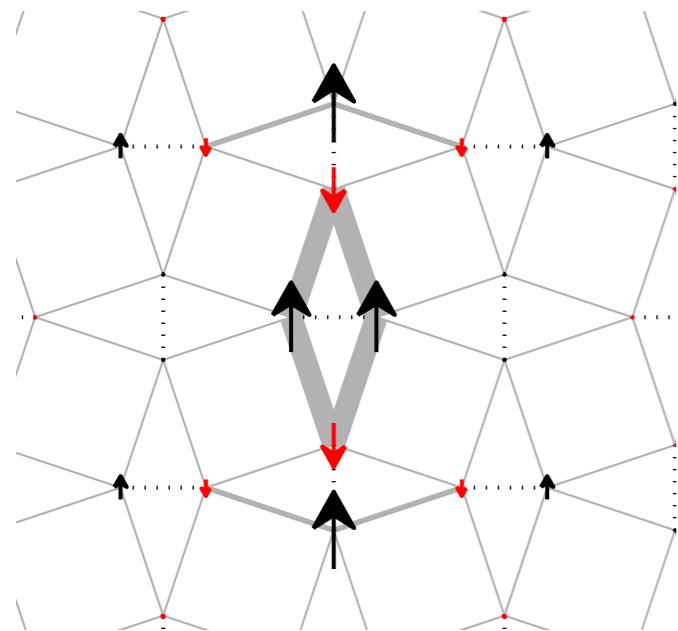
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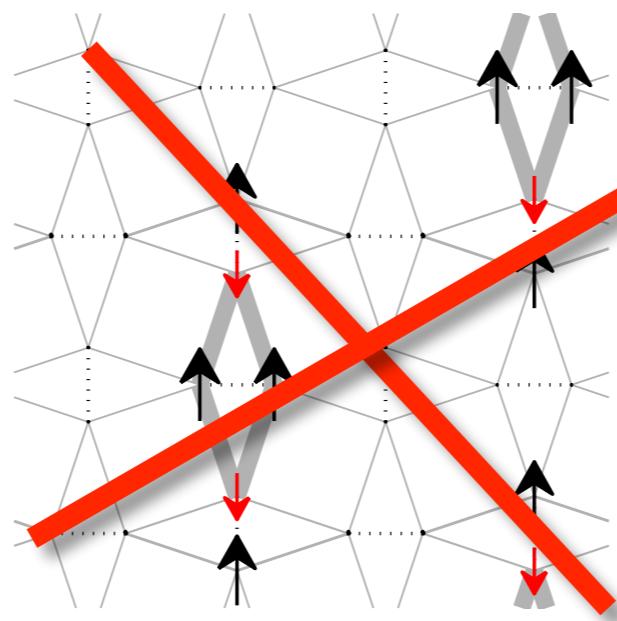
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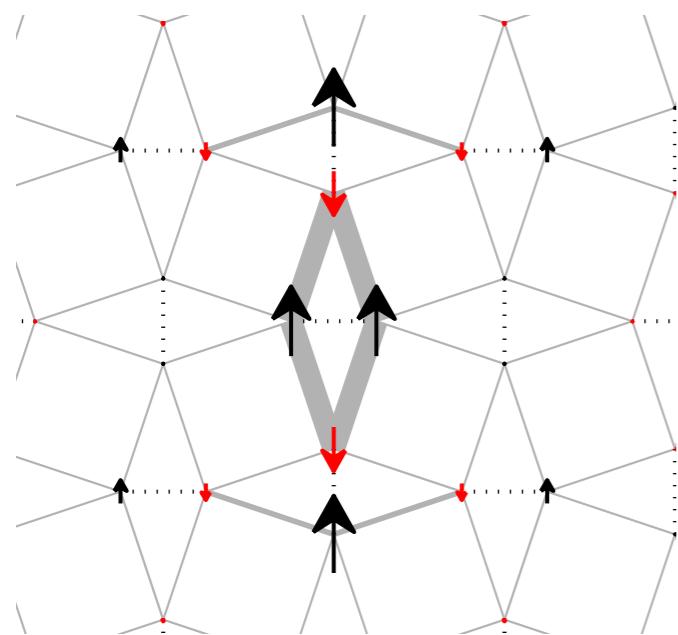
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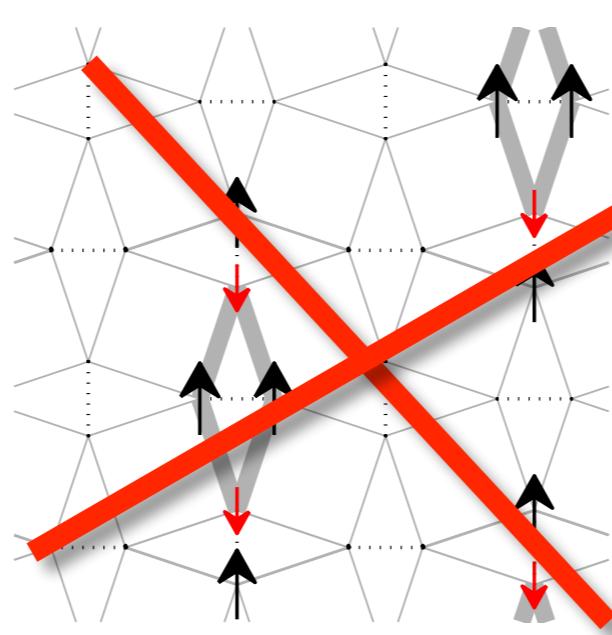
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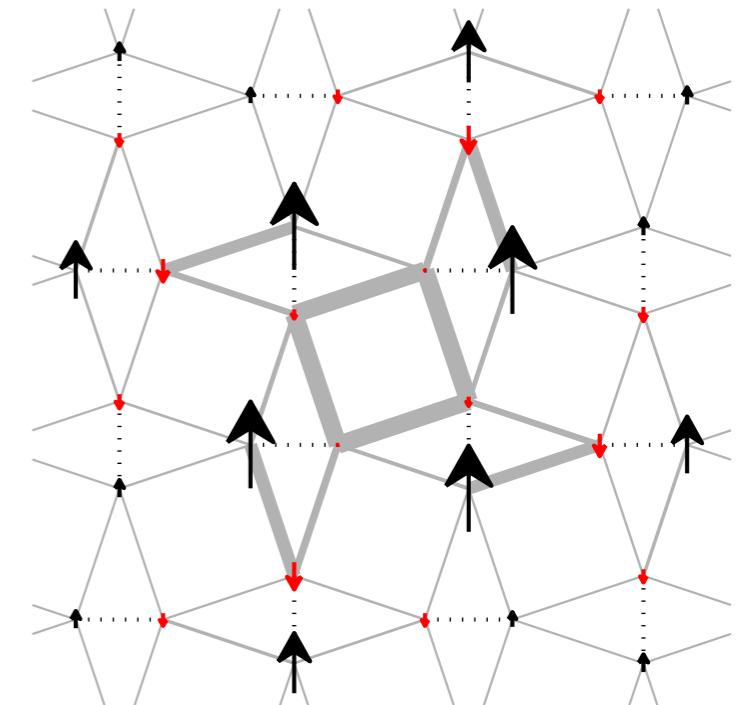
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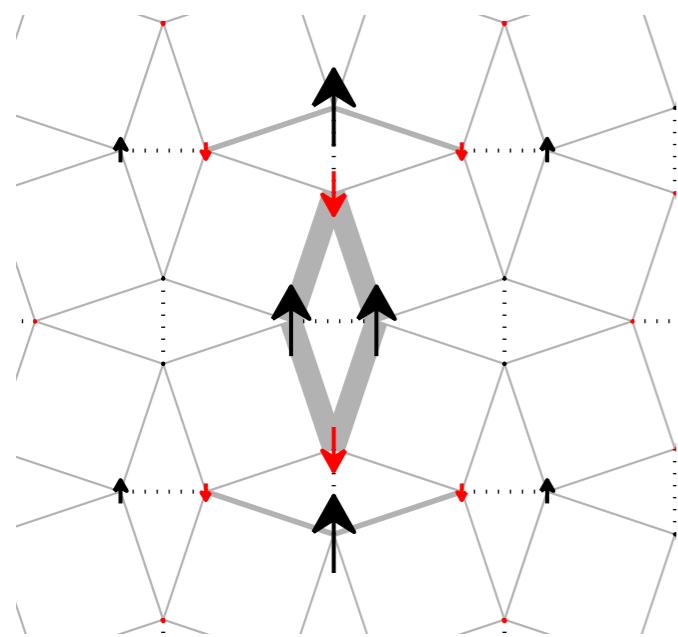


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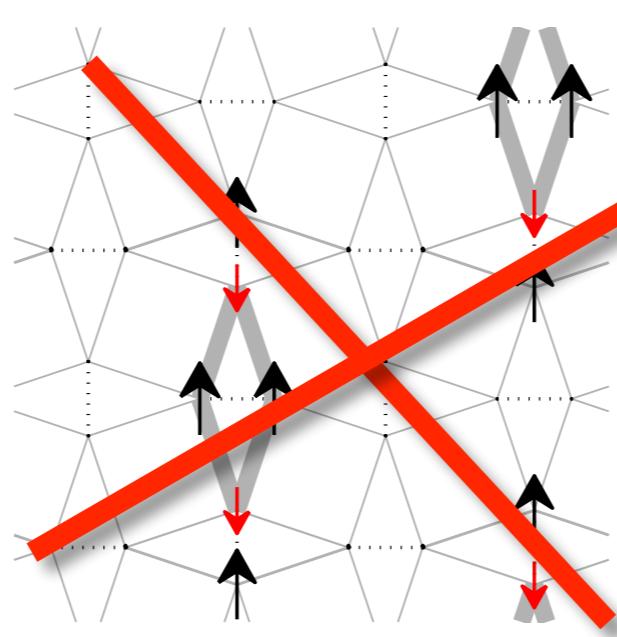
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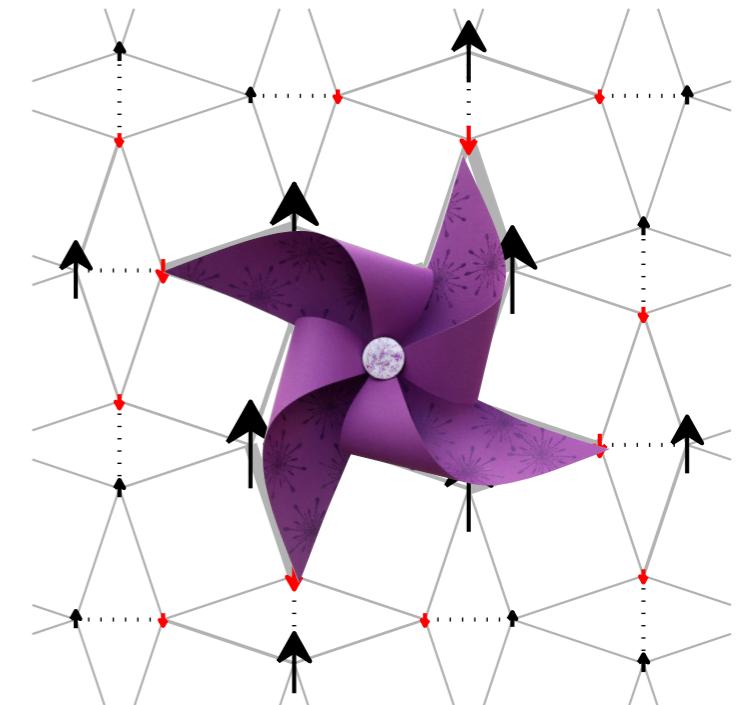
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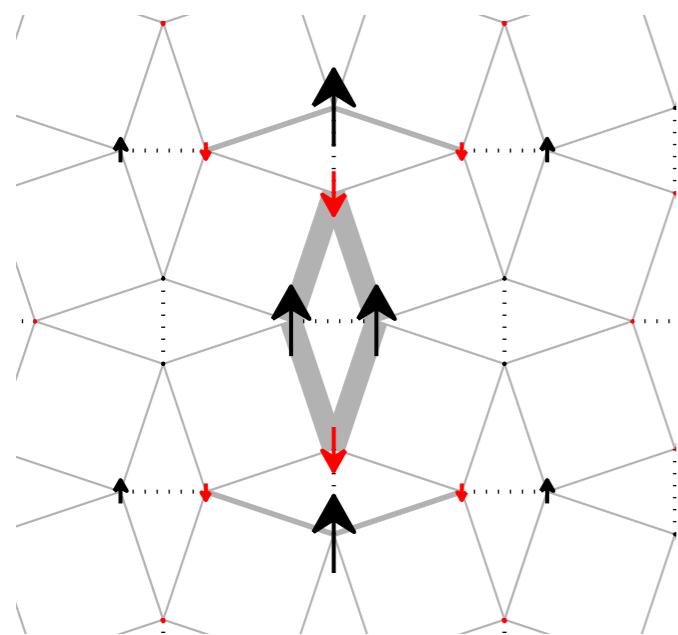


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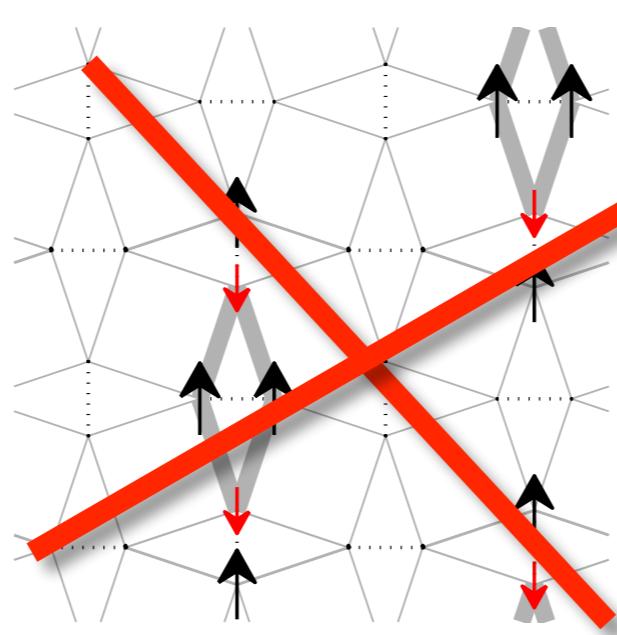
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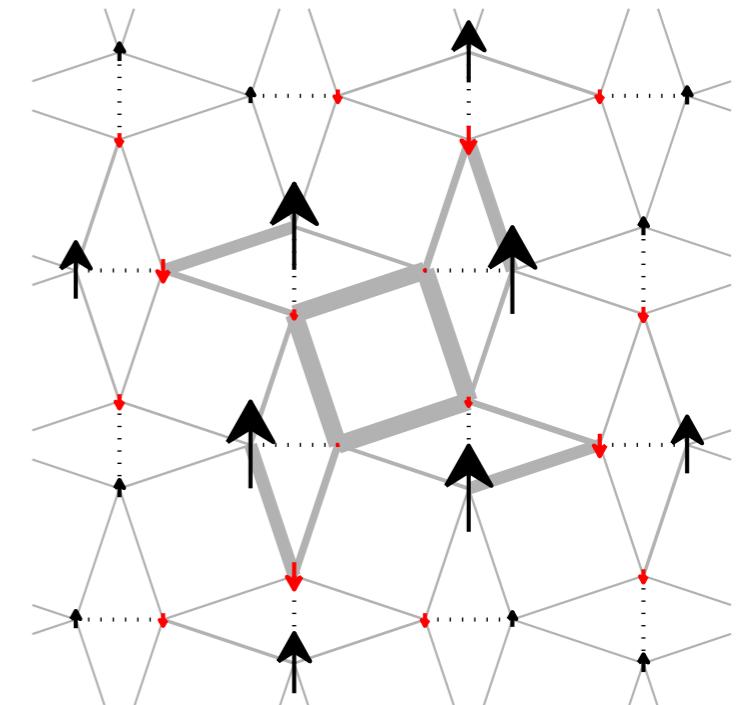
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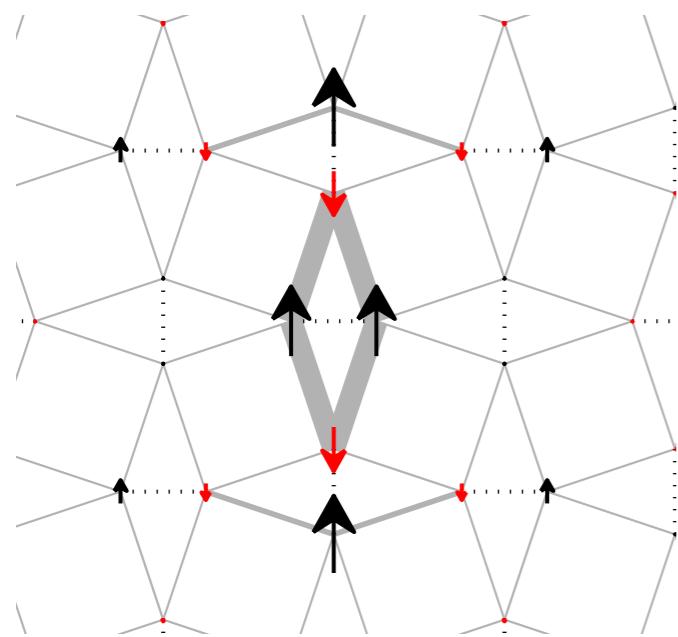


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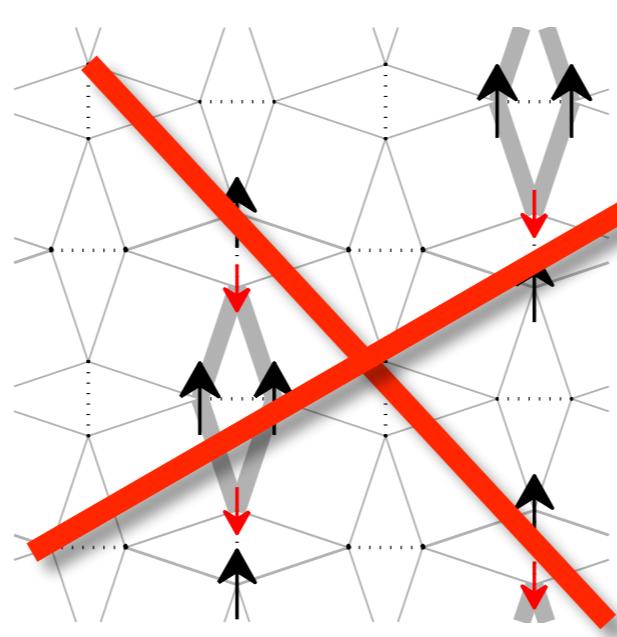
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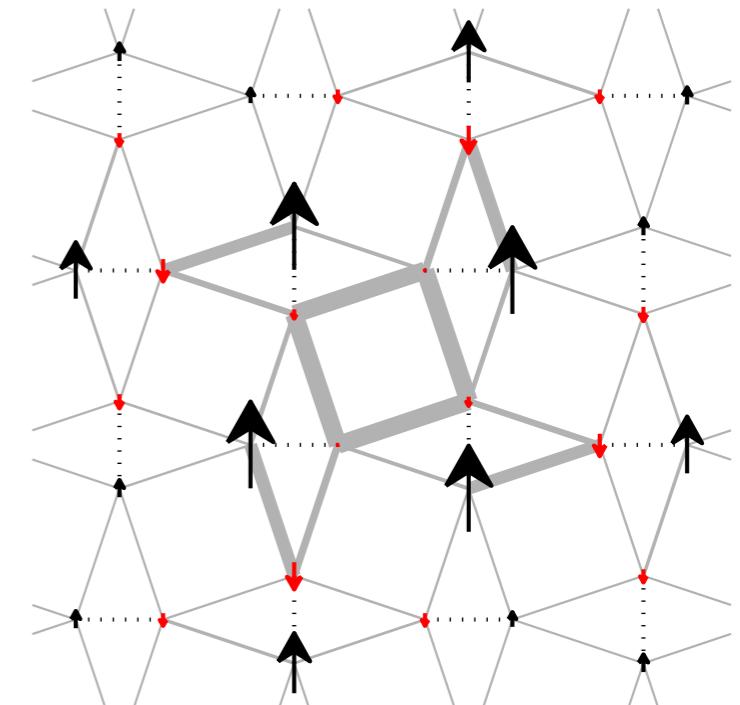
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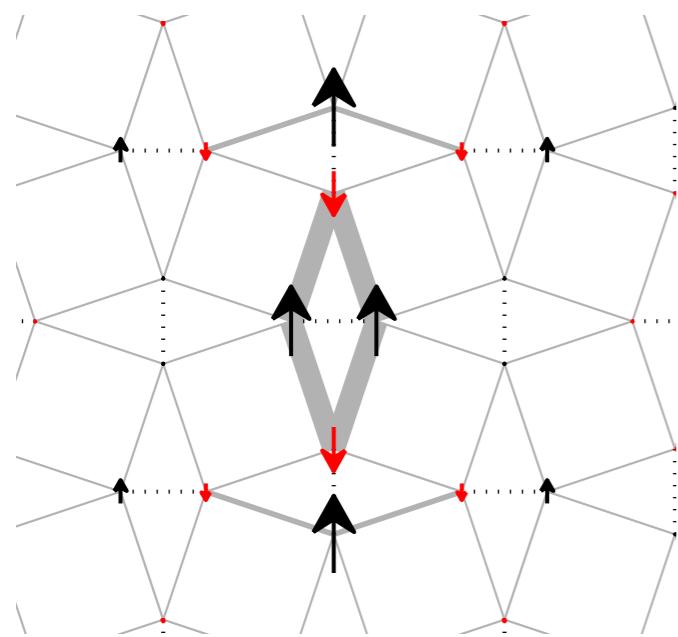
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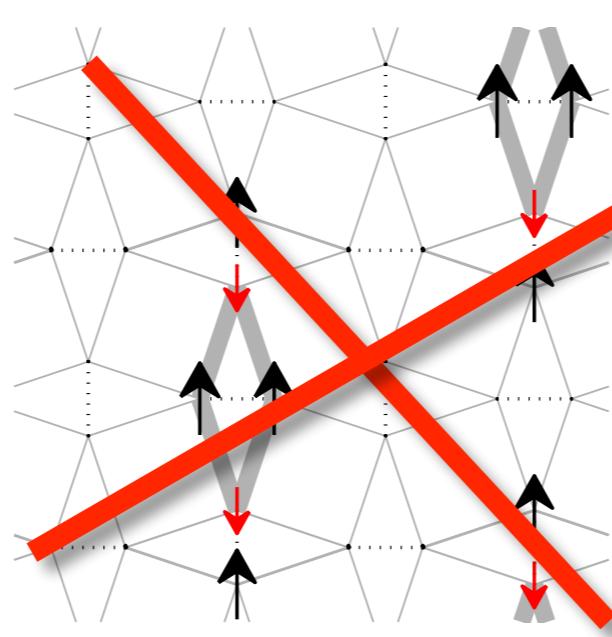
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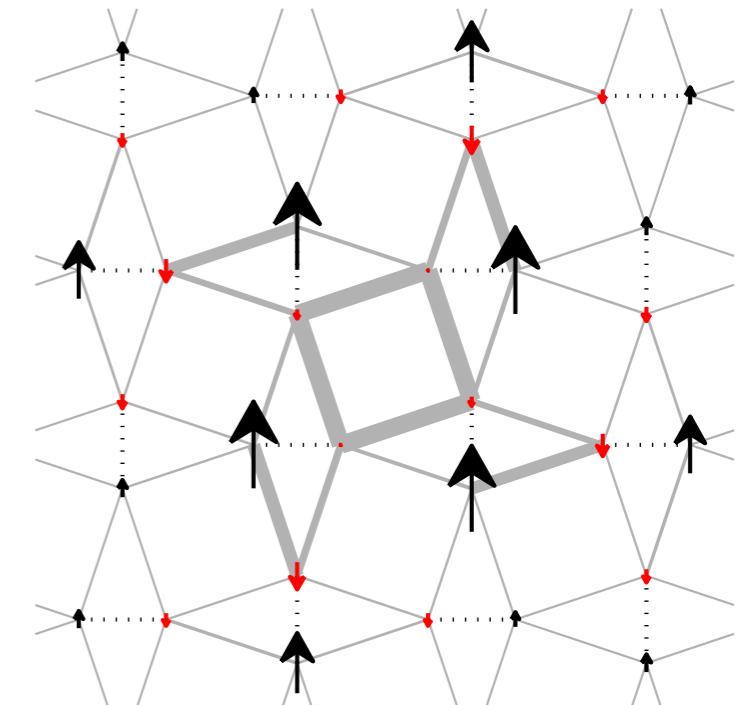


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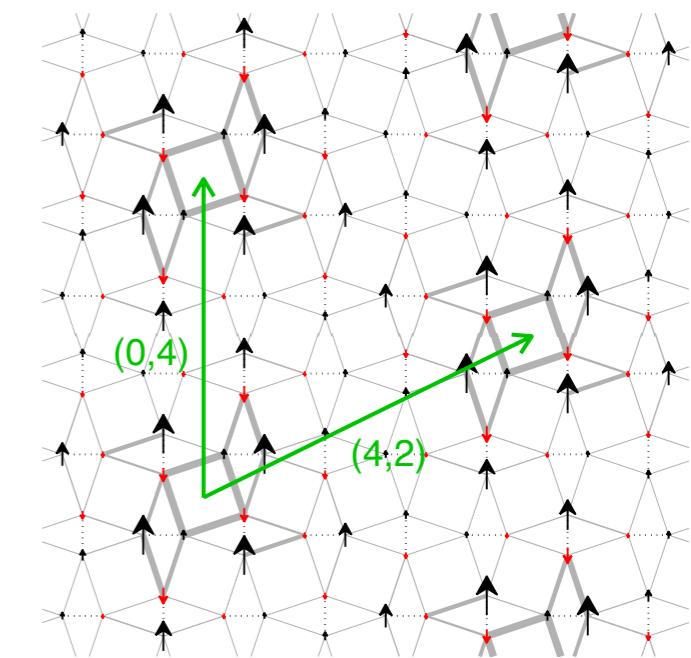
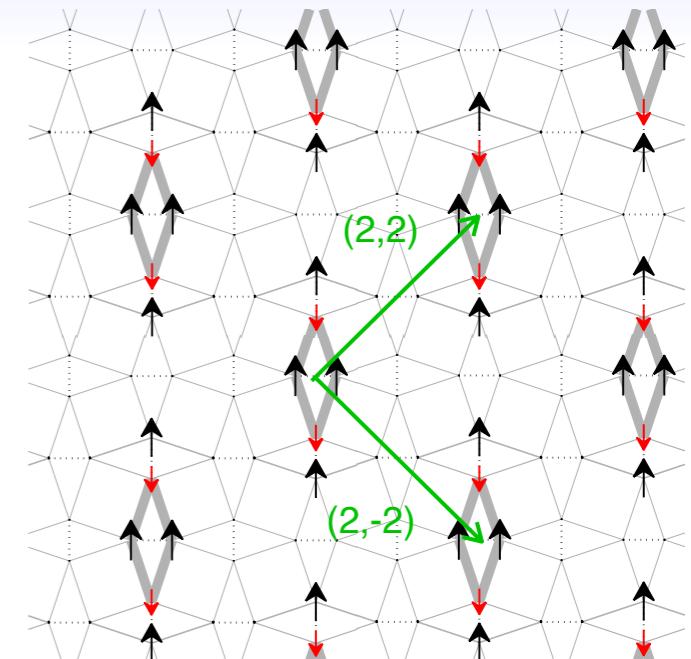
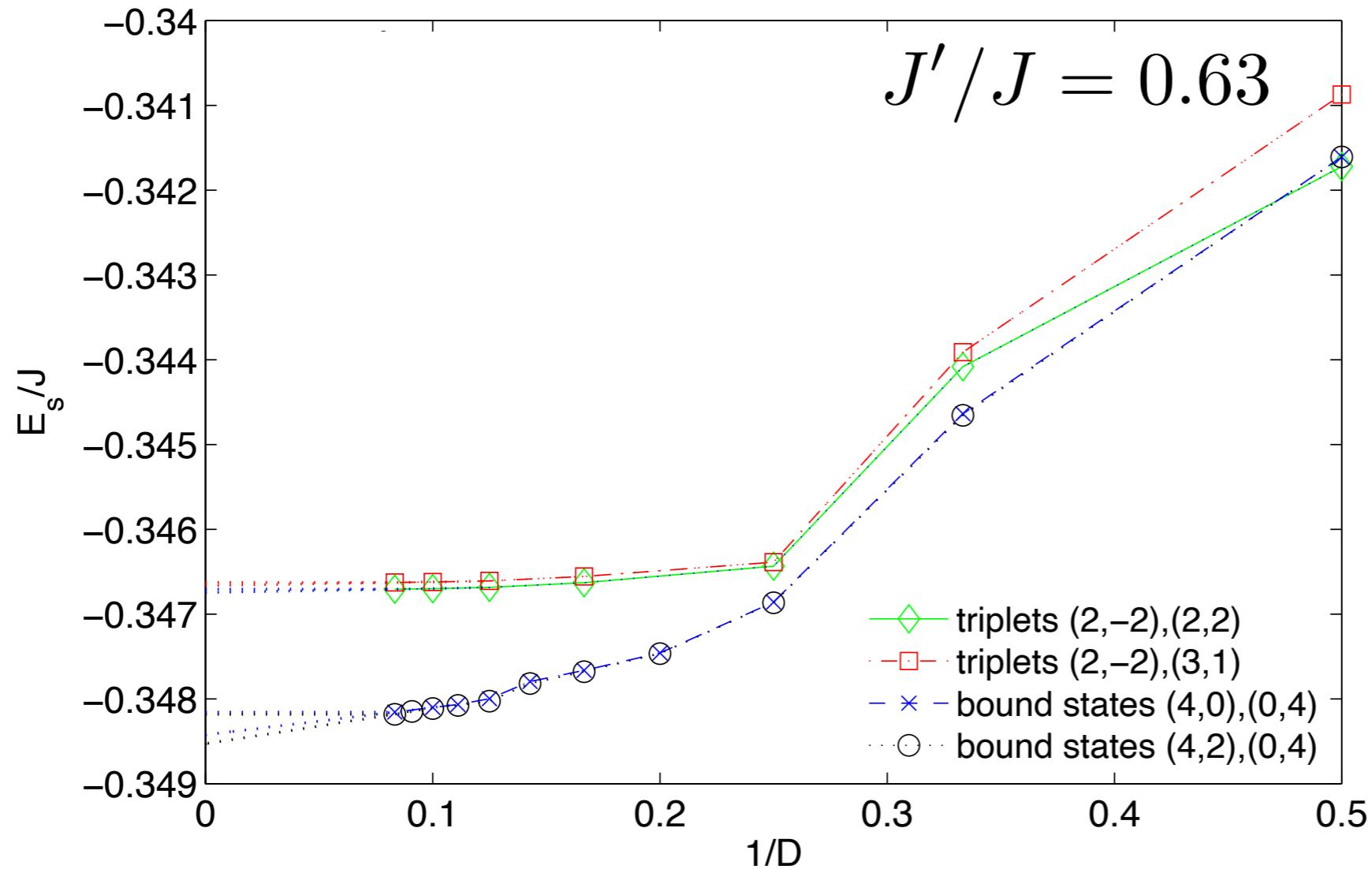
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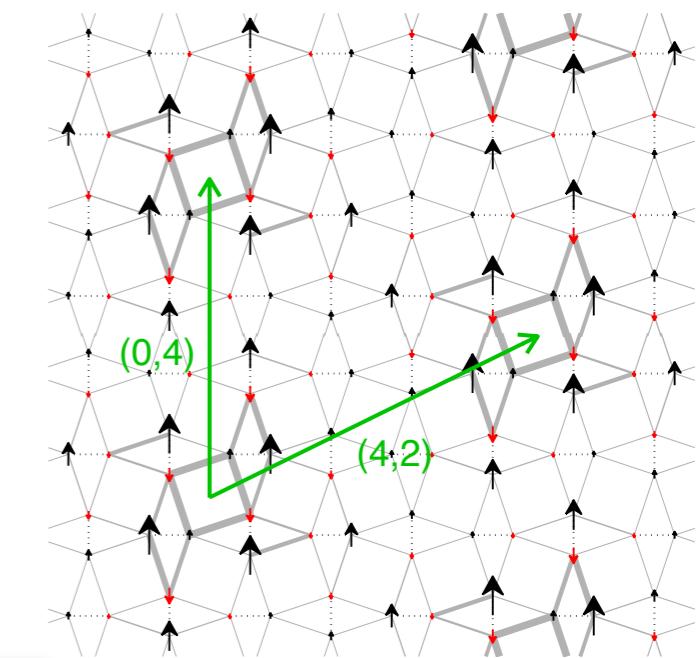
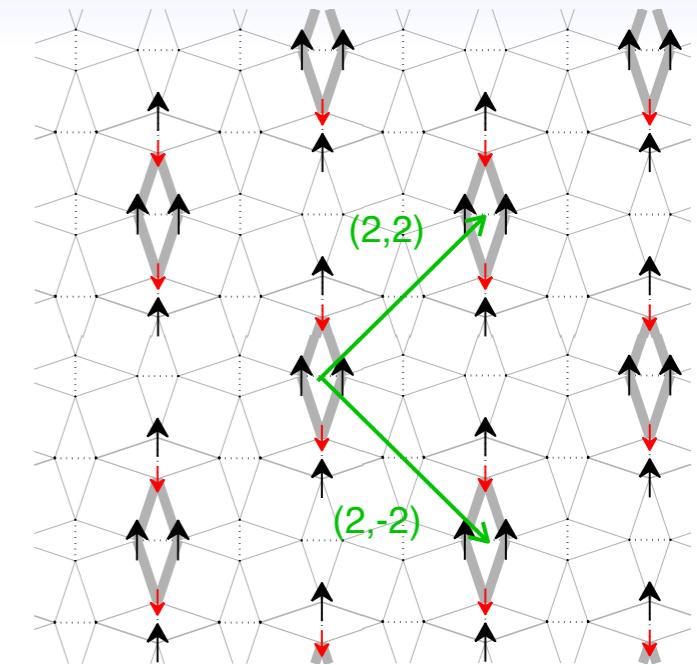
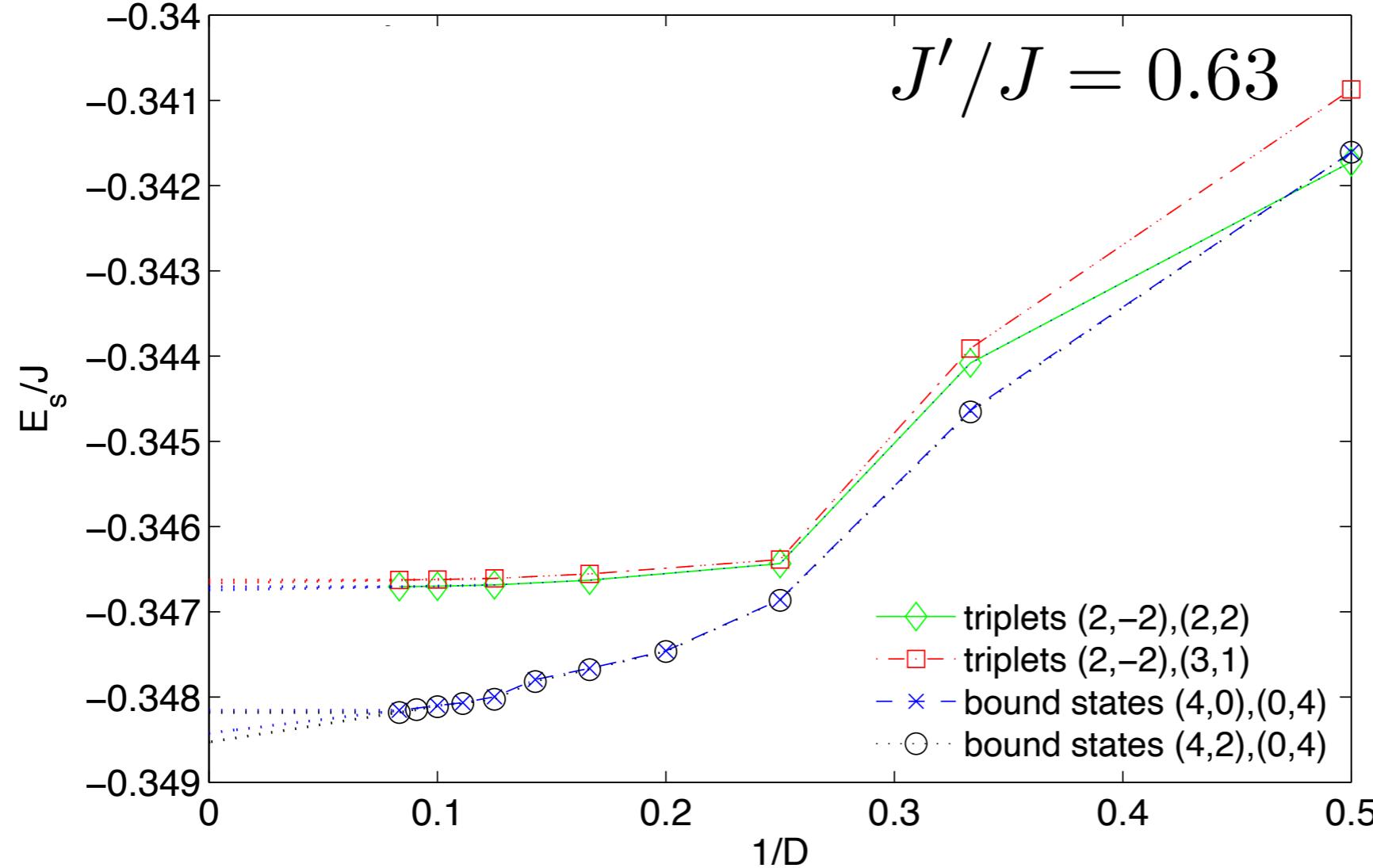
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- Crystals of bound states instead of crystals of triplets!!

# Example: 1/8 plateau

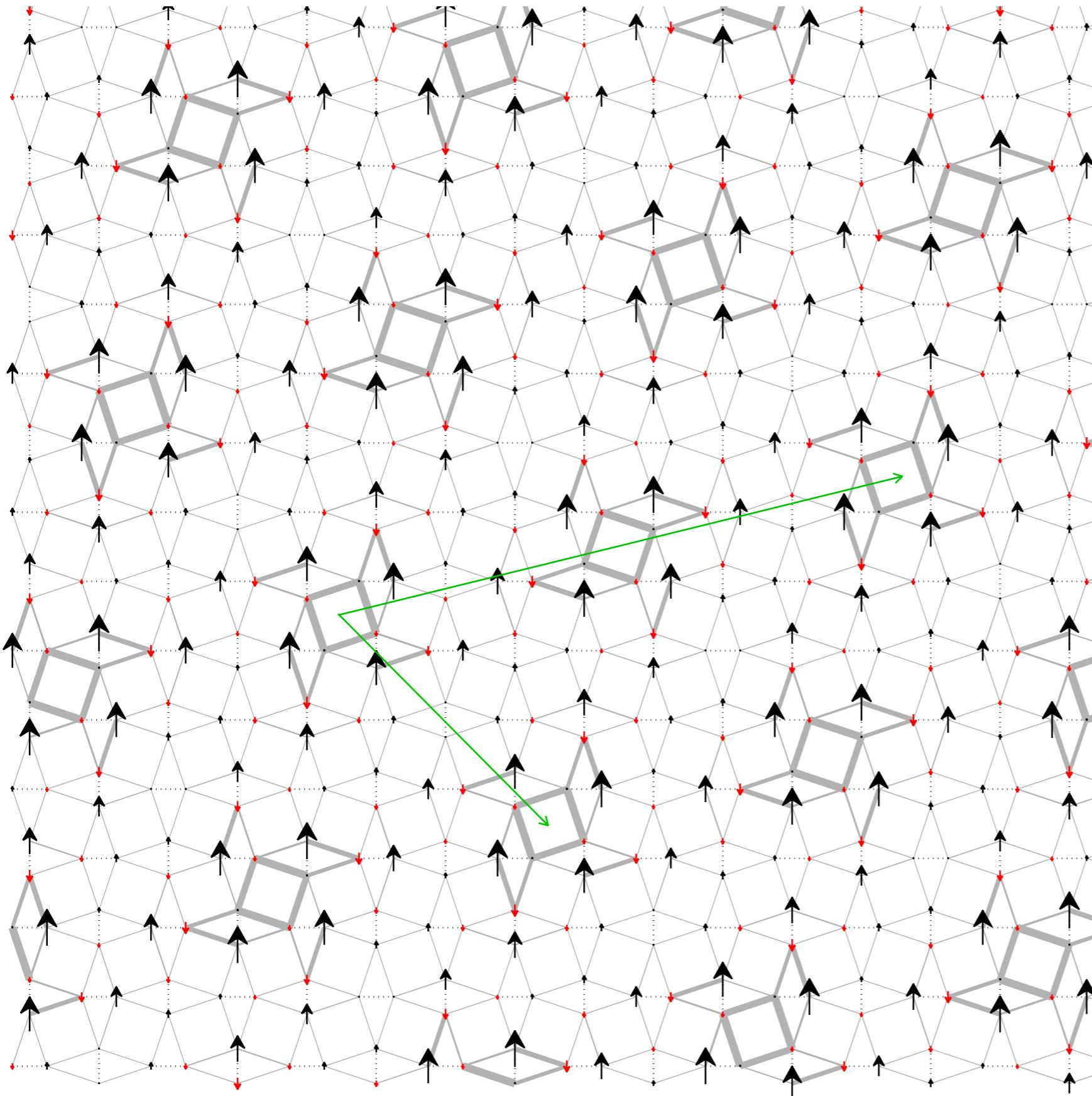


# Example: 1/8 plateau



- All the proposed triplet crystals have a higher energy than the crystals made of bound states!
- Similar results found for other plateaus below 1/4

# 2/15 plateau

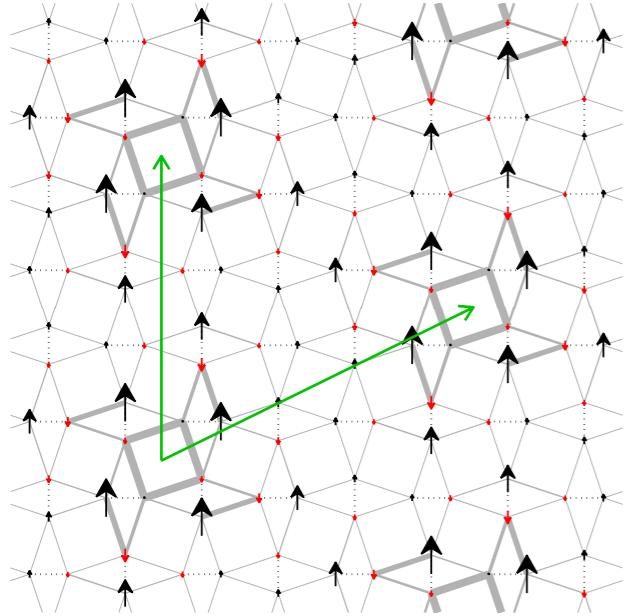


Unit cell with 30 tensors (60 sites)

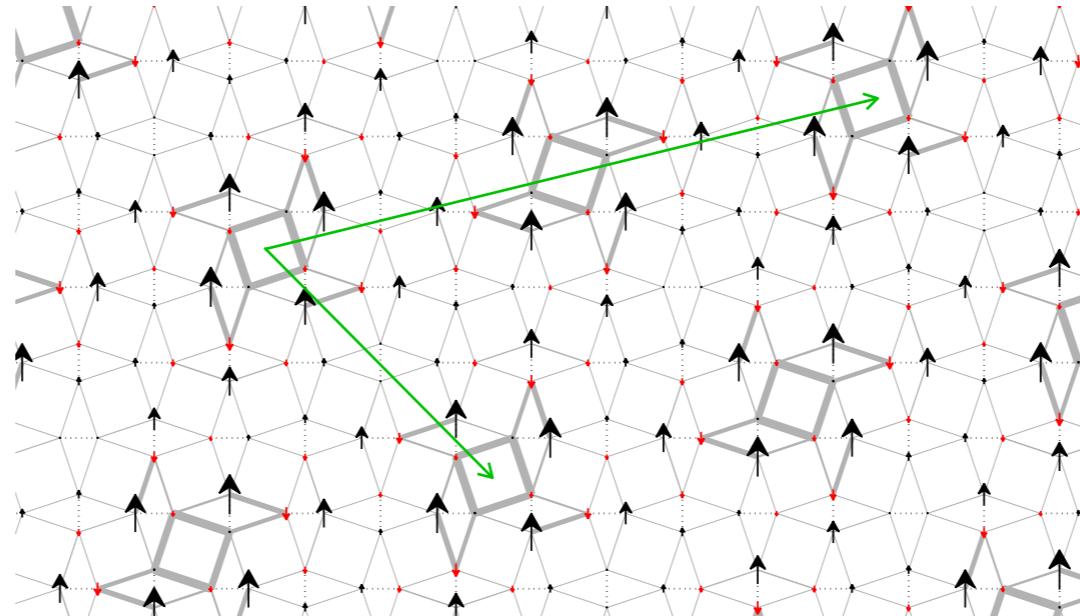
Regular pattern of bound states!

# Computing the energies of all possible crystals

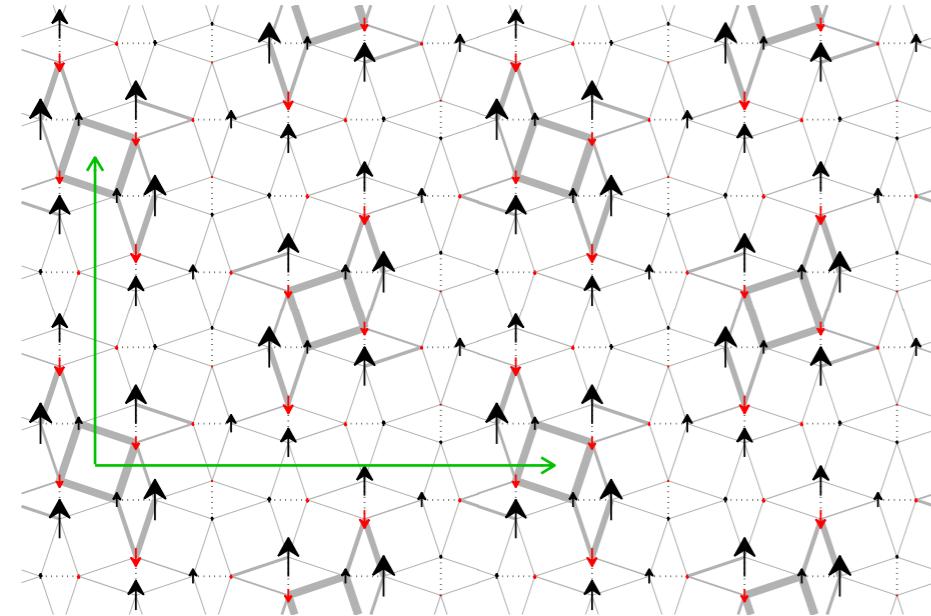
1/8 rhomboid : (4,2),(0,4)



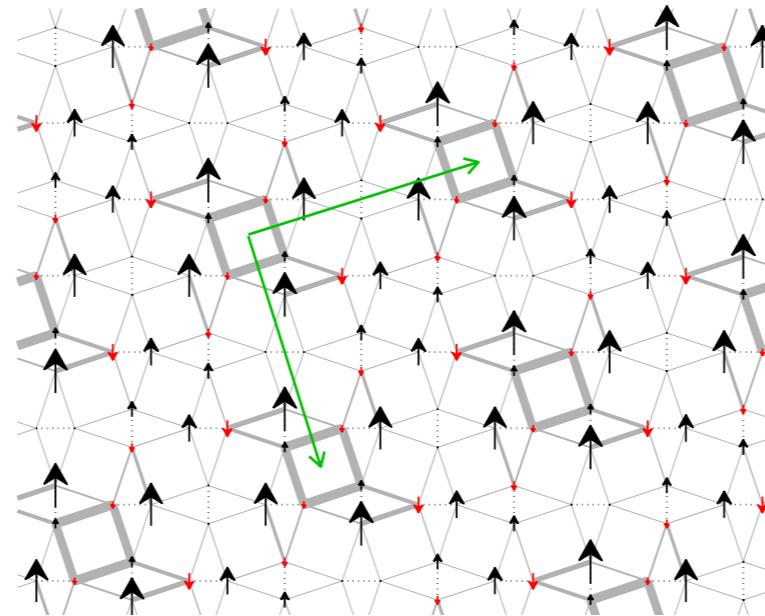
2/15 : (3,-3),(8,2)



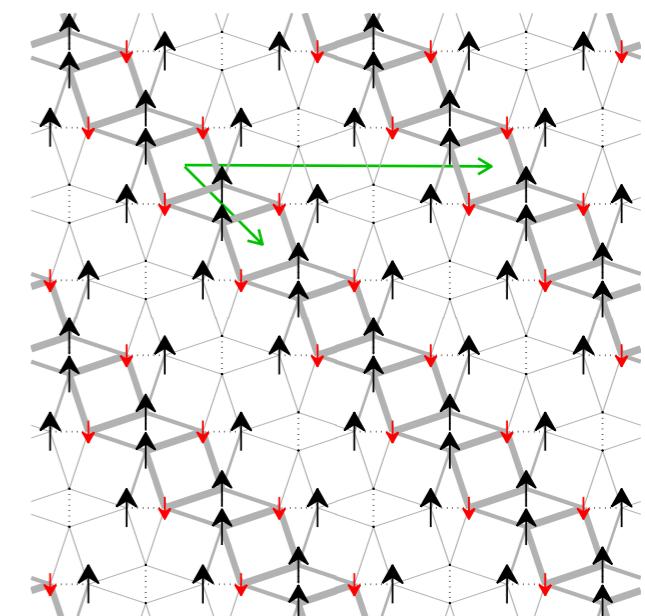
1/6 rectangular : (6,0),(0,4)



1/5 : (1,-3),(3,1)

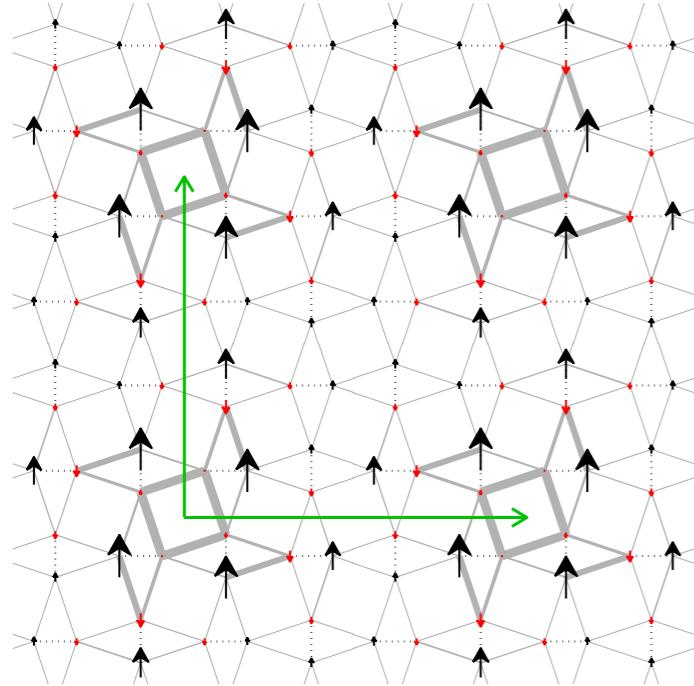


1/4 : (1,-1),(4,0)

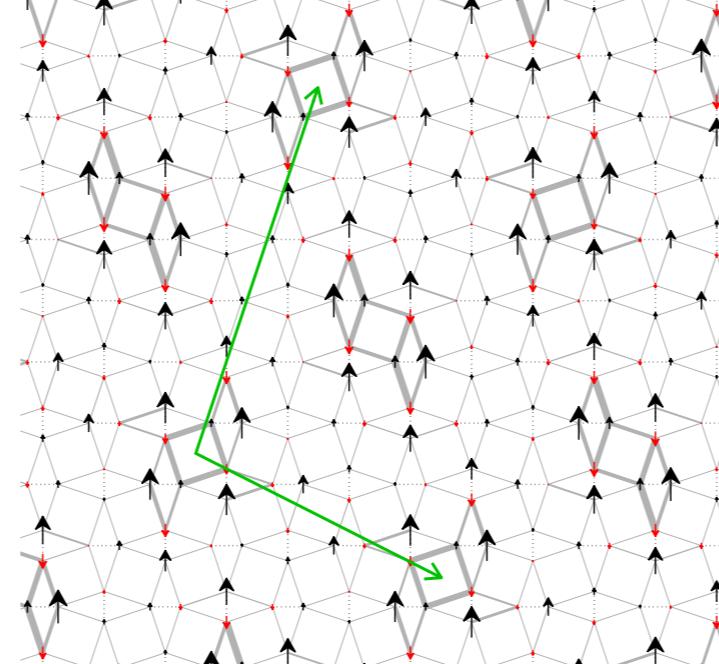


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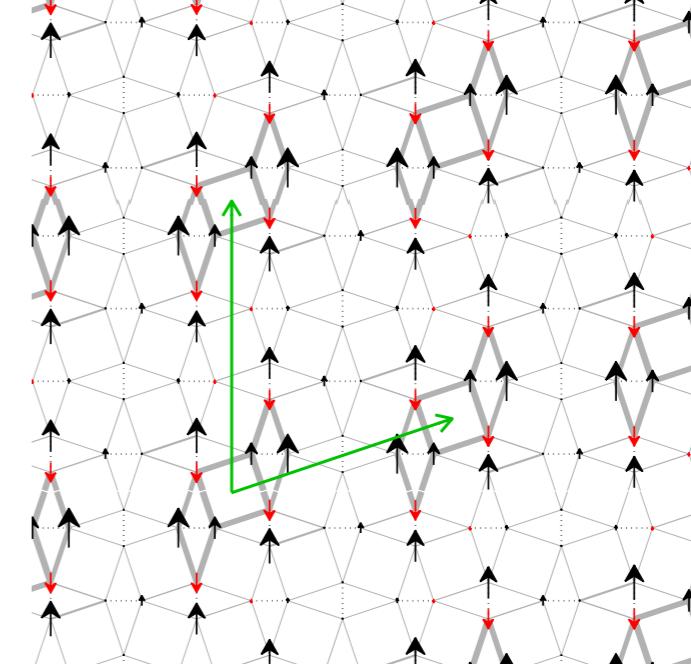
1/8 square : (4,0),(0,4)



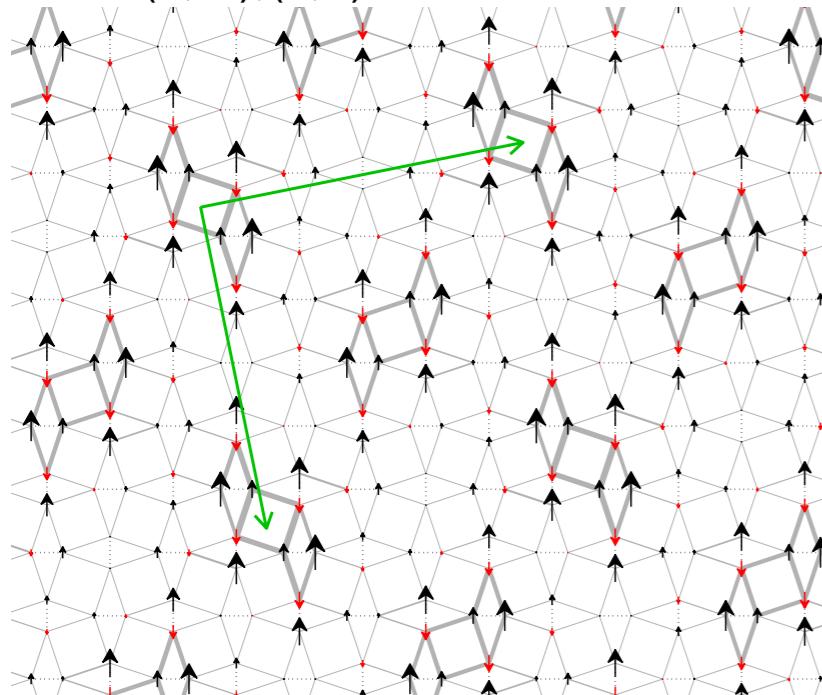
1/7 : (4,-2),(2,6)



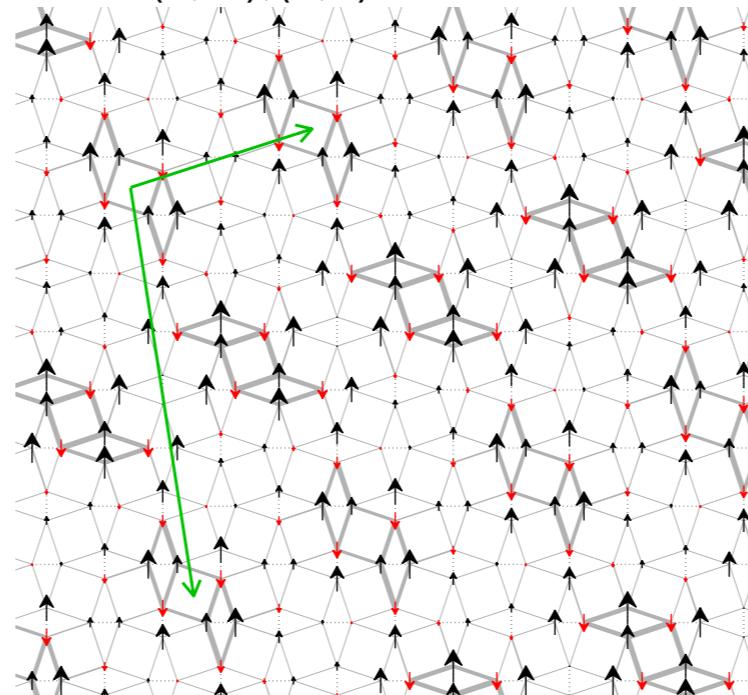
1/6 rhomboid : (3,1),(0,4)



2/13 : (1,-5),(5,1)

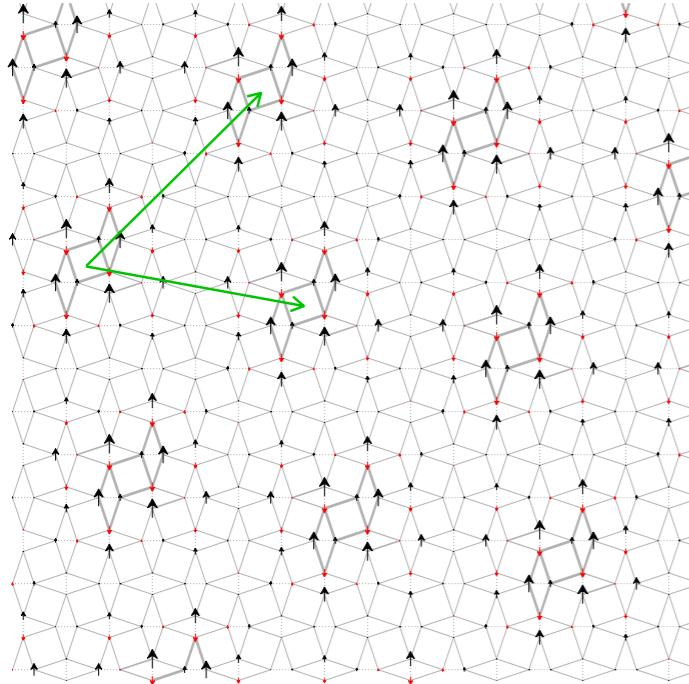


2/11 : (1,-7),(3,1)

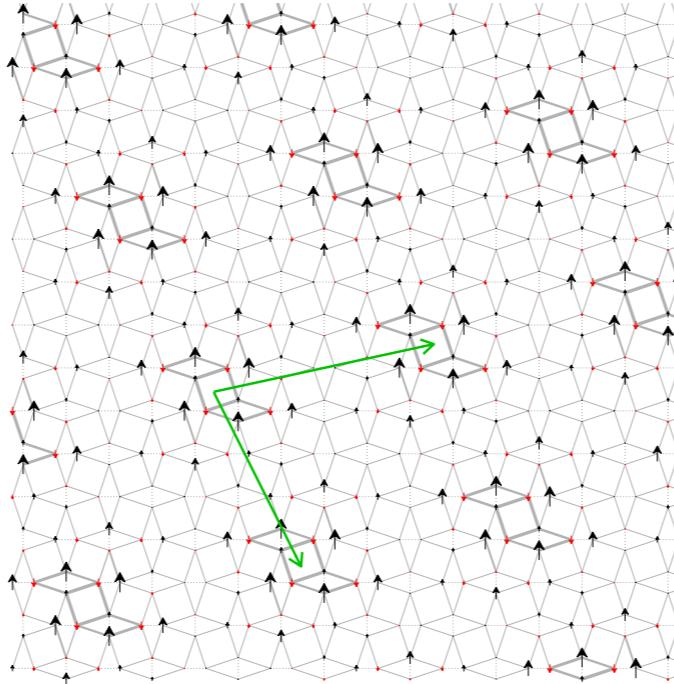


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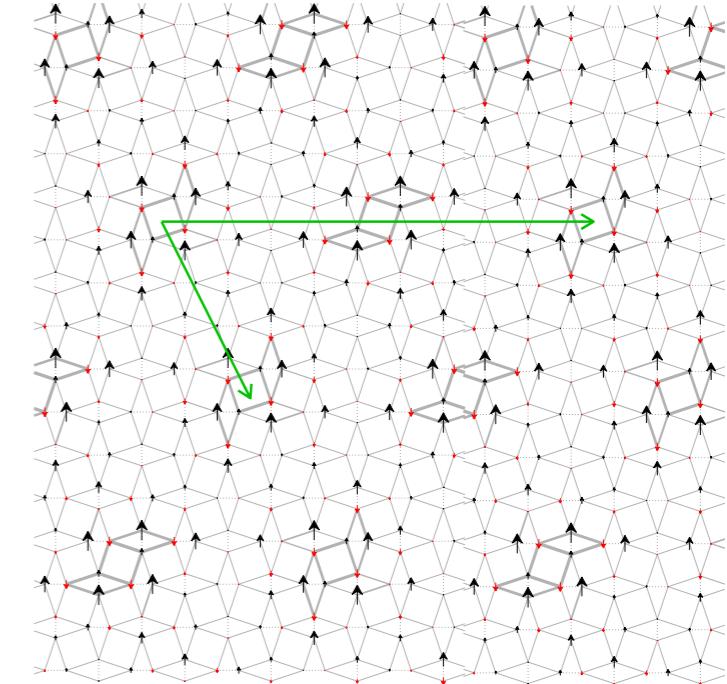
1/12 : (1,-5),(5,-1)



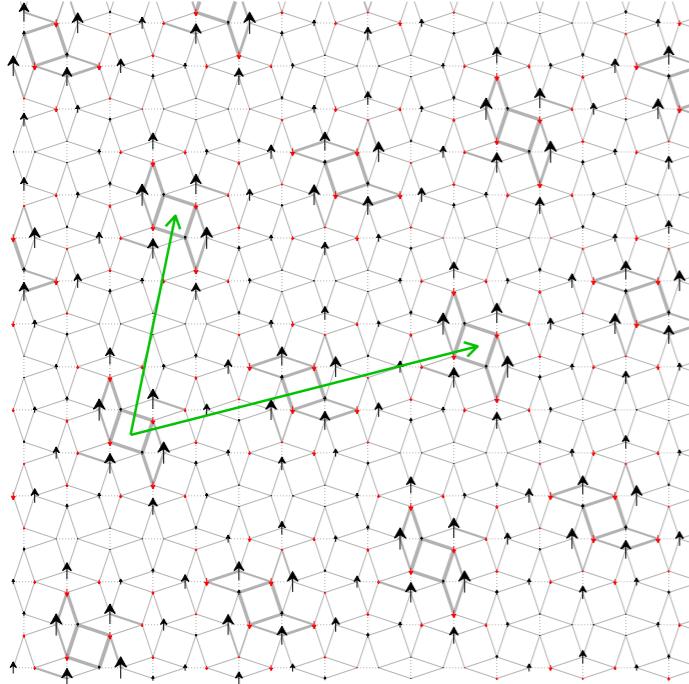
1/11 : (2,-4),(5,1)



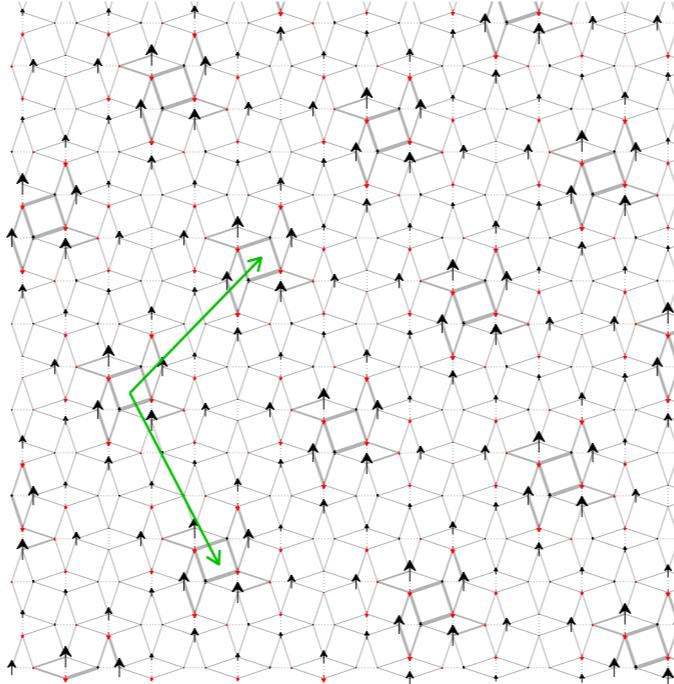
1/10 : (2,-4),(10,0)



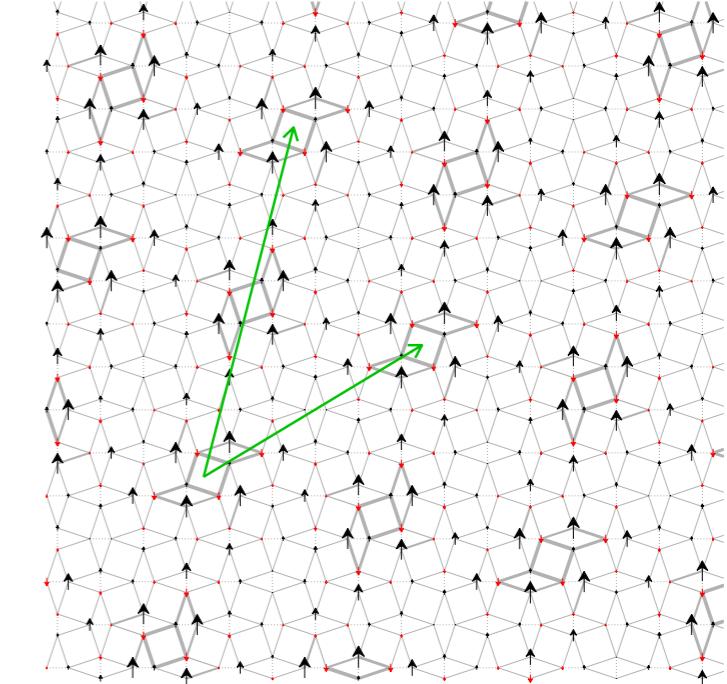
2/19 : (8,2),(1,5)



1/9 : (2,-4),(3,3)

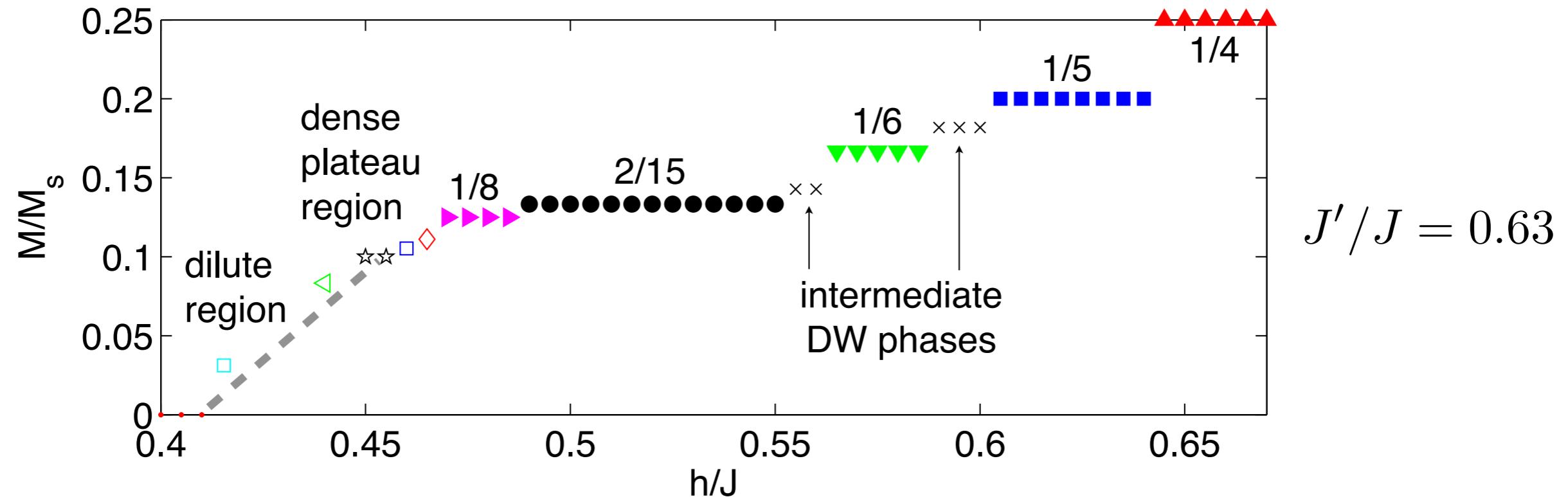


2/17 : (5,3),(2,8)



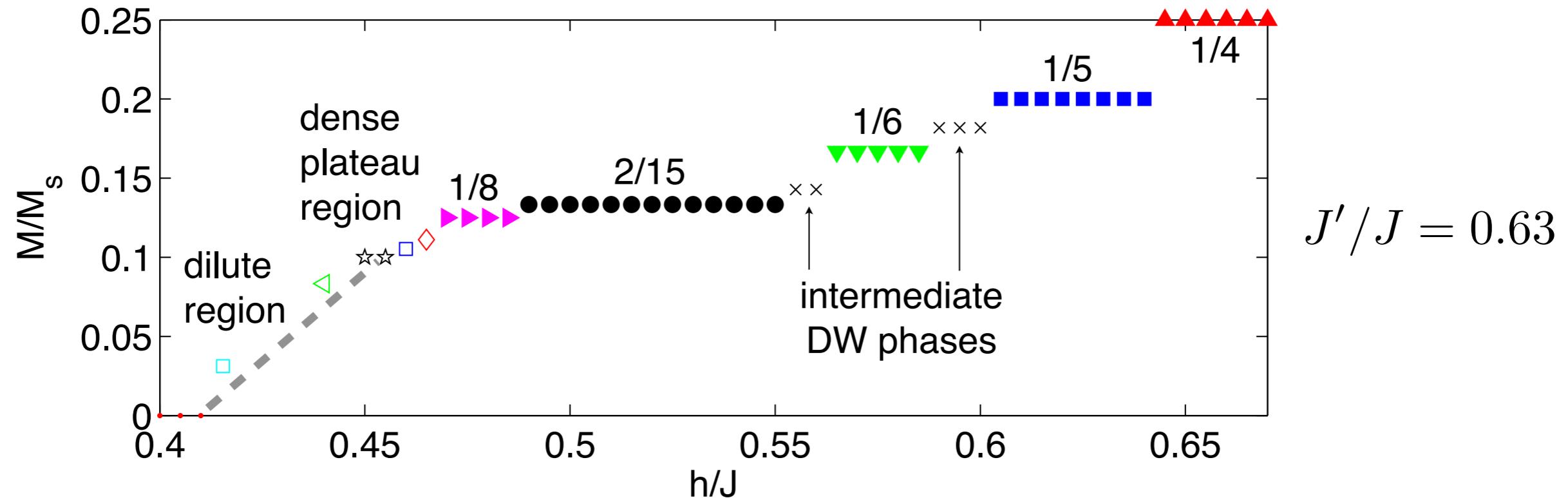
# Magnetization curve obtained with iPEPS

PC, F. Mila, PRL 112 (2014)



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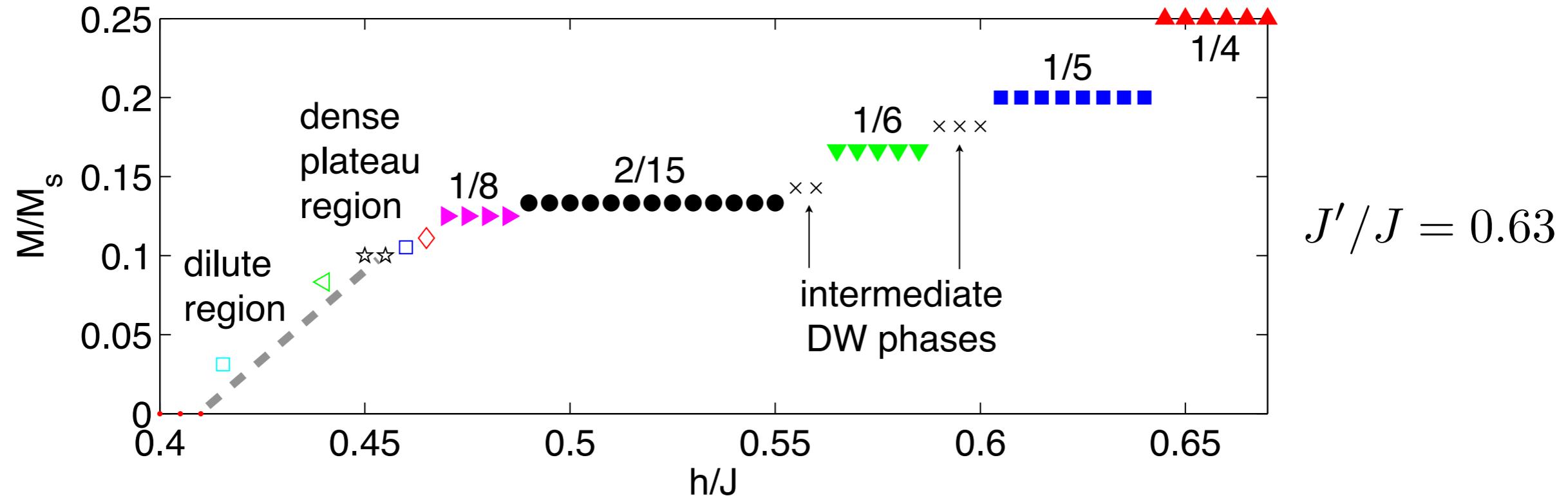


★ Sizable plateaus found at:  $1/8, 2/15, 1/6, 1/5, 1/4, 1/3, 1/2$

[ $1/5$  plateau vanishes upon adding a small (but realistic) DM interaction]

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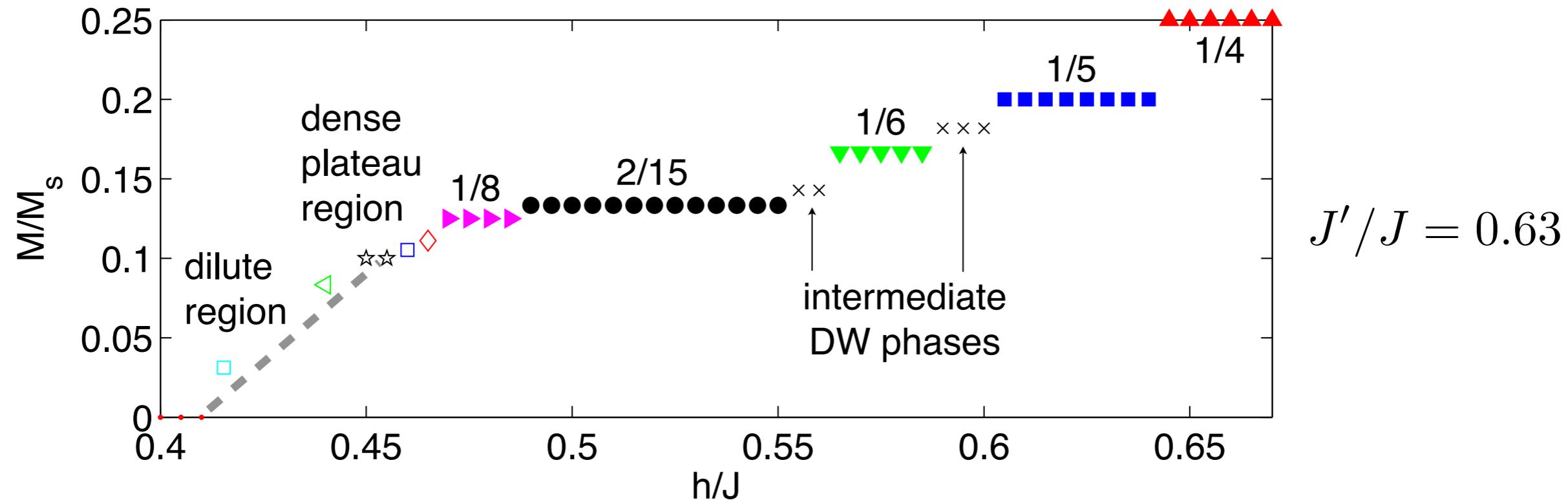


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★ Sequence in agreement with experiments

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PC, F. Mila, PRL 112 (2014)



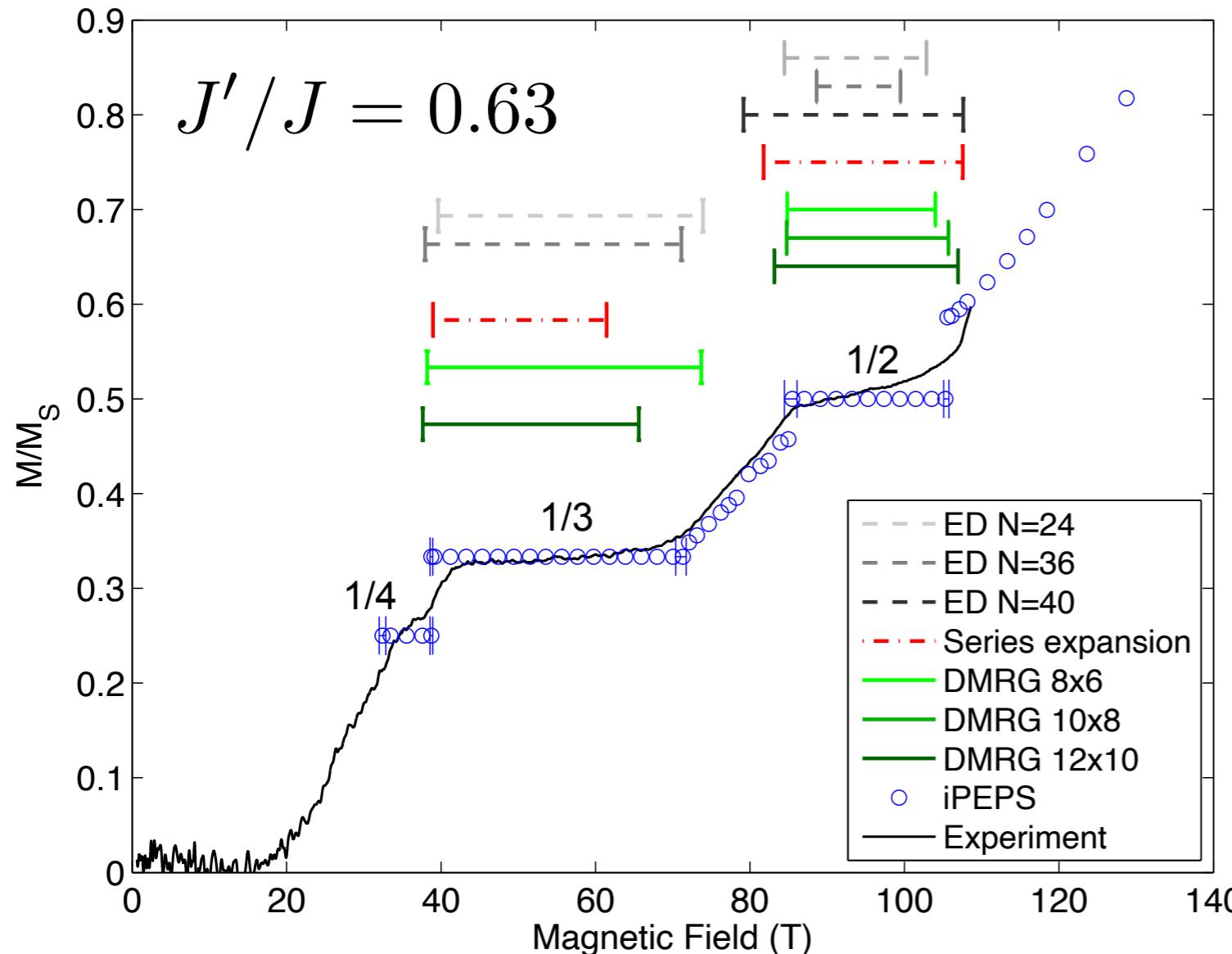
- ★ Sizable plateaus found at:  $1/8, 2/15, 1/6, 1/5, 1/4, 1/3, 1/2$   
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- ## ★ Sequence in agreement with experiments

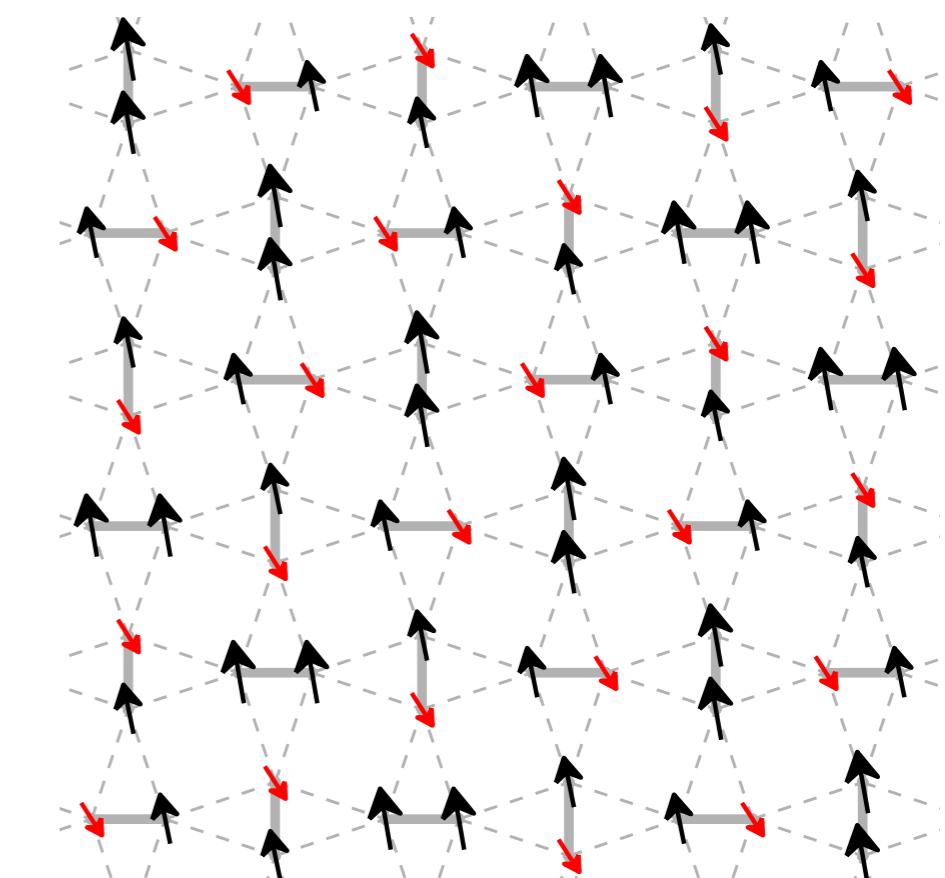
- # ★ New understanding of the magnetization process in $\text{SrCu}_2(\text{BO}_3)_2$

# $\text{SrCu}_2(\text{BO}_3)_2$ in ultra-high magnetic fields up to 118T

Matsuda, Abe, Takeyama, Kageyama, PC, Honecker, Manmana, Foltin, Schmidt & Mila, PRL 111 (2013)

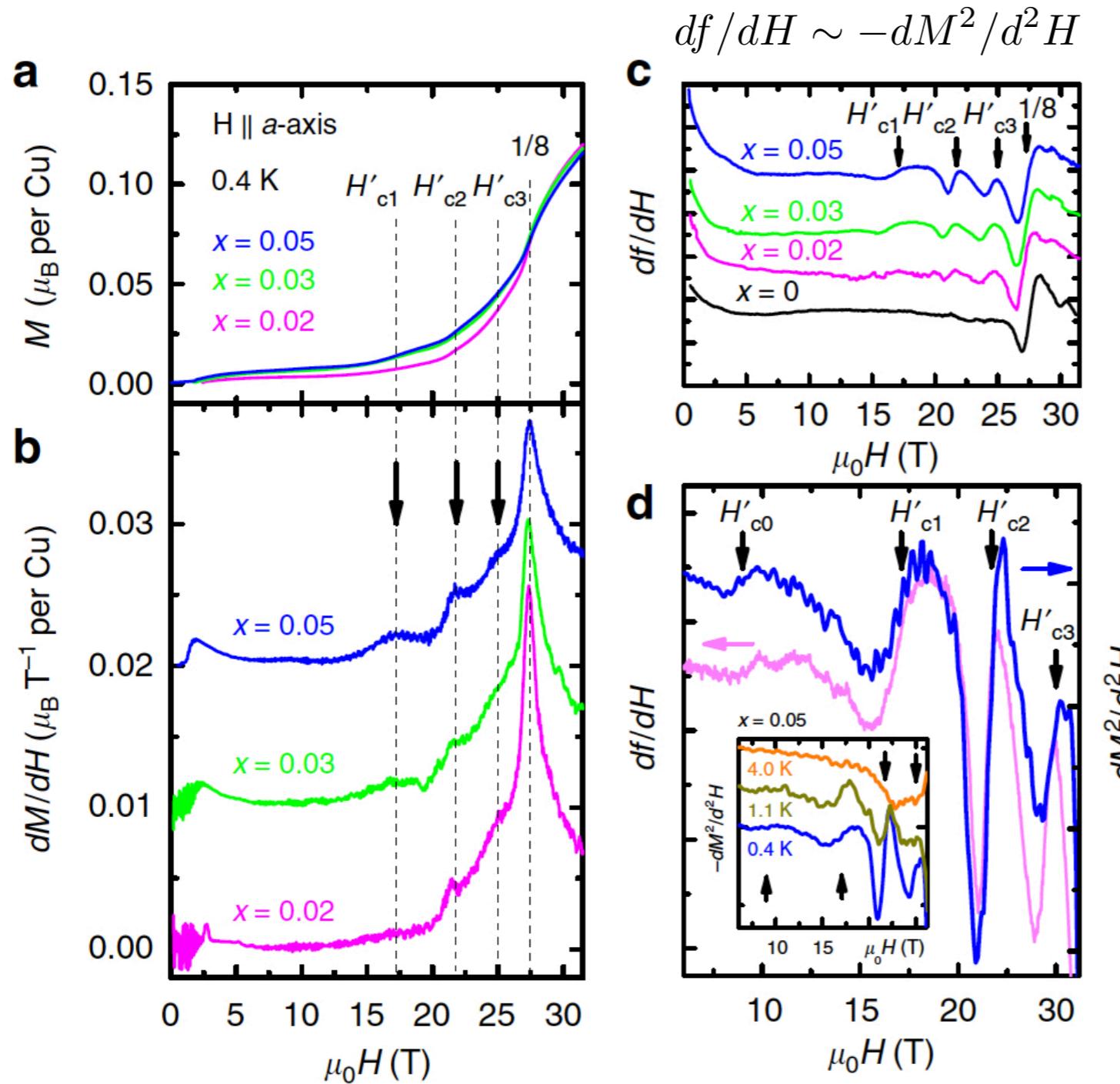


- Best fit with experiments for  $J'/J = 0.63$  using iPEPS, DMRG, ED, series expansion
- Supersolid phases at high fields



# SrCu<sub>2-x</sub>Mg<sub>x</sub>(BO<sub>3</sub>)<sub>2</sub> in a magnetic field

Shi, Steinhardt, Graf, PC, Weickert, Harrison, Jaime, Marjerrison, Dabkowska, Mila, Haravifard, Nature Communications 10, 2439 (2019).



$$H'_{C0} \sim 9T, \quad H'_{C1} \sim 17.1T, \quad H'_{C2} \sim 21.7T, \quad H'_{C3} \sim 25T$$

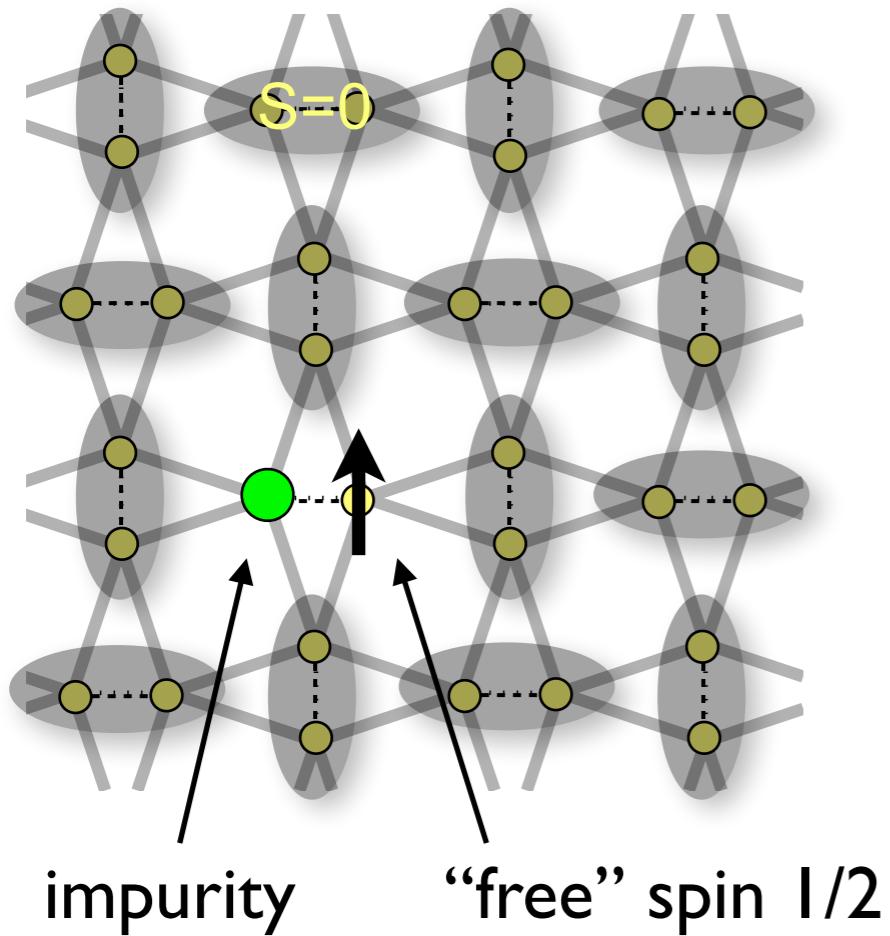
★ New anomalies appearing below the  $1/8$  plateau at fields which are absent at zero doping

★ Use iPEPS to gain insights into the nature of these anomalies

# $\text{SrCu}_{2-x}\text{Mg}_x(\text{BO}_3)_2$ in a magnetic field

Shi, Steinhardt, Graf, PC, Weickert, Harrison, Jaime, Marjerrison, Dabkowska, Mila, Haravifard, Nature Communications 10, 2439 (2019).

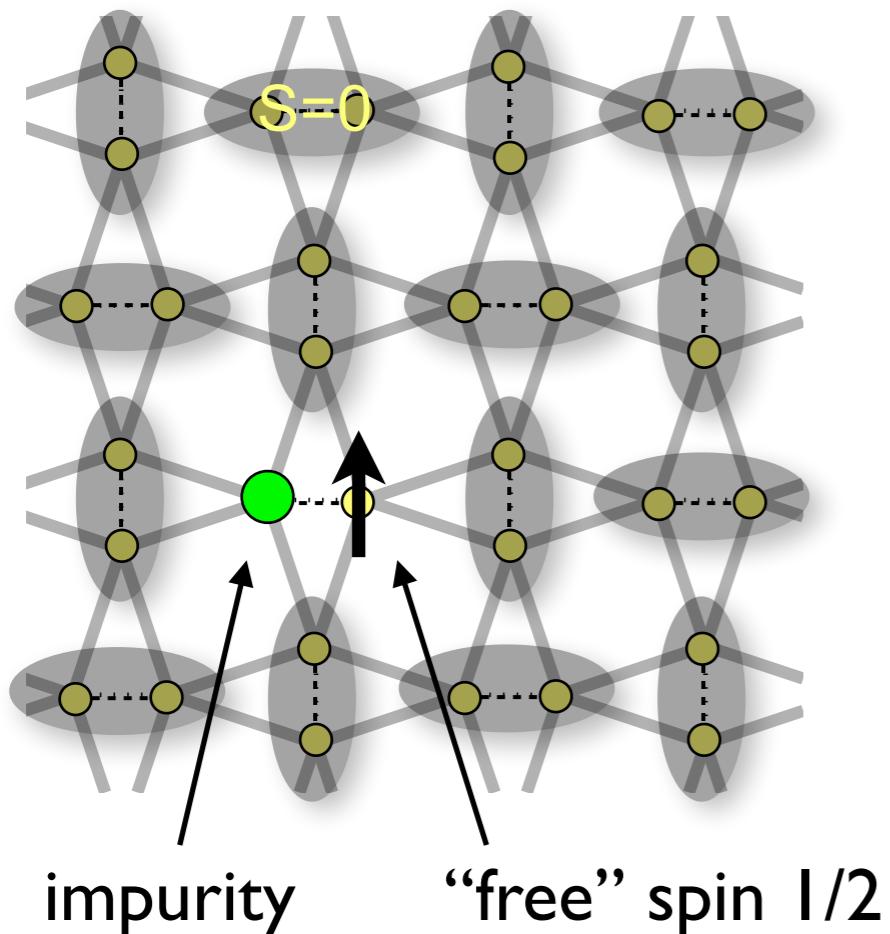
$$\hat{H} = J' \sum_{\langle i,j \rangle} S_i \cdot S_j + J \sum_{\langle\langle i,j \rangle\rangle_{\text{dimer}}} S_i \cdot S_j - h \sum_i S_i^z \quad \text{with non-magnetic impurities}$$



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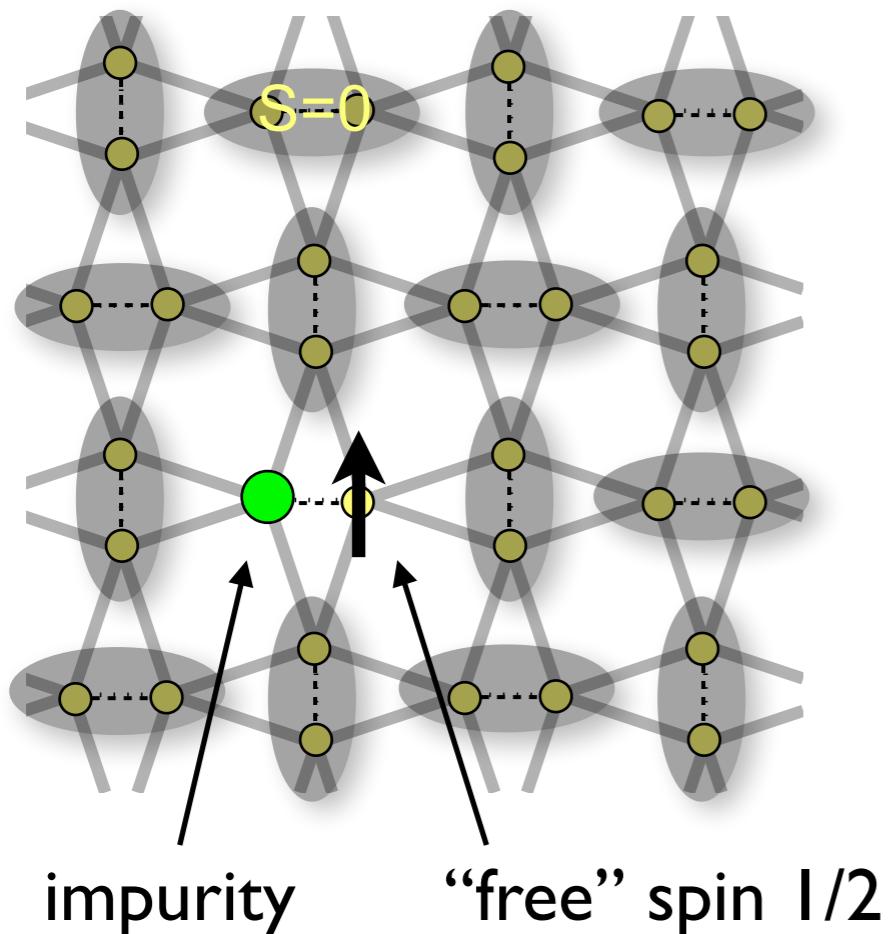


★ The free spin next to an isolated impurity aligns with an arbitrarily small field

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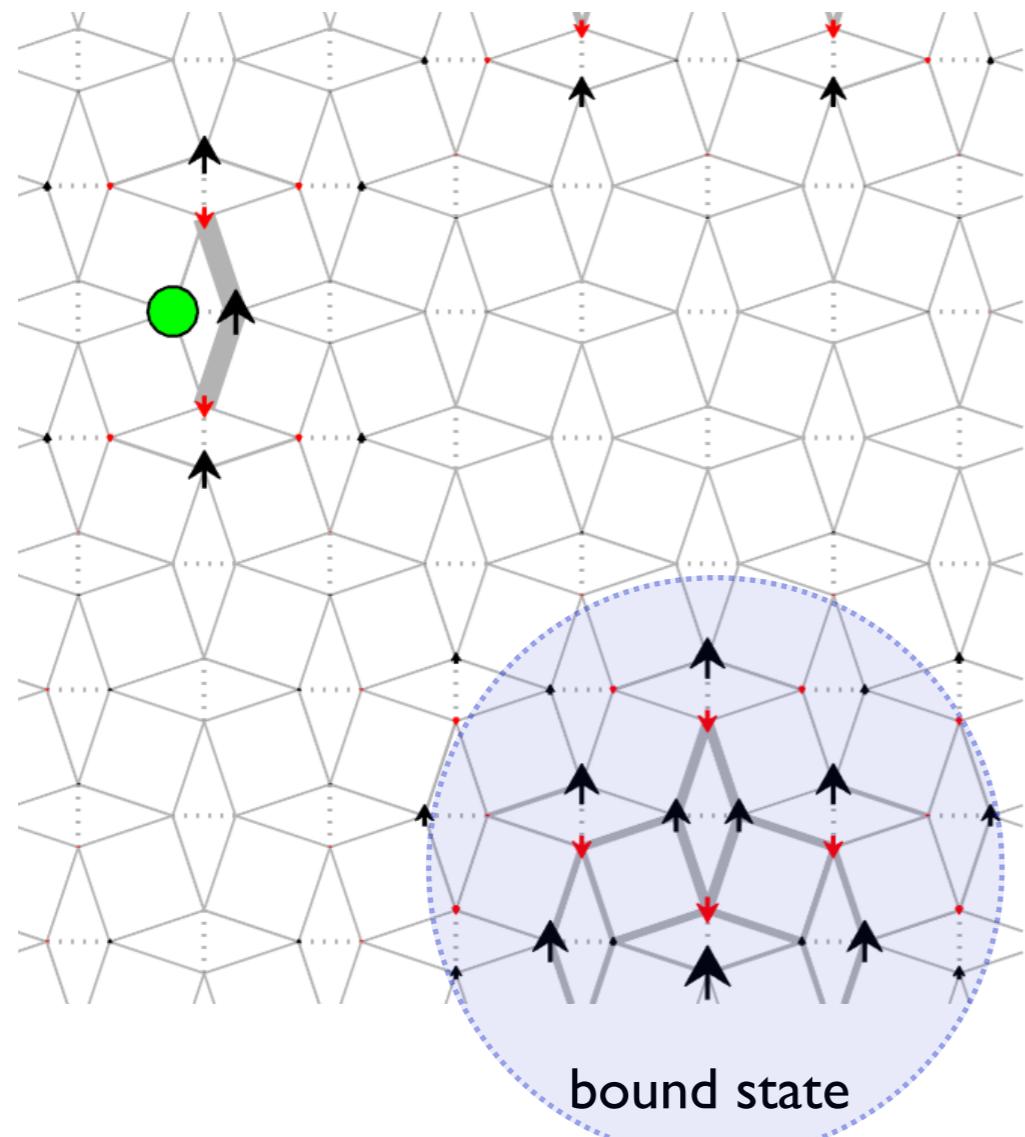


★ The free spin next to an isolated impurity aligns with an arbitrarily small field

★ Magnetization process in the presence of the impurities?

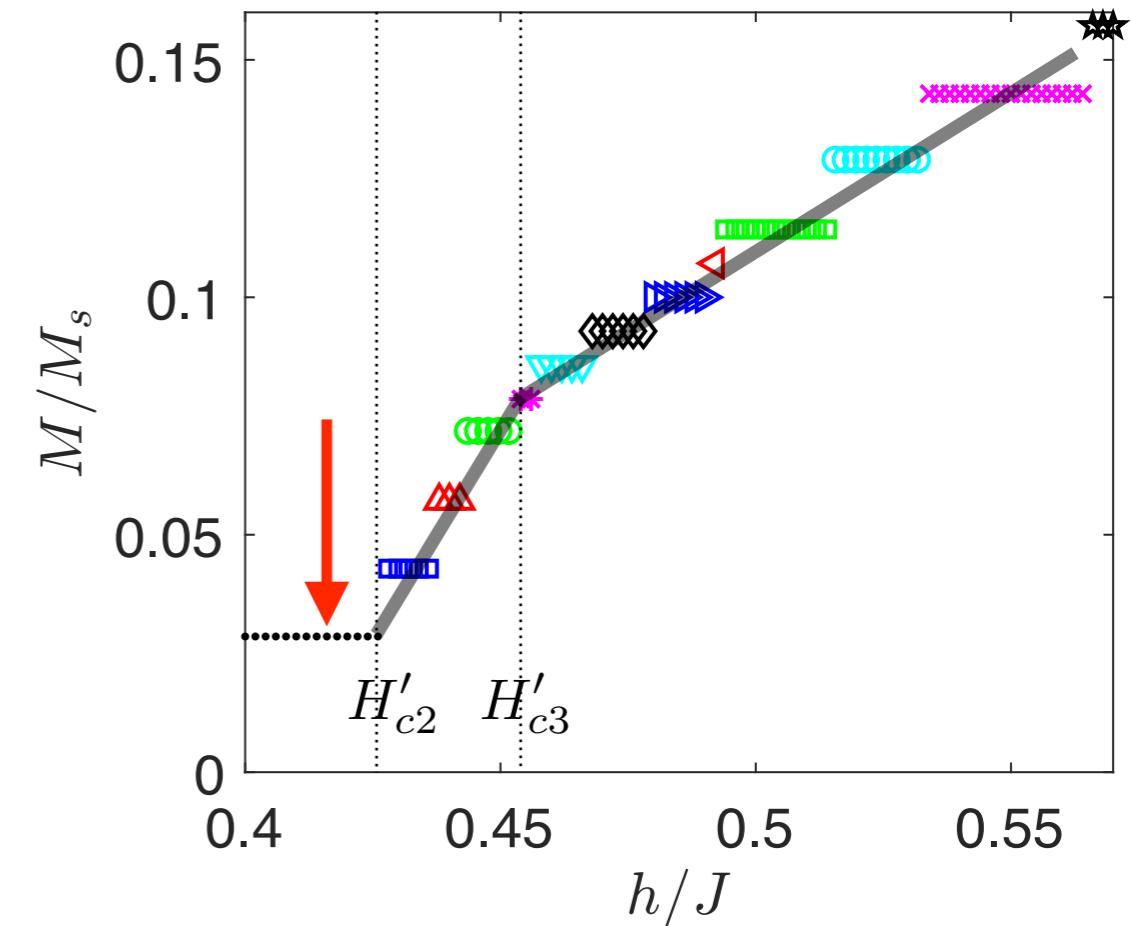
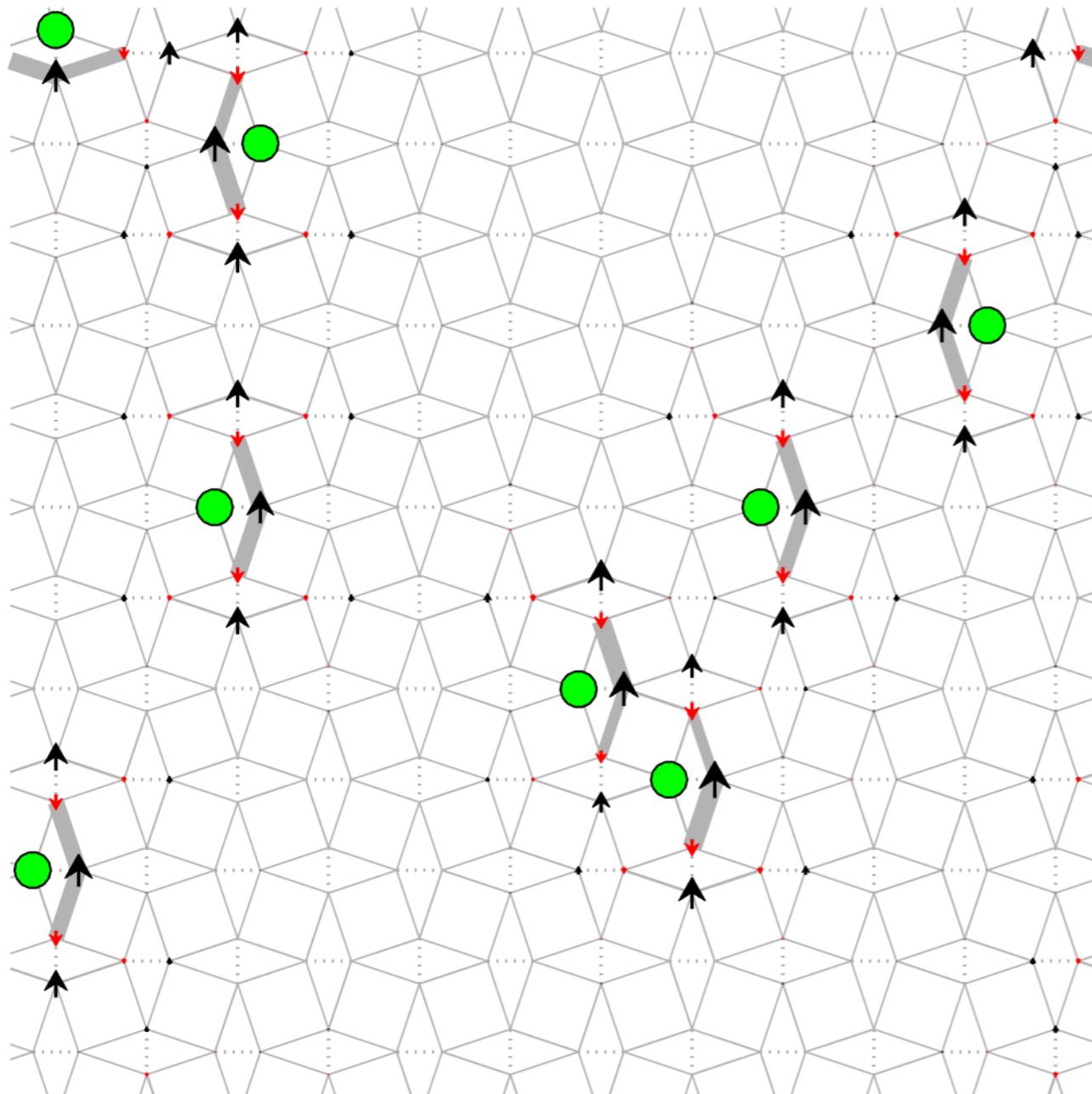
# Key question I: Bound states attracted or repelled?

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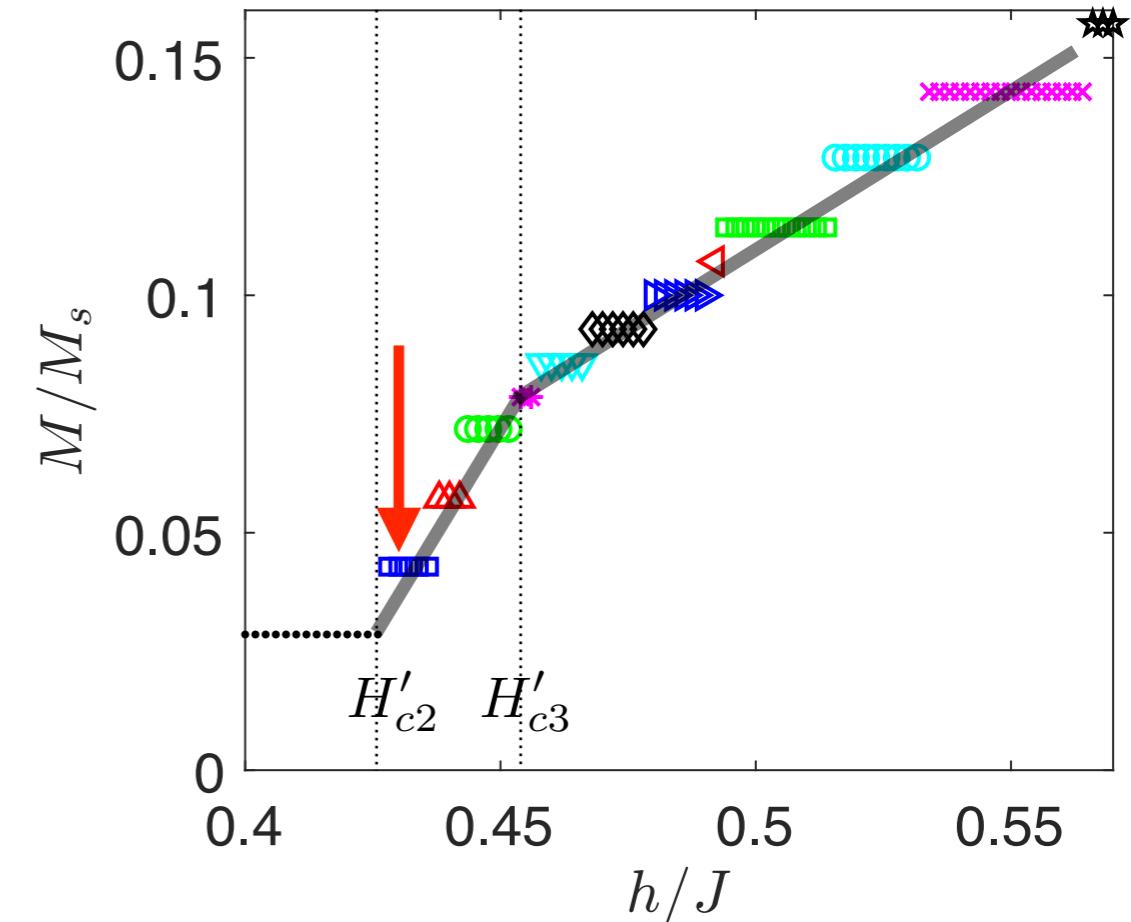
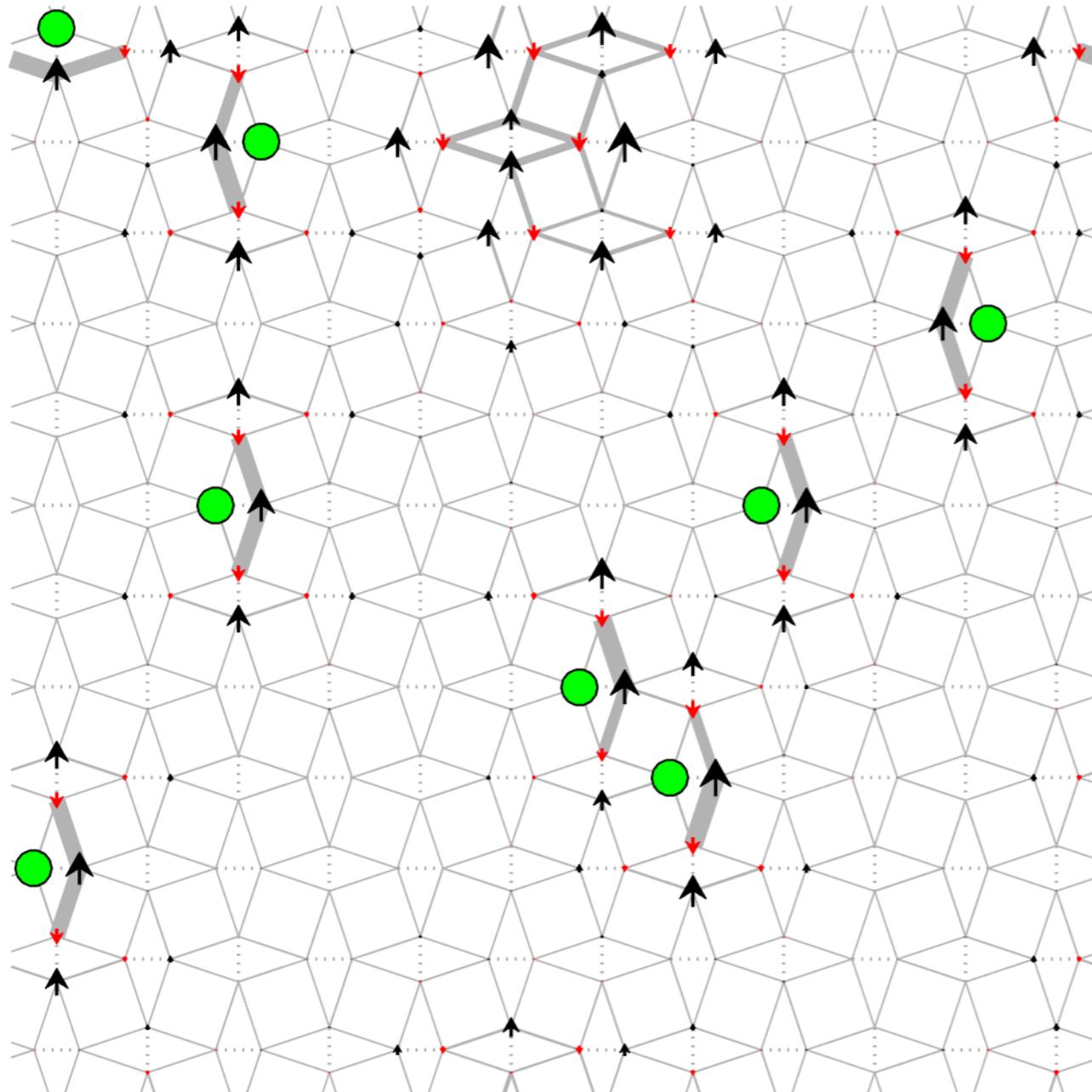


★ A bound state gets repelled  
by an impurity site

# Key question 2: how does the lattice get filled?



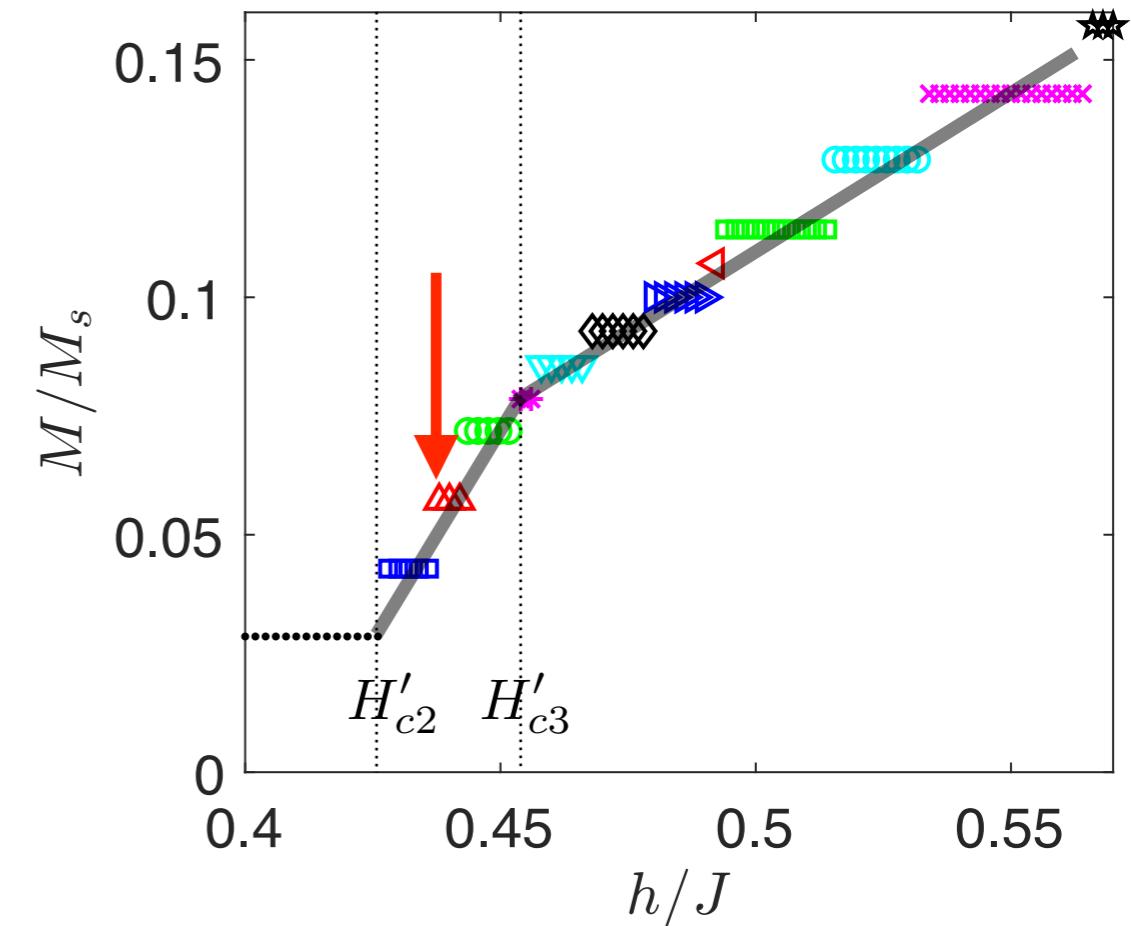
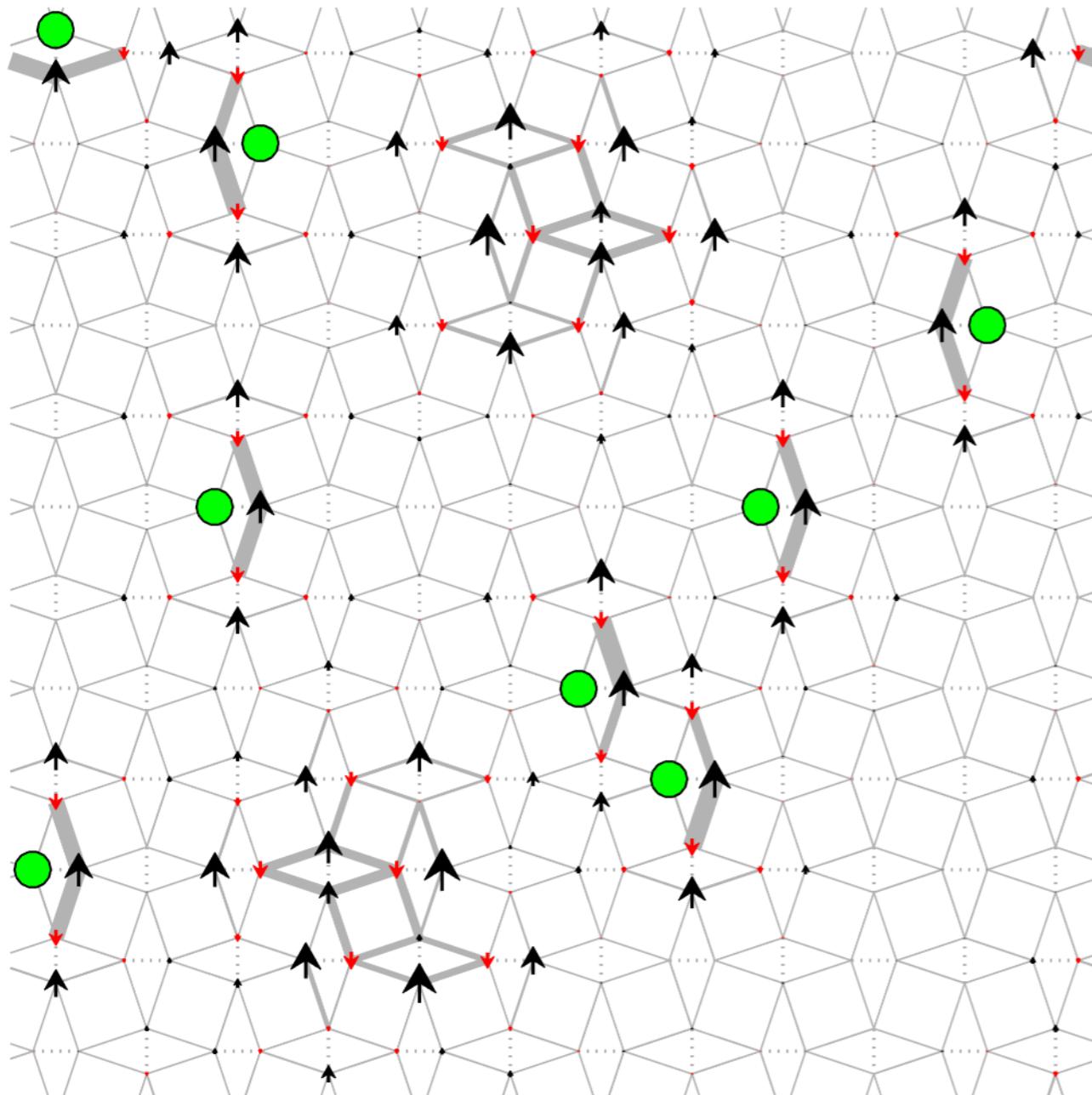
# Key question 2: how does the lattice get filled?



★ At  $H'_{c2} \sim 21.8T$ : appearance of first bound states

Jump in magnetization leads to anomaly in experiments at  $H'_{c2} \sim 21.7T$

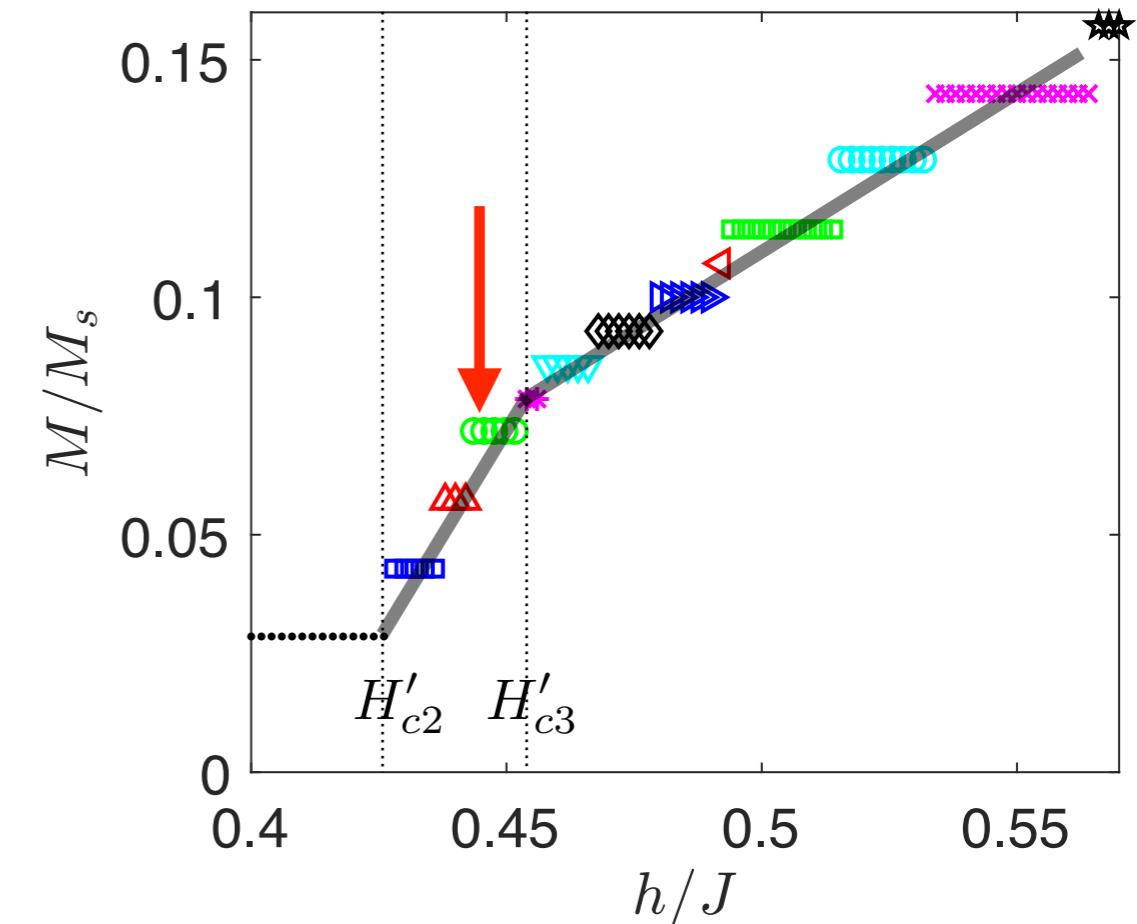
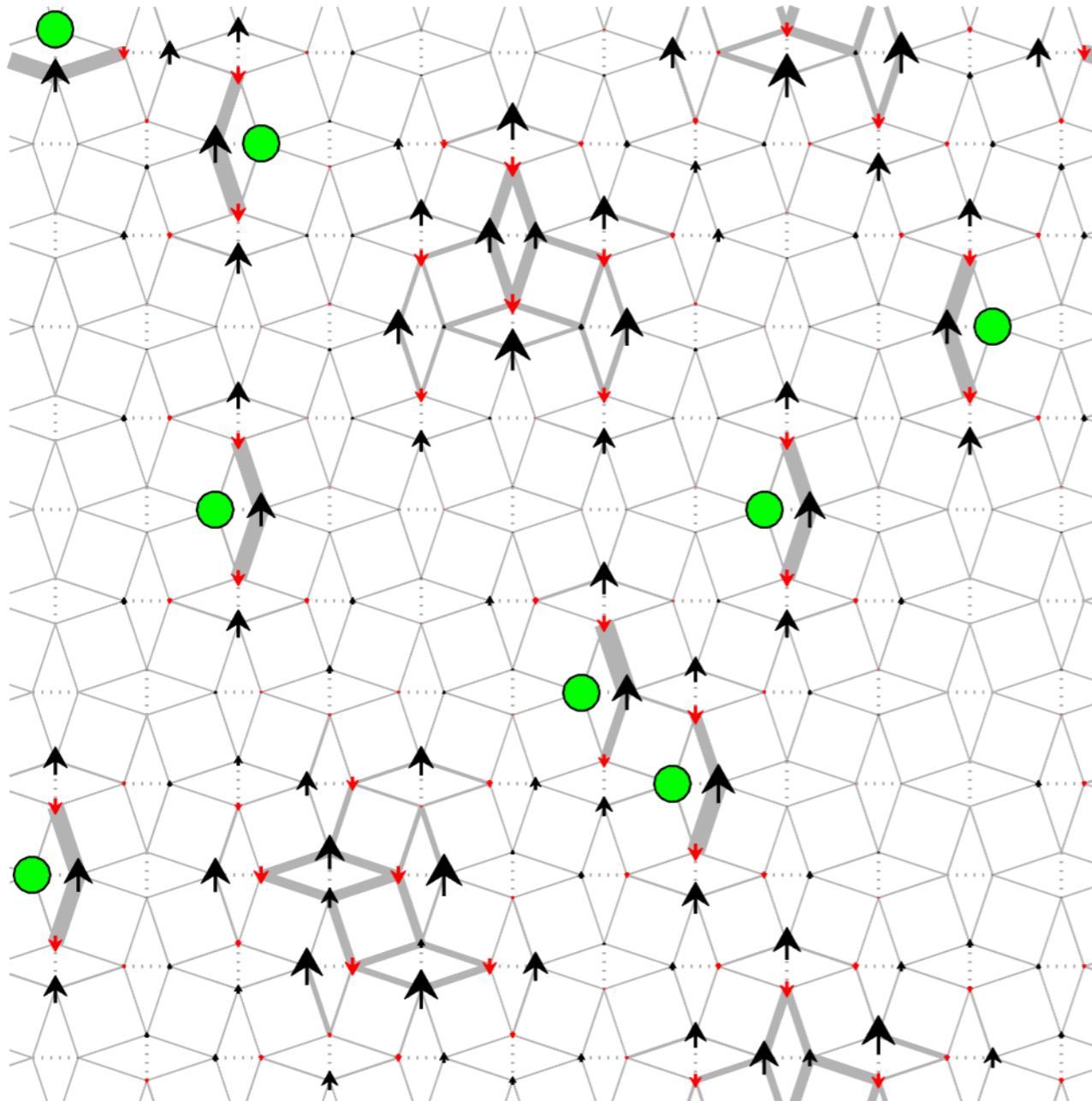
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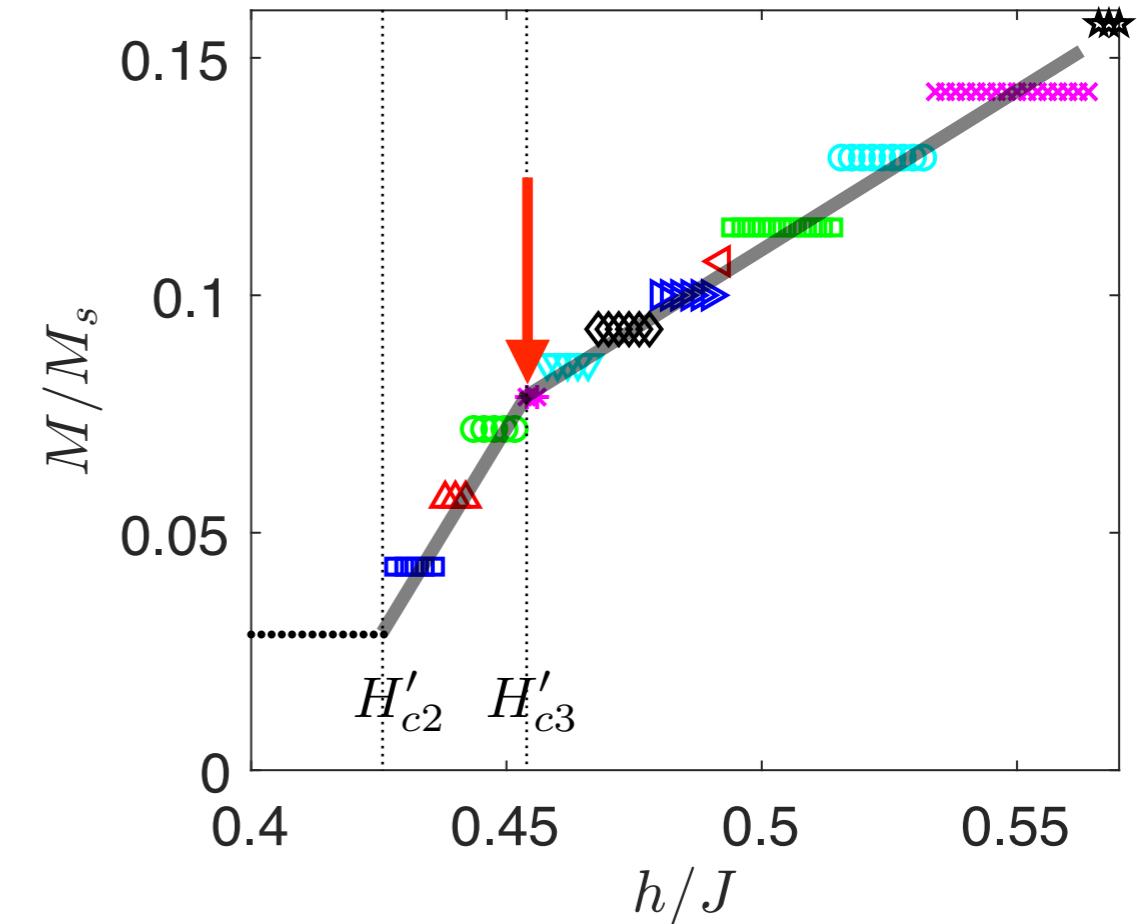
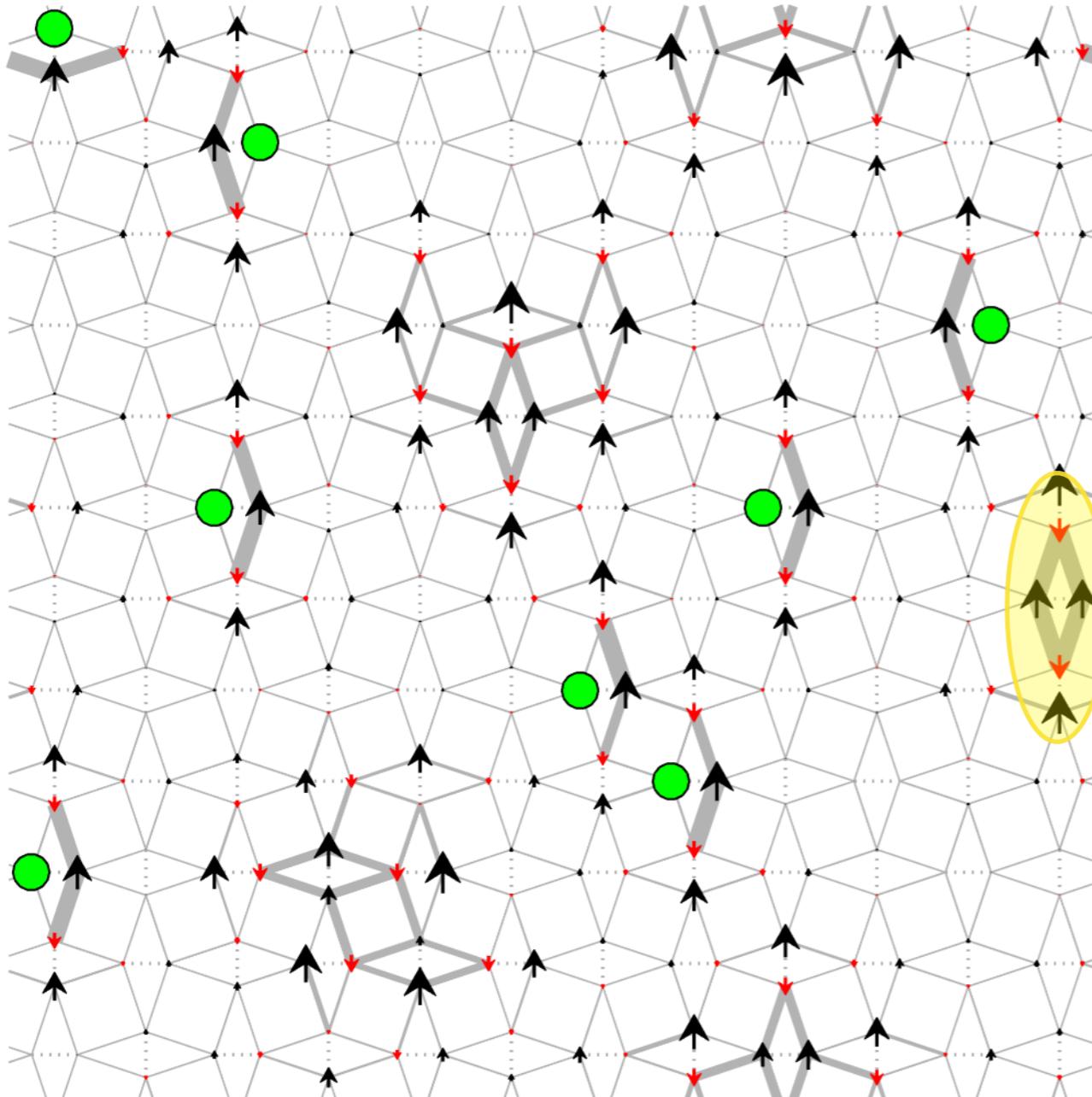
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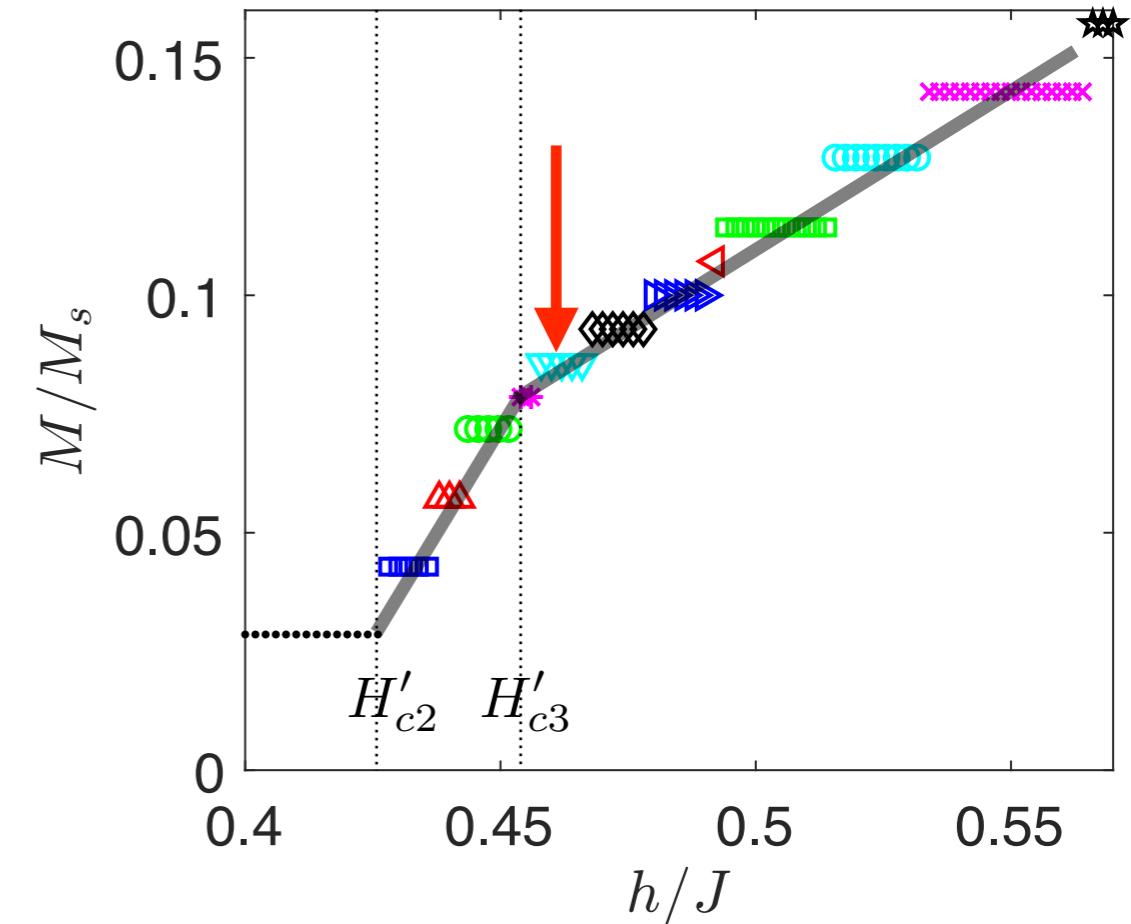
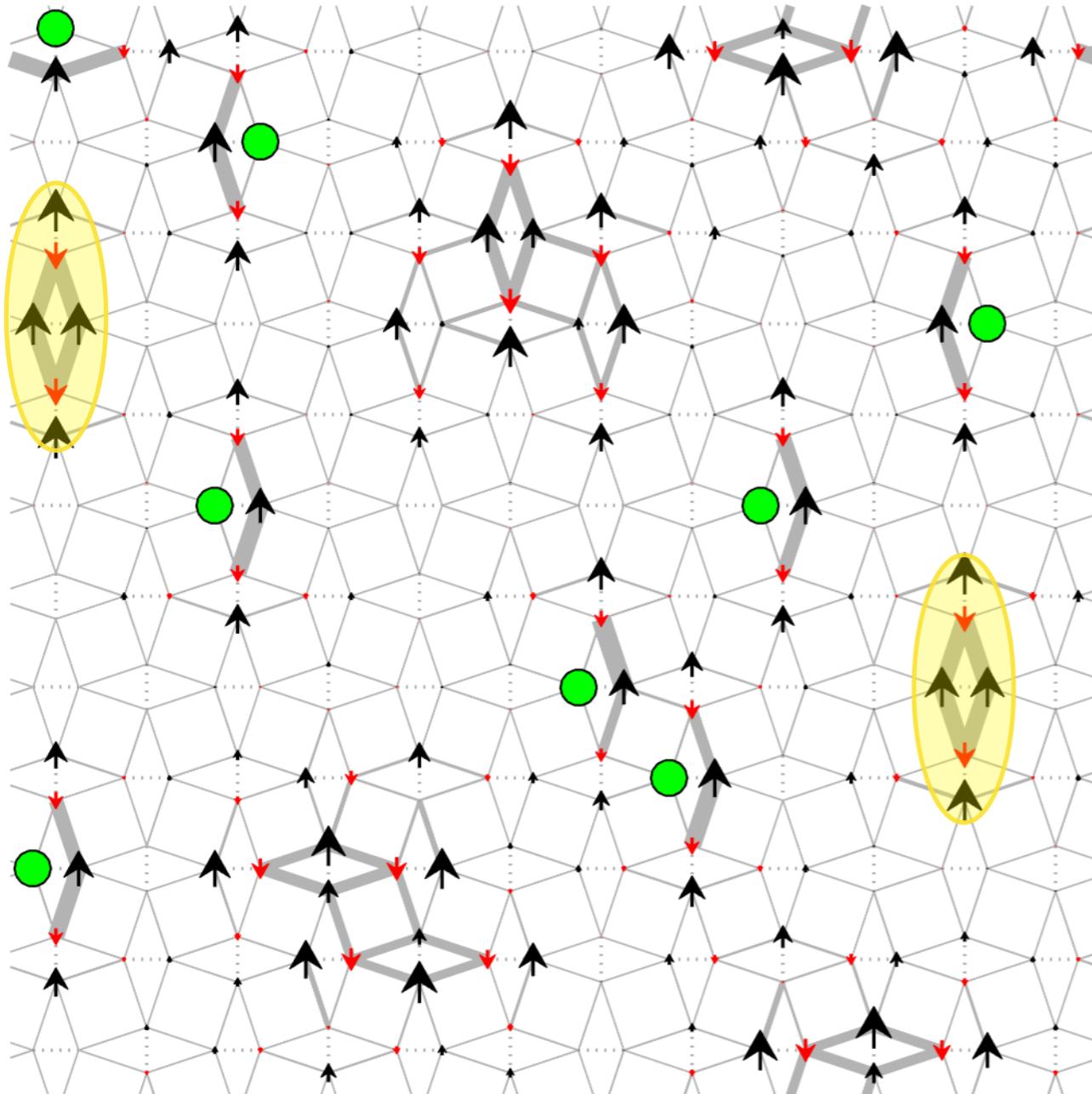
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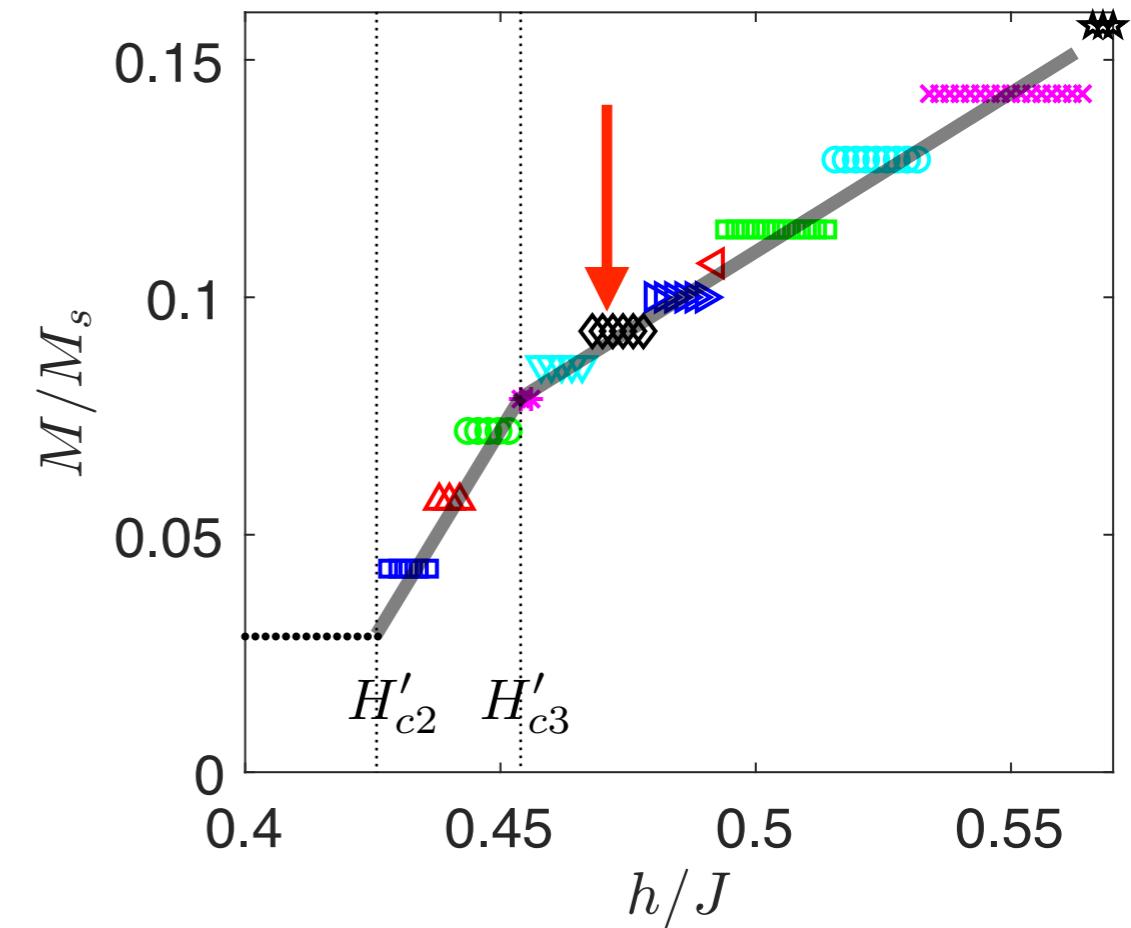
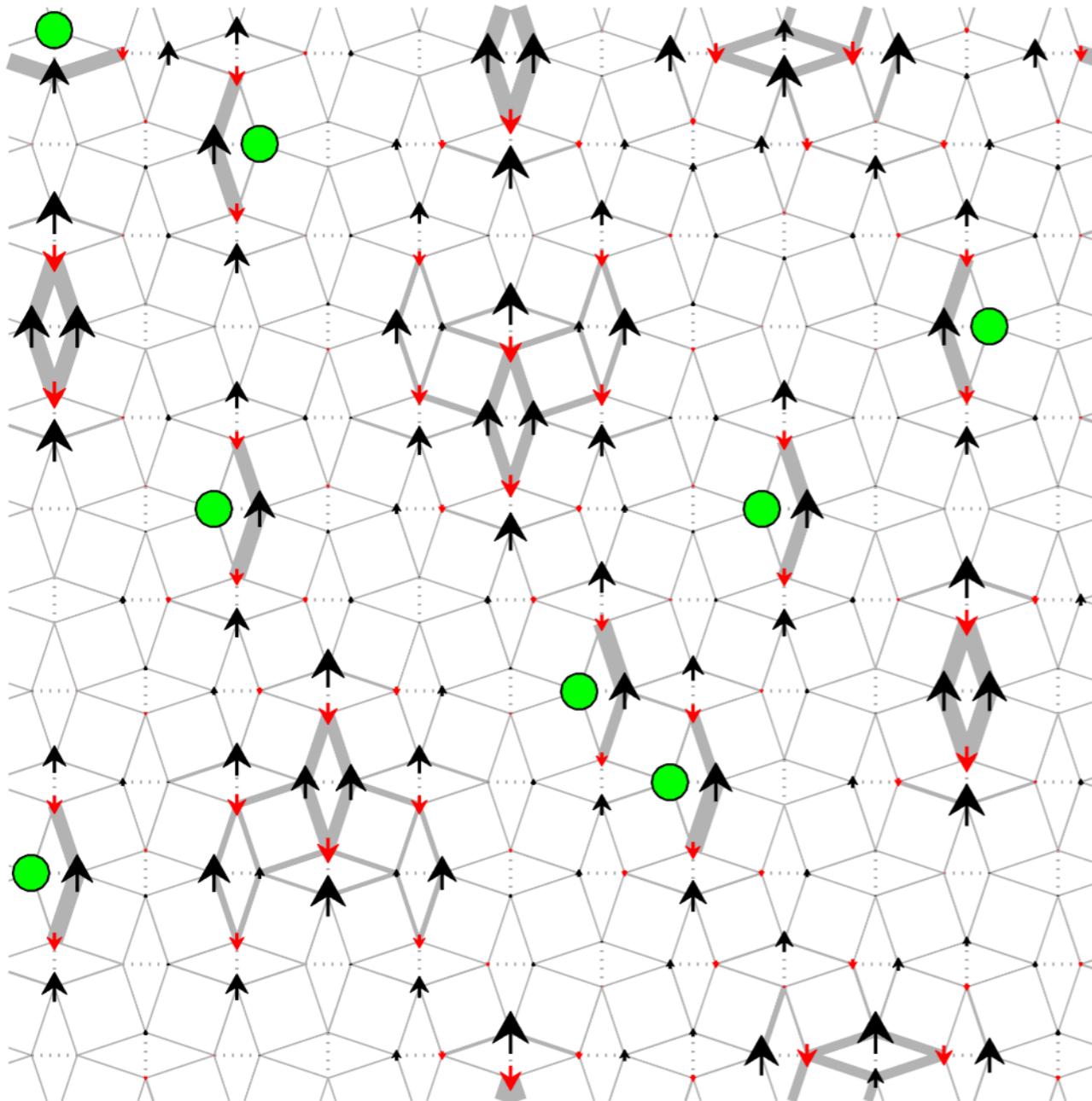
★ At  $H'_{c3} \sim 23.2T$  : appearance of additional triplets  
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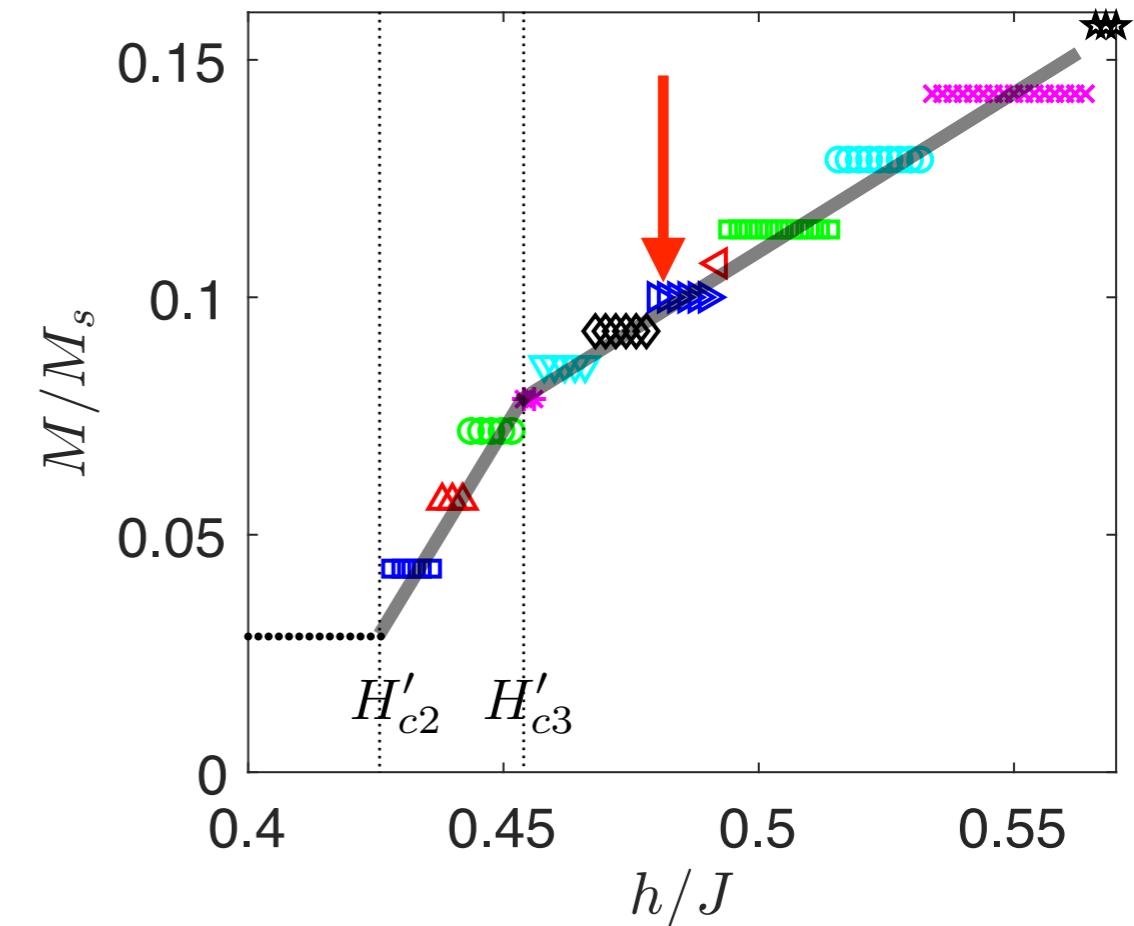
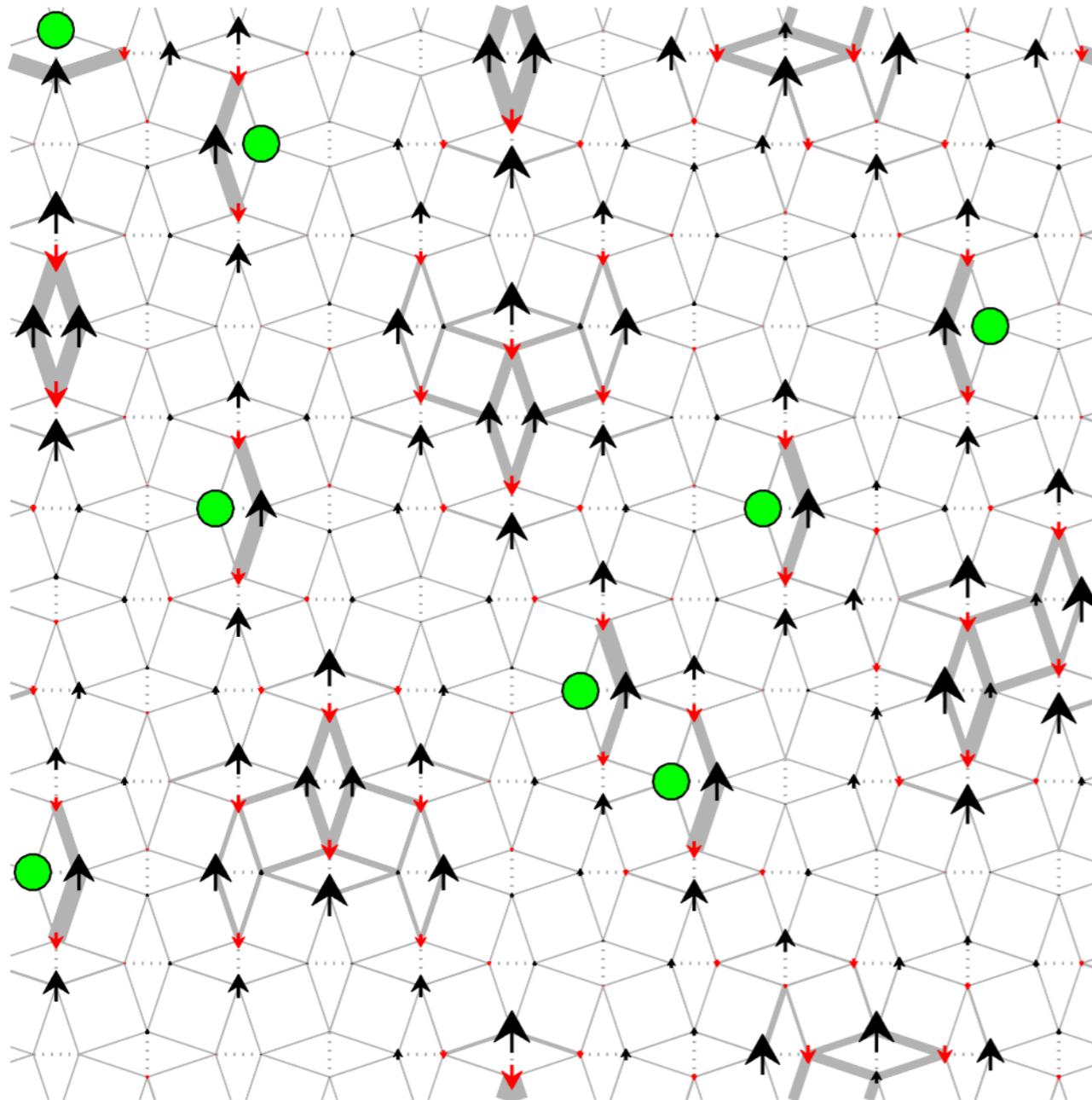
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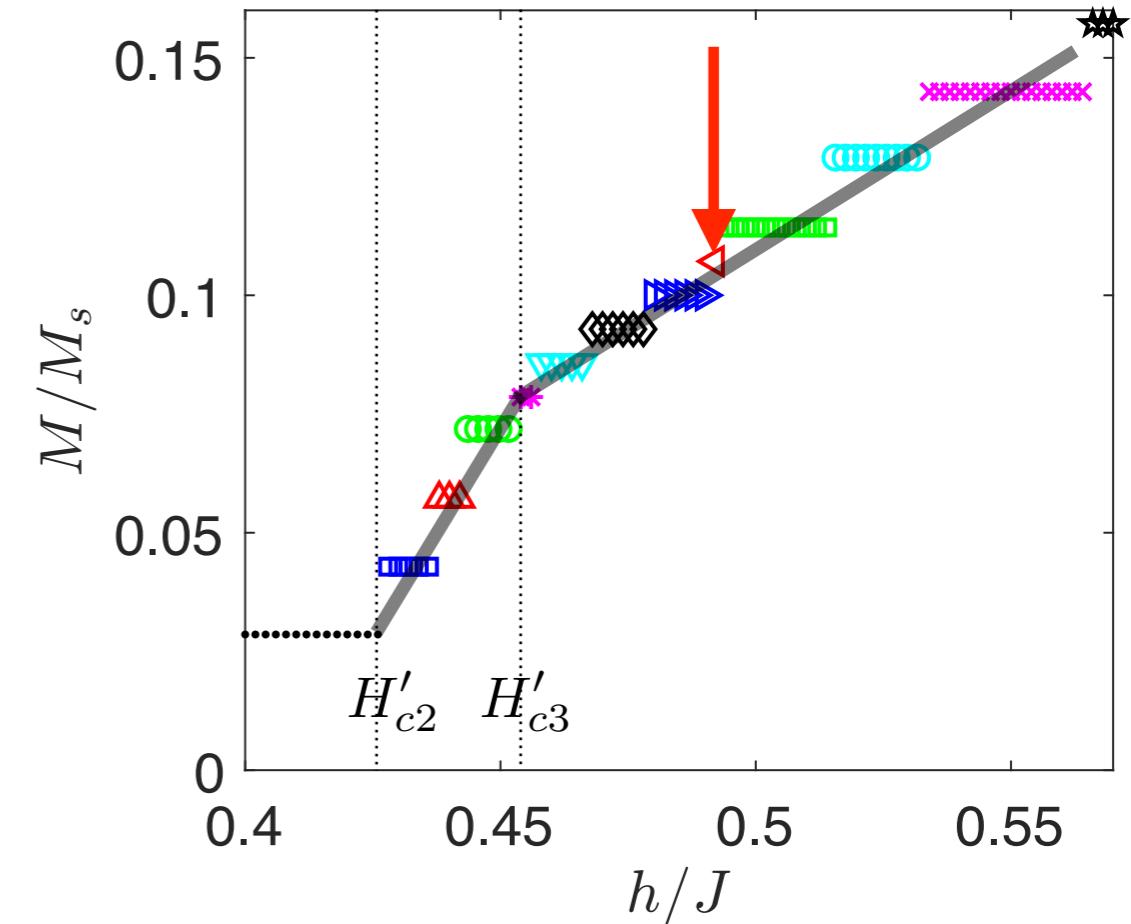
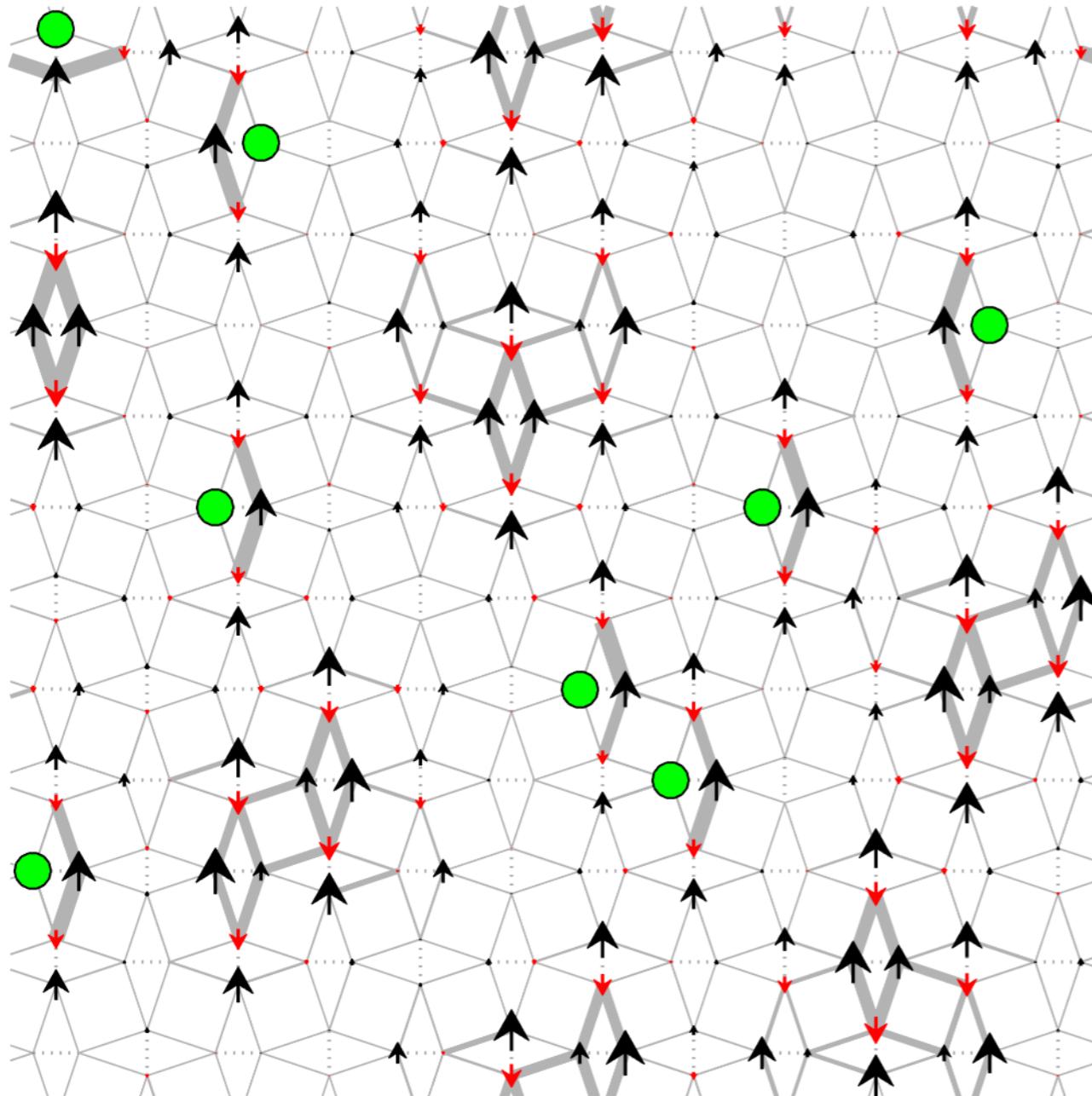
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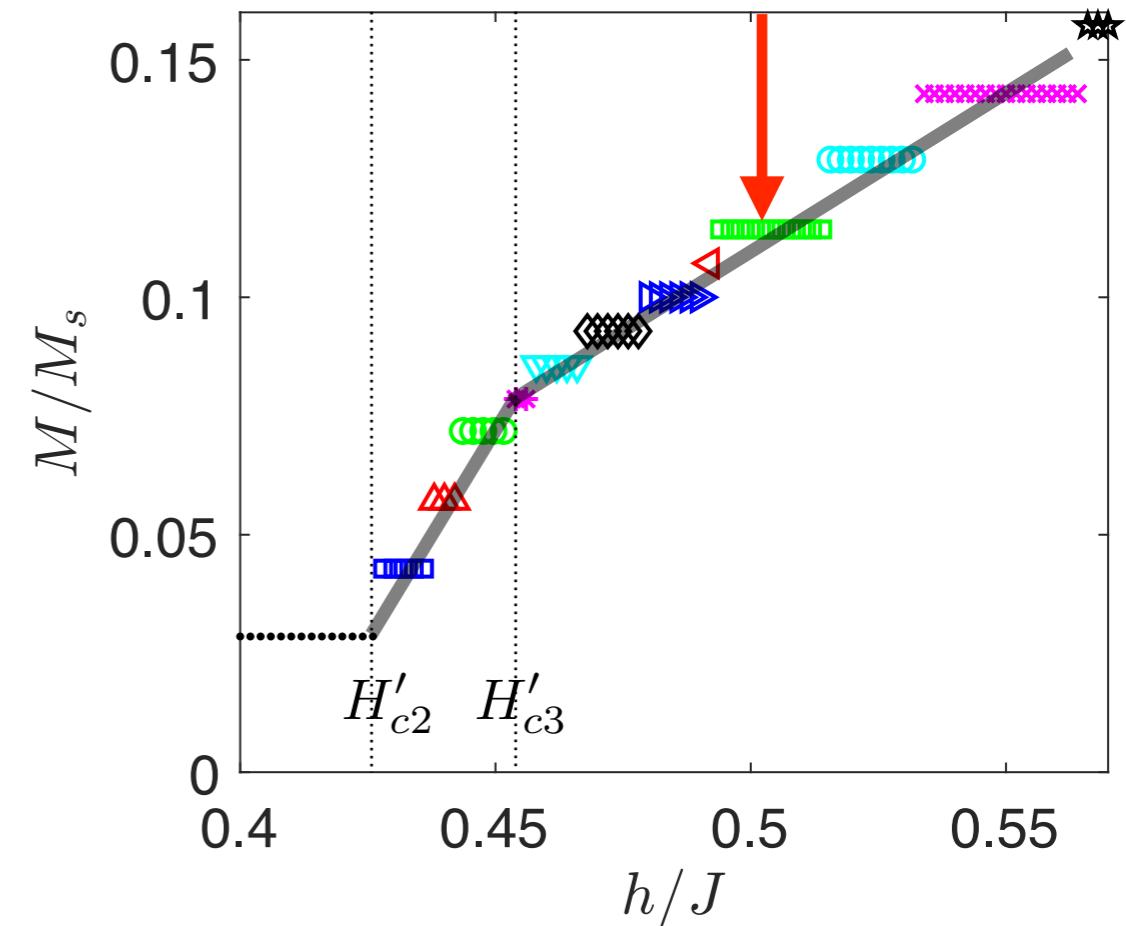
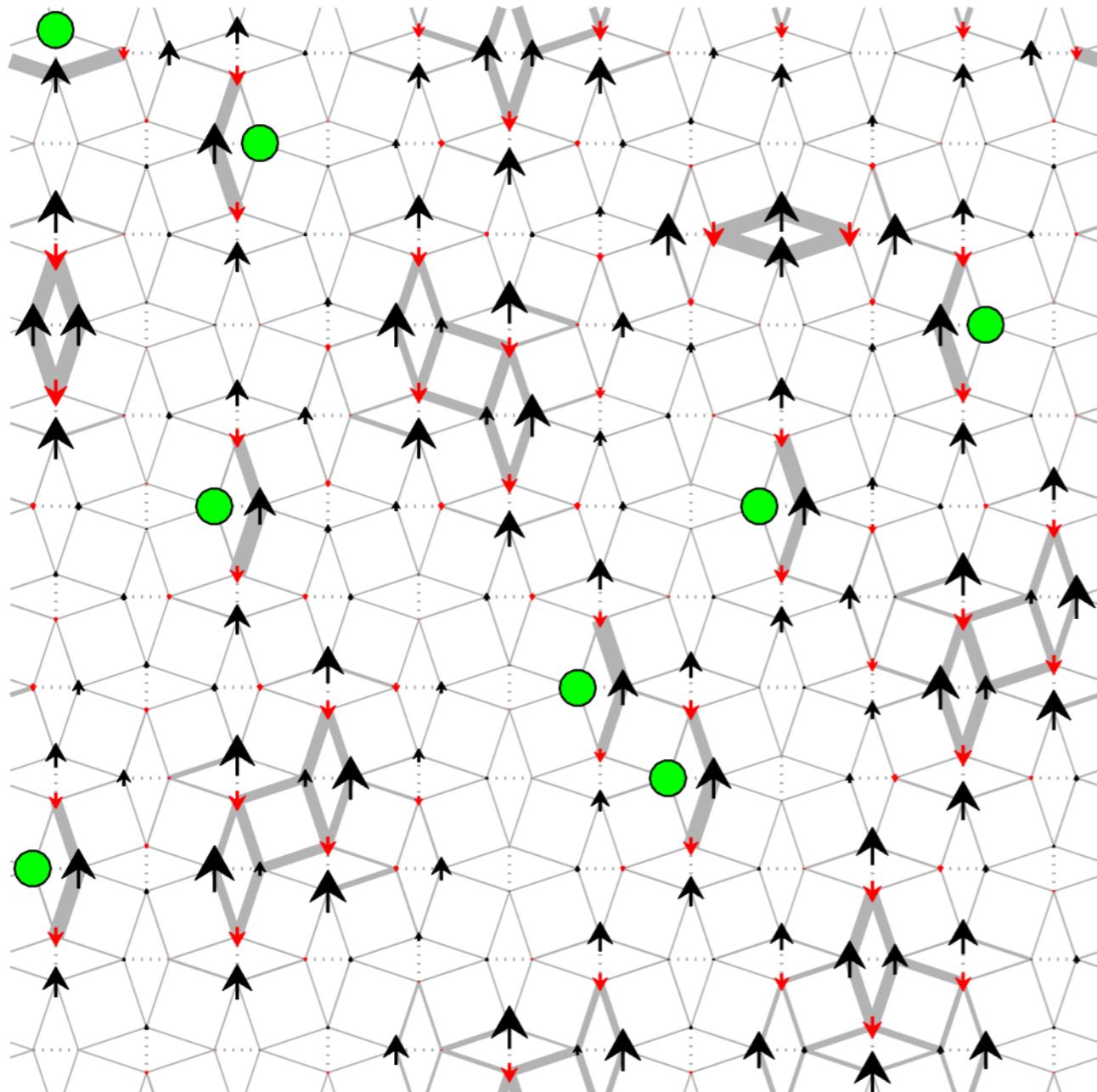
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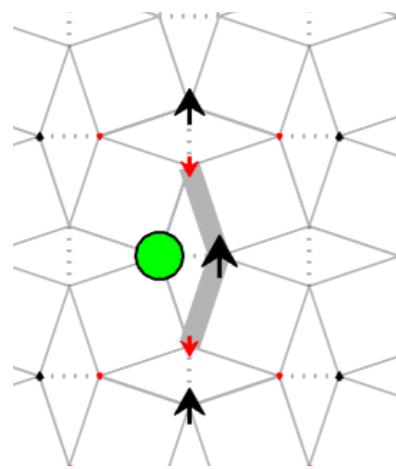
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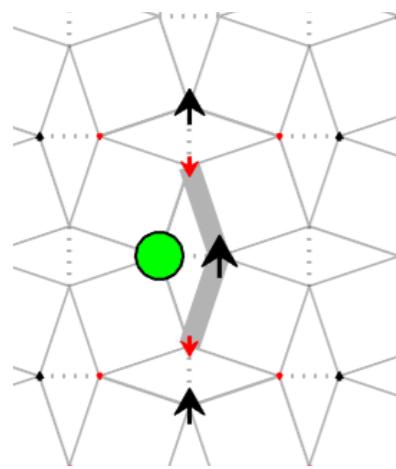
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# Key question 3: what are the anomalies below $H'c_2$ ?



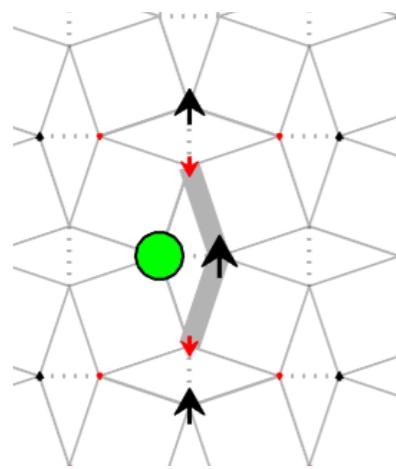
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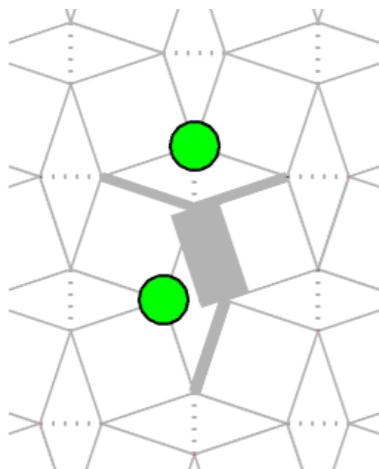


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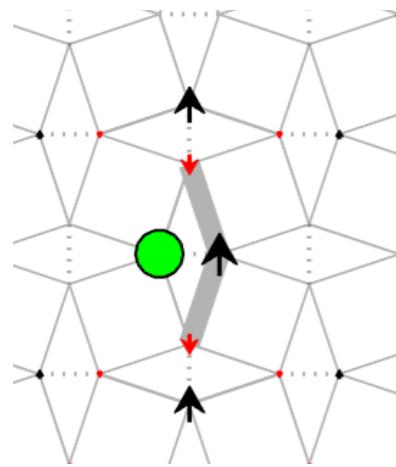


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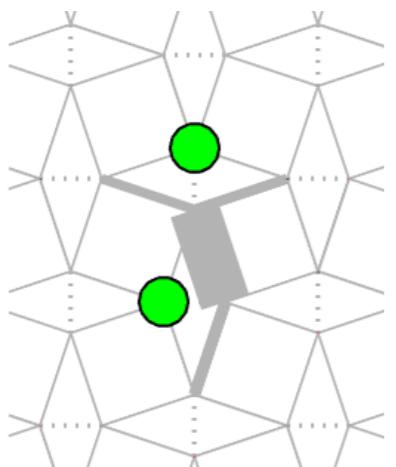


- 2 neighboring impurities: attached spins couple together
- It takes a finite energy to excite this impurity pair configuration, i.e. only beyond a certain critical value of  $h$

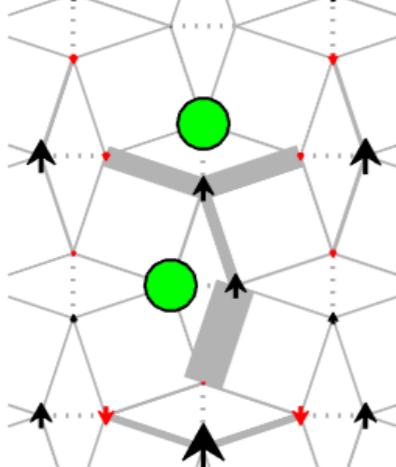
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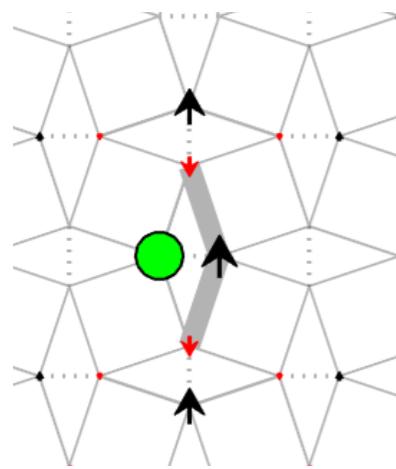


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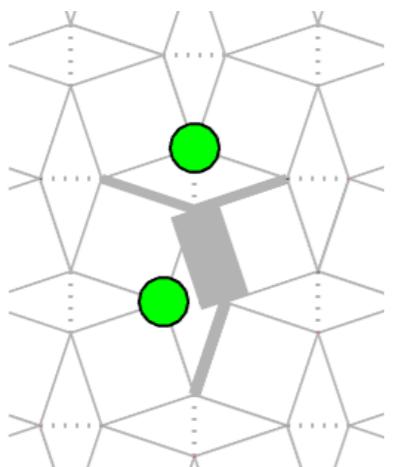


★ Excitation energy:  $0.238J$  ( $\sim 12.1\text{ T}$ )  
well below  $H'c_2$ !

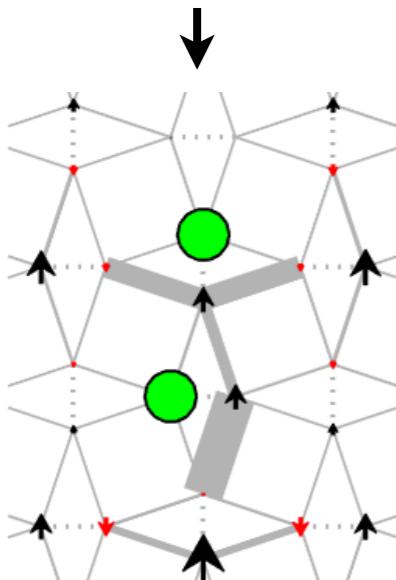
# Key question 3: what are the anomalies below $H'_{C2}$ ?



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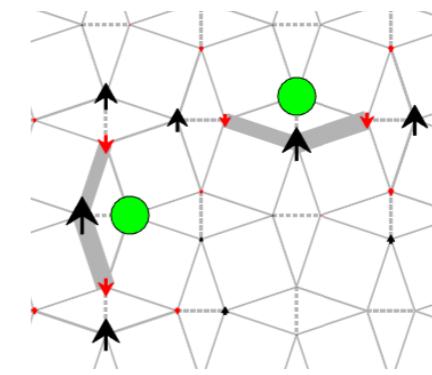
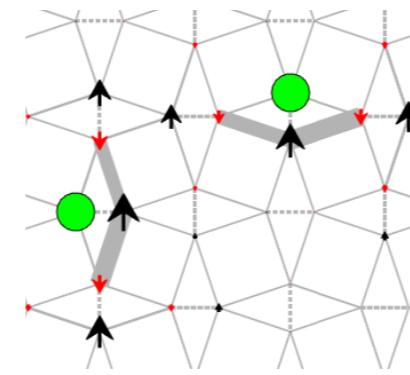
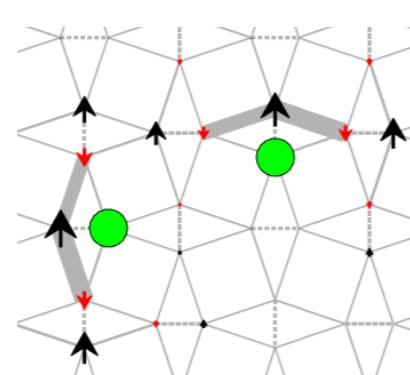
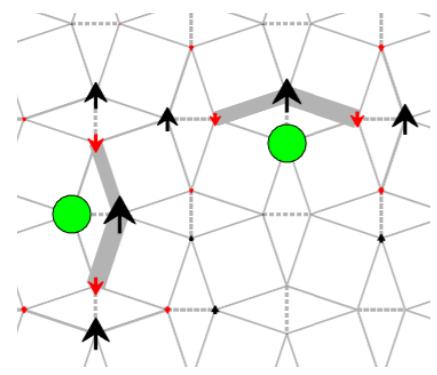
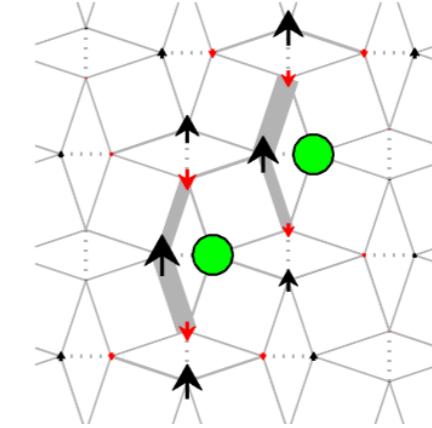
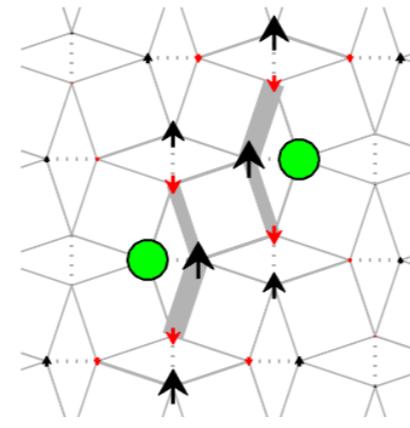
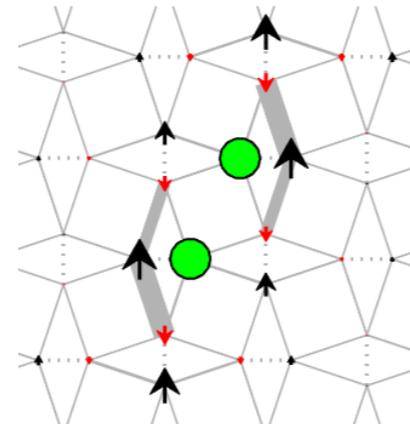
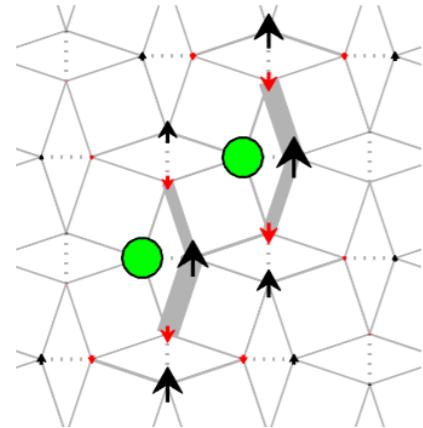


★ Excitation energy:  $0.238J$  ( $\sim 12.1T$ )  
well below  $H'_{C2}$ !

★ Lower than experimental value ( $H'_{C1} \sim 17.1T$ ),  
but, additional impurities will increase theoretical value

# Key question 3: what are the anomalies below $H'c_2$ ?

★ More 2-impurity configurations with lower excitation energies:



...

★ These values change in the presence of additional nearby impurities!

## Summary: Mg-doped Shastry-Sutherland model

- ✓ Using large unit-cell iPEPS simulations we obtained a qualitative / semi-quantitative understanding of the novel anomalies observed in Mg-doped  $SrCu_2(BO_3)_2$
- ✓  $H'_{c0}, H'_{c1}$  : excitation of impurity-pair (multi-impurity) configurations
- ✓  $H'_{c2}$  : appearance of localized bound states (jump in magnetization)
- ✓  $H'_{c3}$  : appearance of additional triplets in the lattice (change of slope)

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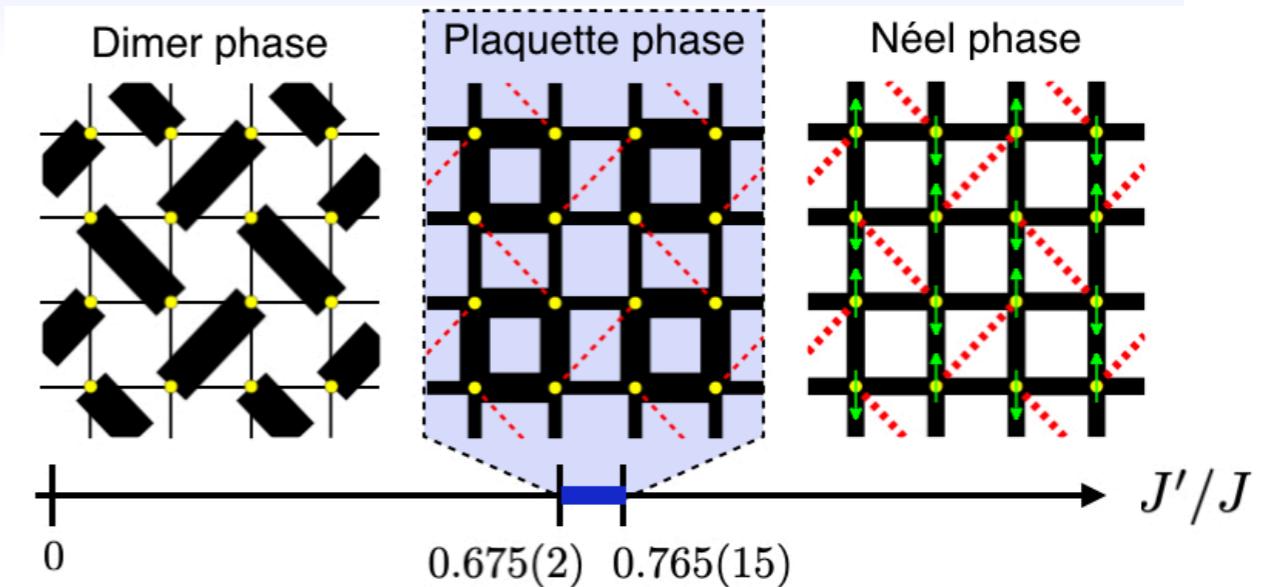
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- ✓ *Room for improvement:*  
larger unit cells, taking disorder averages, more accurate model, ...

# SrCu<sub>2</sub>(BO<sub>3</sub>)<sub>2</sub> under pressure

- ▶ Applying pressure: *change ratio of J'/J*

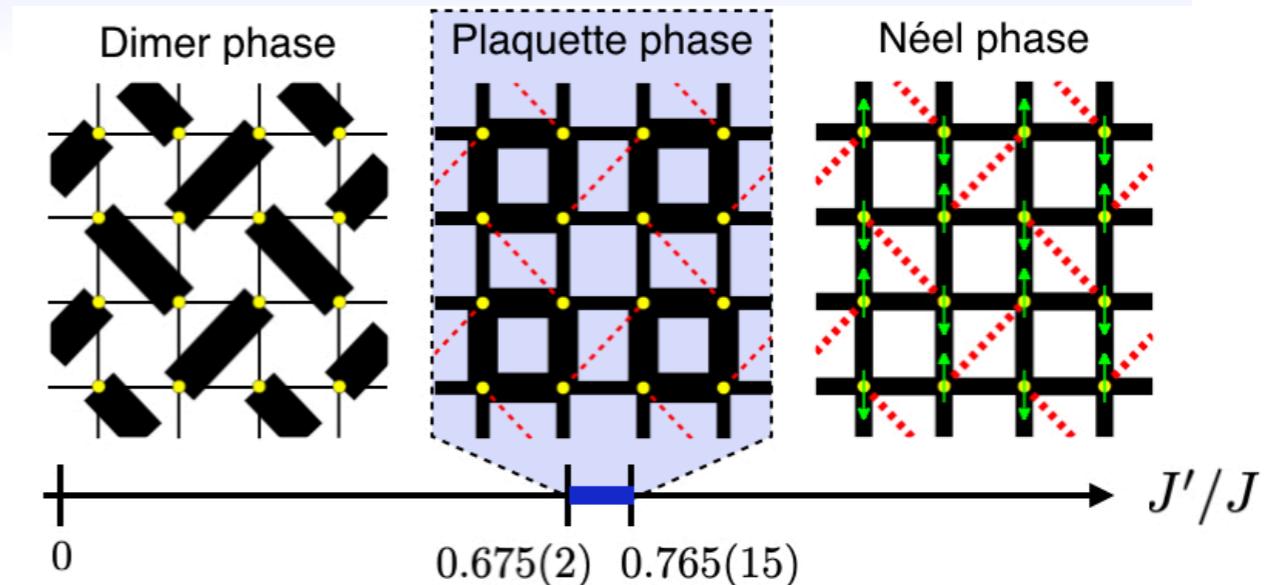
$$\hat{H} = J' \sum_{\langle i,j \rangle} S_i \cdot S_j + J \sum_{\langle\langle i,j \rangle\rangle_{\text{dimer}}} S_i \cdot S_j$$



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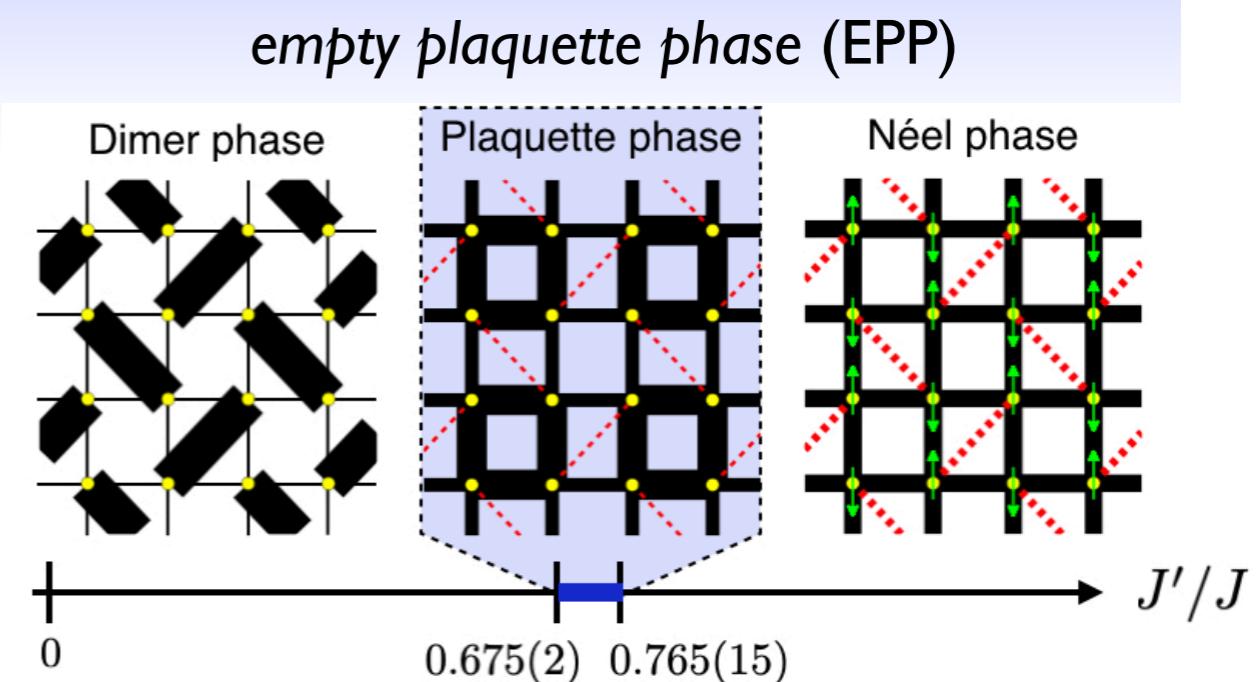
- ▶ Experiments:  
Phase transition into a gapped  
phase around  $\sim 1.7$  GPa

Waki, et al. J. Phys. Soc. Jpn. 76, 073710 (2007).  
Haravifard, et al. Nat. Commun. 7, 11956 (2016).  
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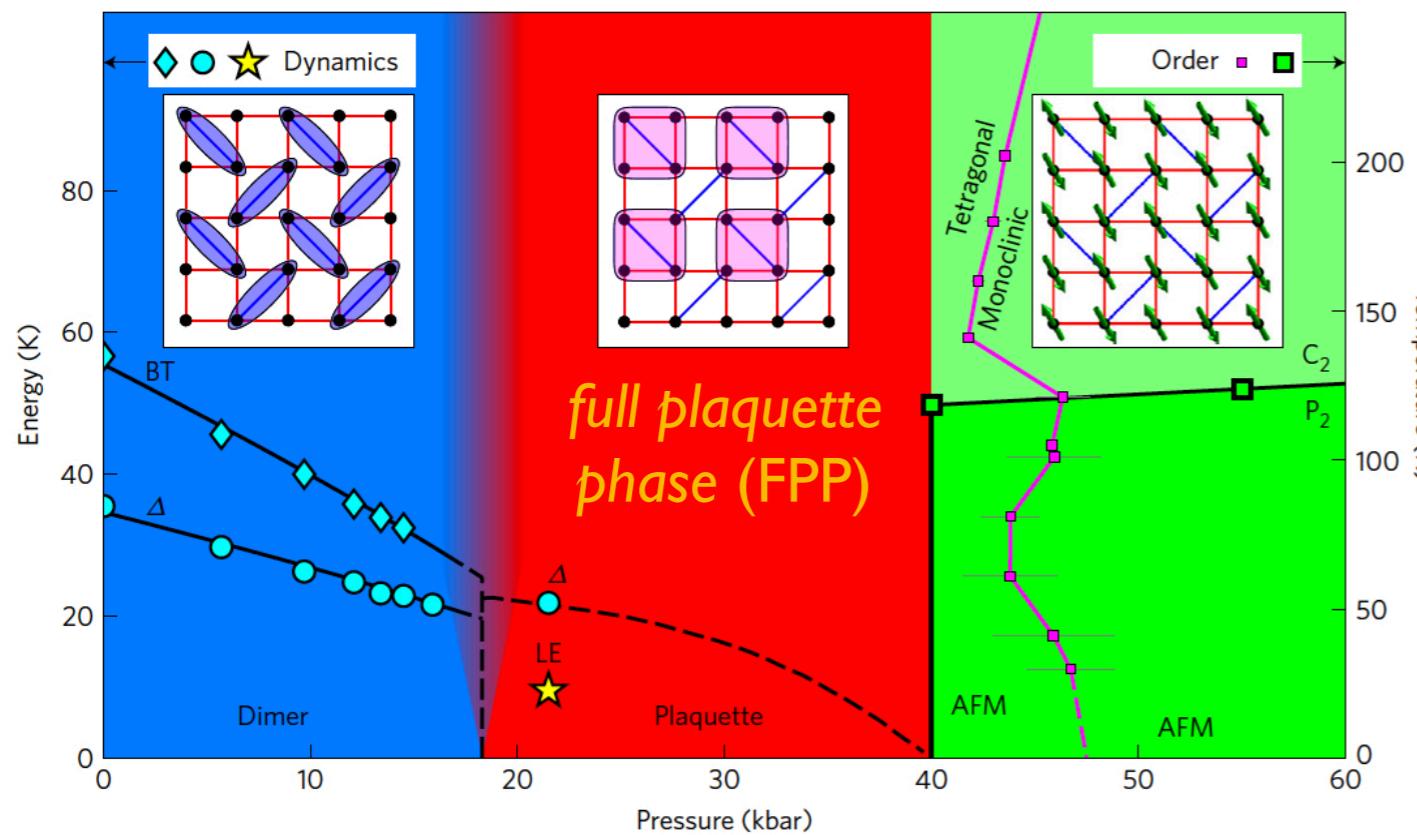
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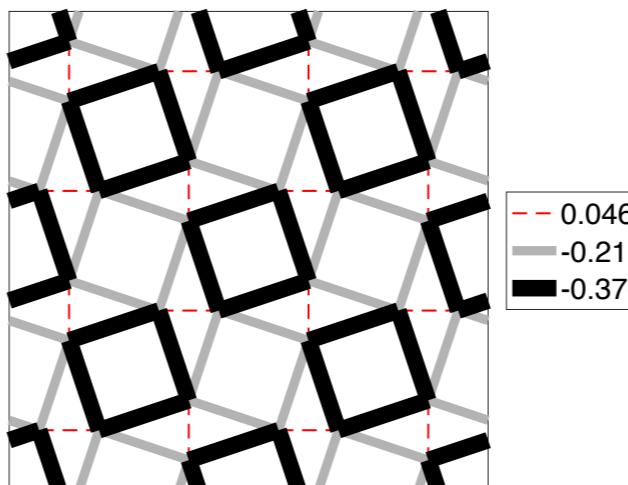
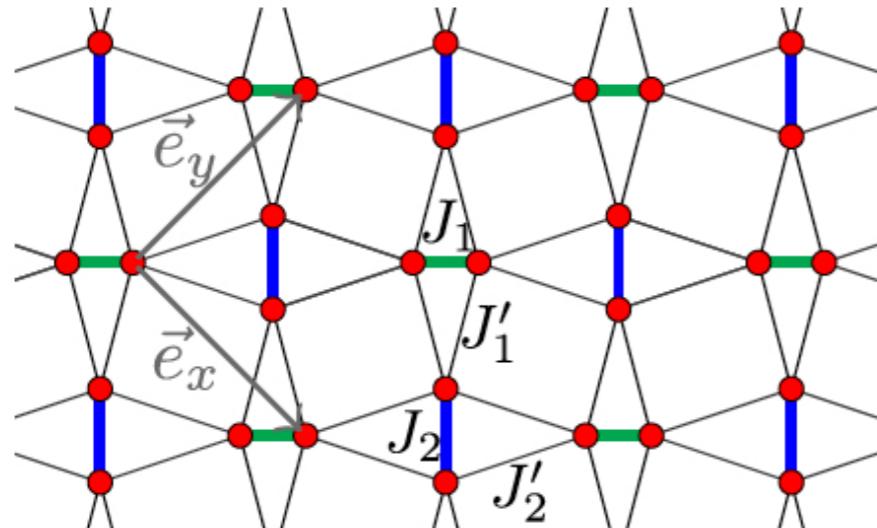
- ▶ However: data from inelastic neutron scattering experiments: full plaquette phase (FPP), not empty plaquette phase (EPP)

Zayed, et al., Nat. Phys. 13, 962 (2017).

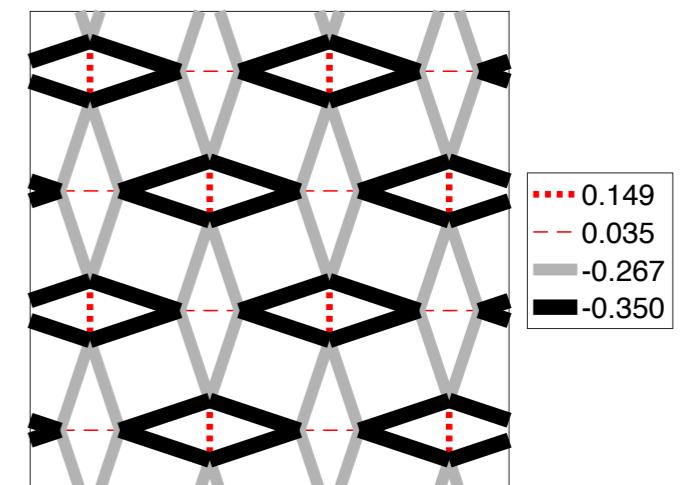
# SrCu<sub>2</sub>(BO<sub>3</sub>)<sub>2</sub> under pressure

Boos, Crone, Niesen, PC, Schmidt & Mila, PRB 100 (2019)

- Distorted Shastry-Sutherland model: competition between *EPP* and *FPP* phase



*EPP*

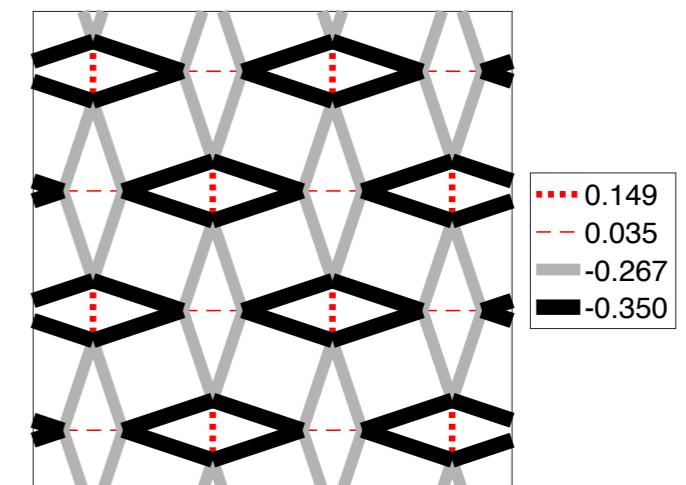
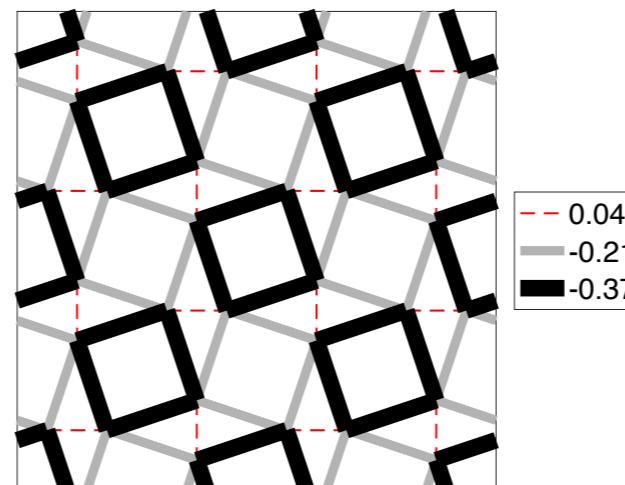
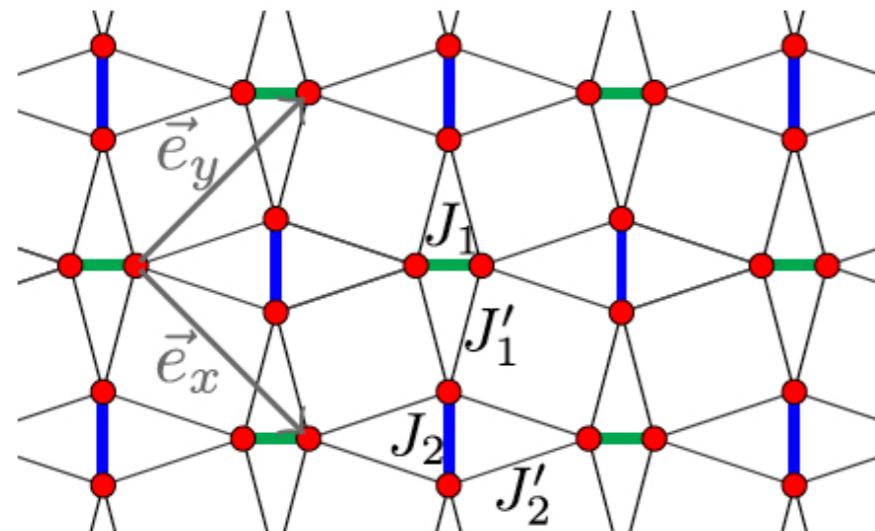


*vs* *FPP*

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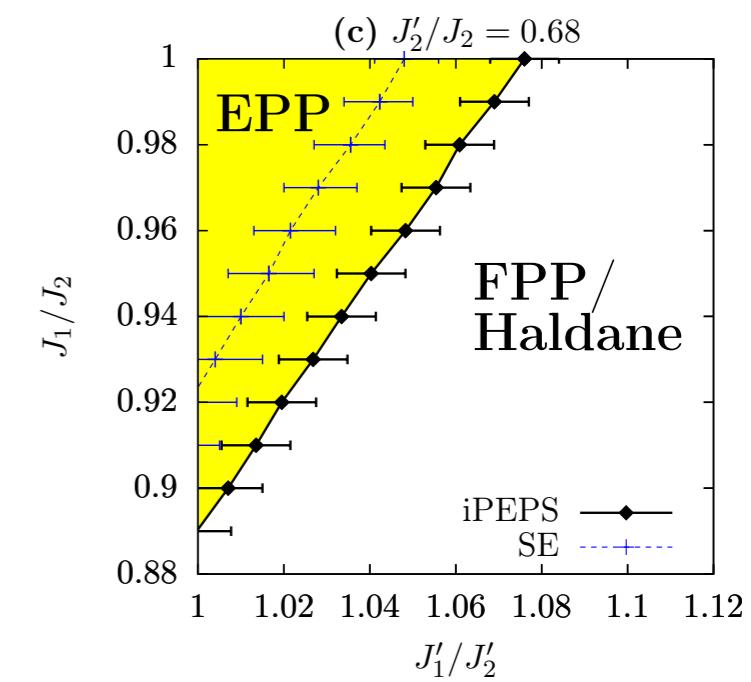
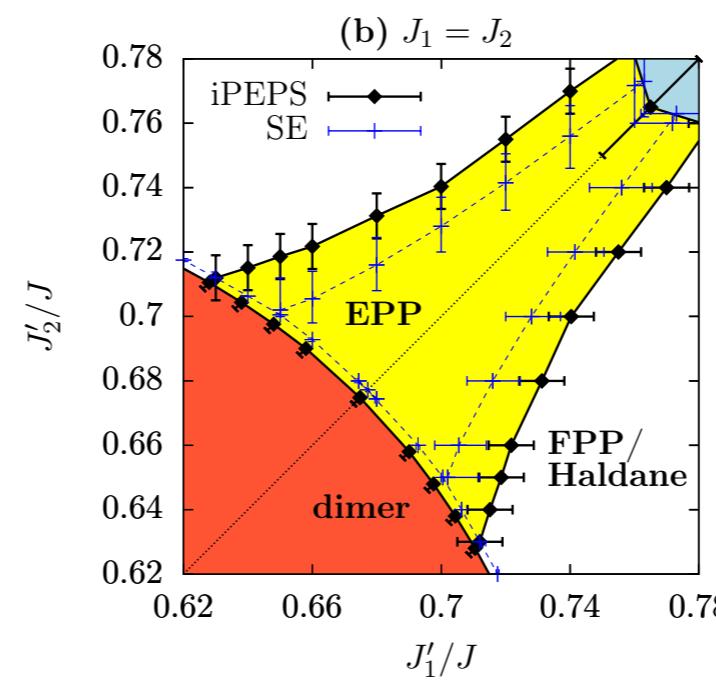
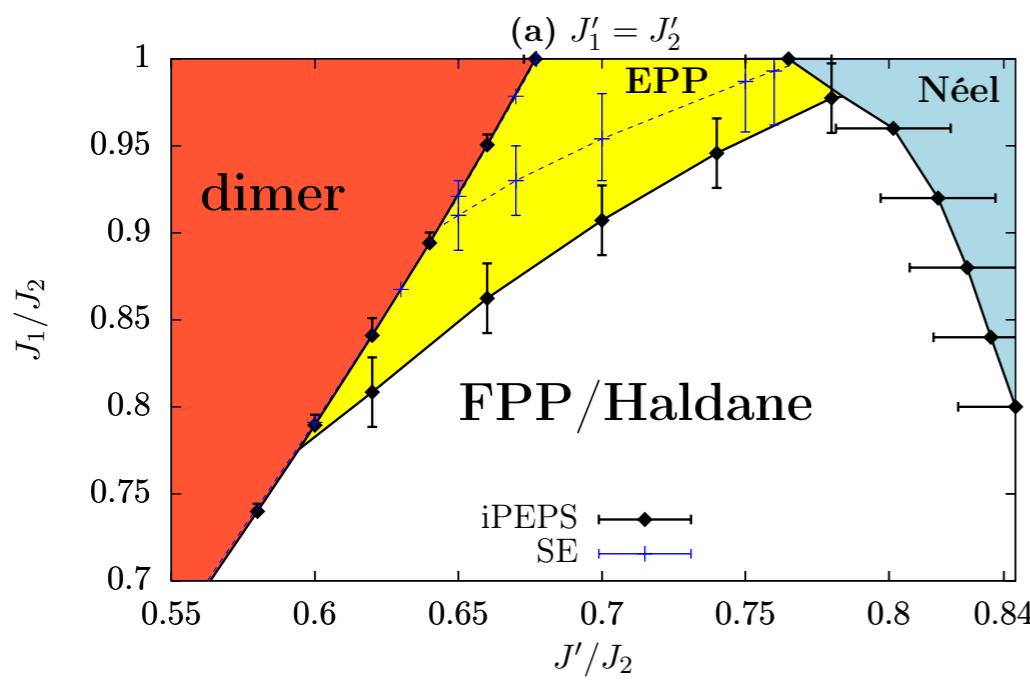
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*EPP*

*vs*

*FPP*

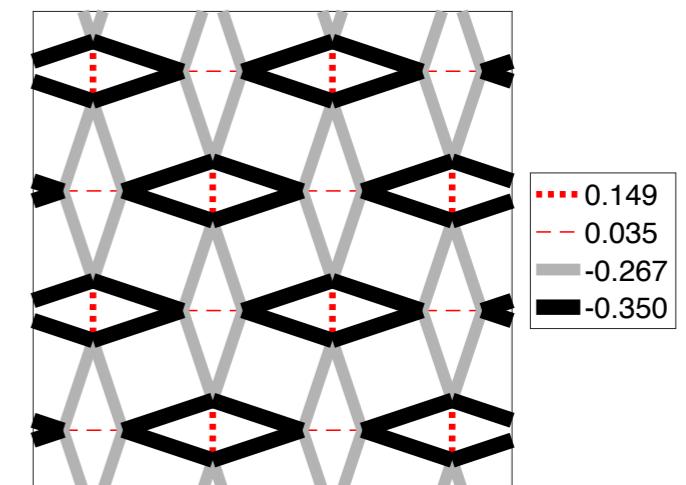
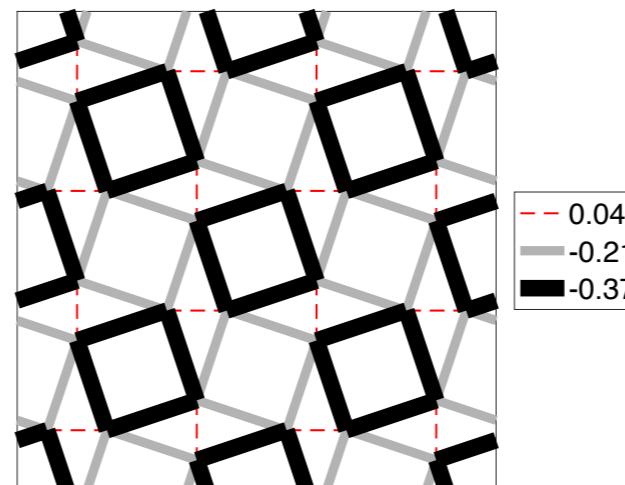
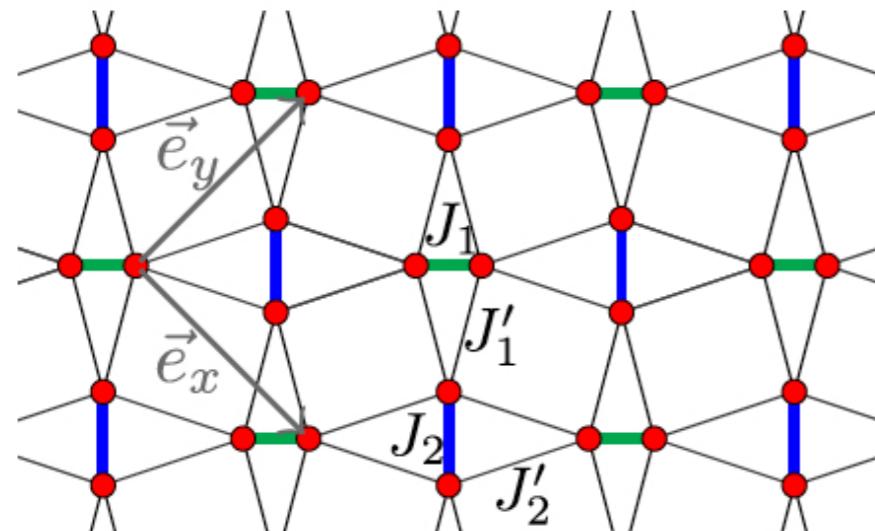


- Small deformation leads to FPP phase!

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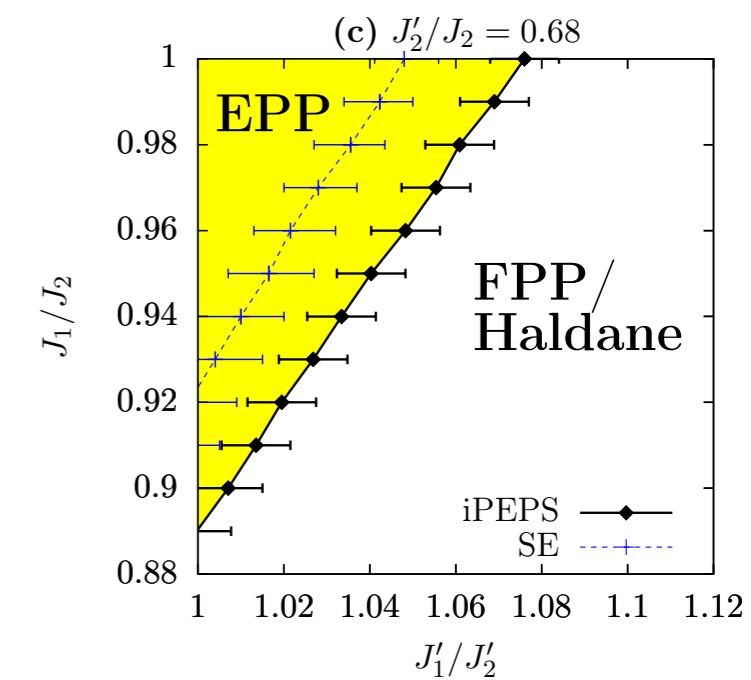
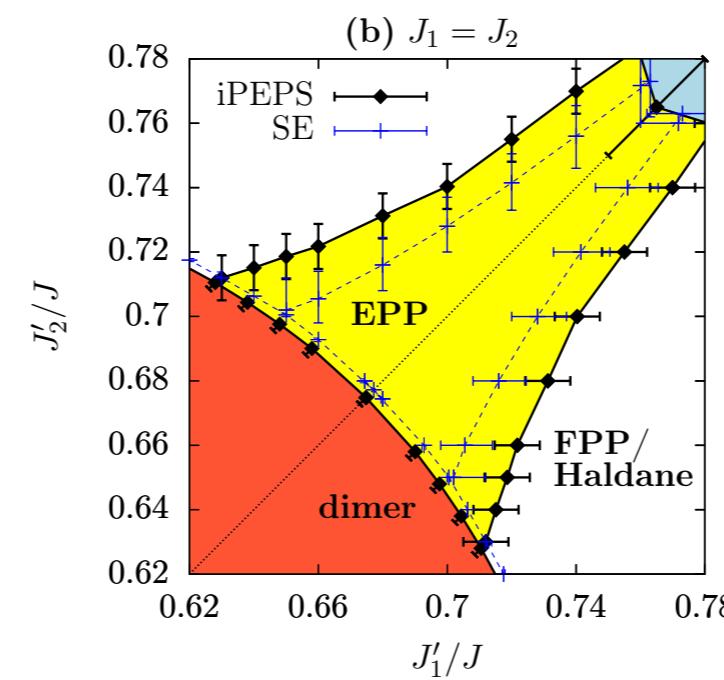
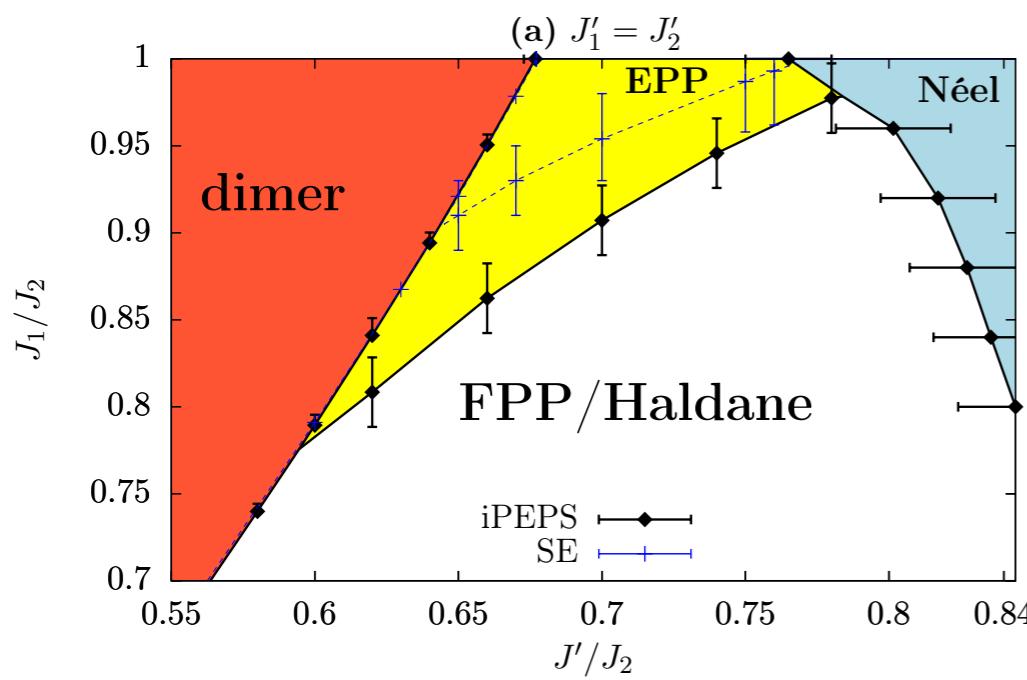
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EPP

vs

FPP



- Small deformation leads to FPP phase!

*But precise model still unclear...*

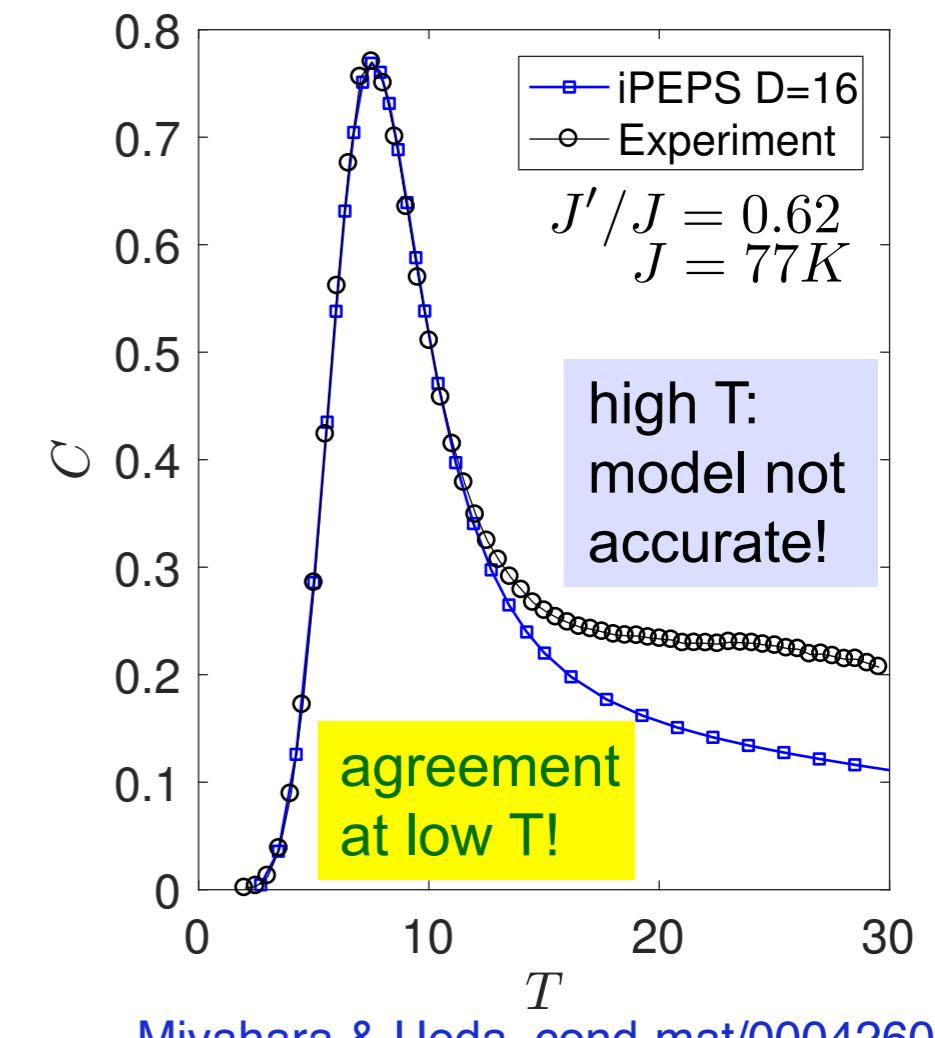
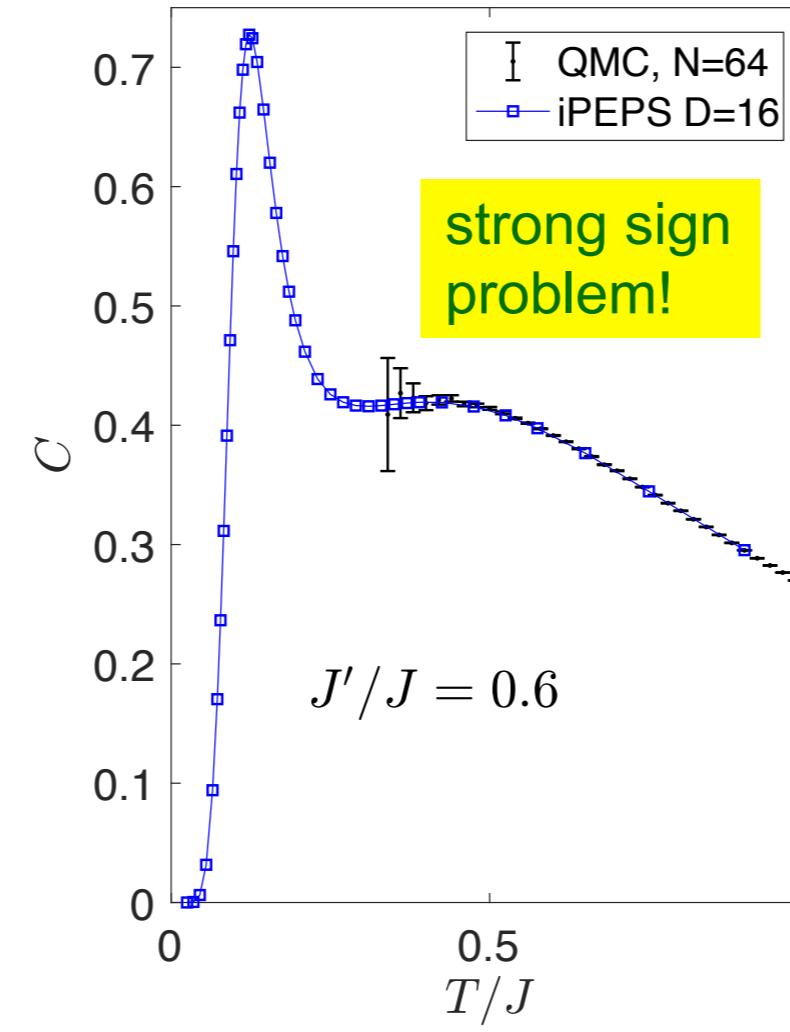
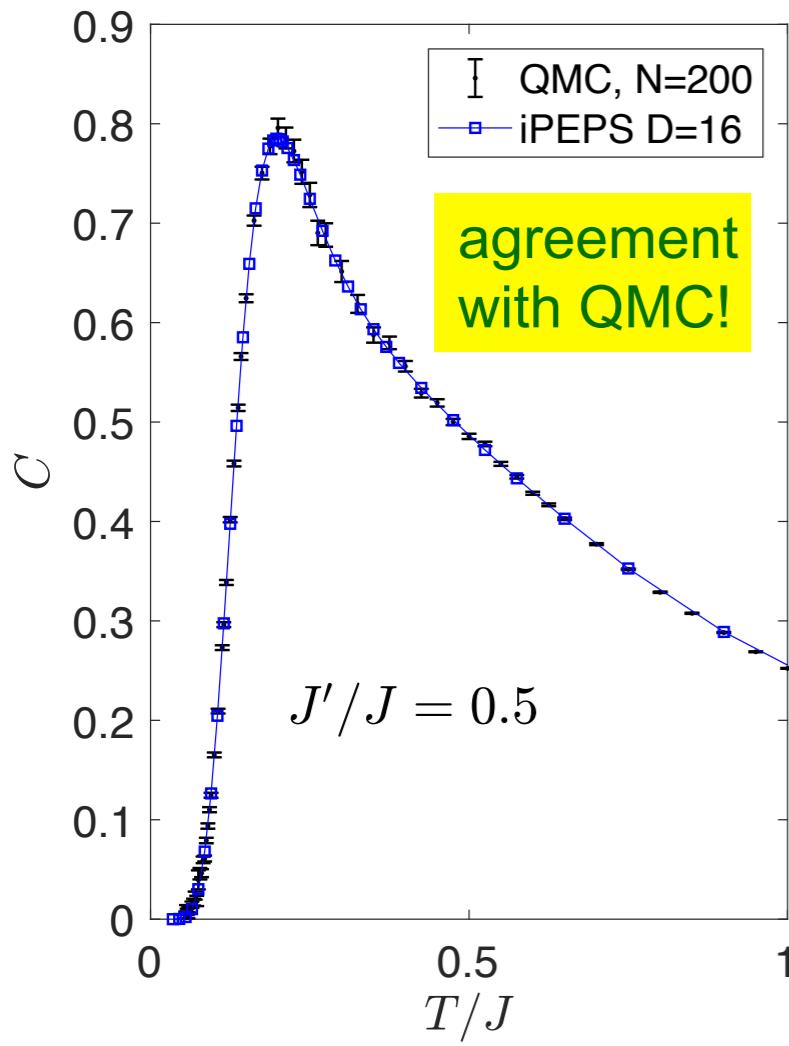
# Finite temperature simulations with iPEPS

- Methodological developments: P. Czarnik, J. Dziarmaga, PC, PRB 99 (2019)

[see also Li et al. PRL 106 (2011); Czarnik et al. PRB 86 (2012); Xie et al., PRB 86 (2012); Czarnik & Dziarmaga PRB 90 (2014); PRB 92 (2015); Czarnik et al. PRB 94 (2016), Dai et al PRB 95 (2017); Kshetrimayum, Rizzi, Eisert, Orus, PRL 122 (2019)]

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Wietek, PC, Wessel, Normand, Mila, and Honecker, PRR I (2019)



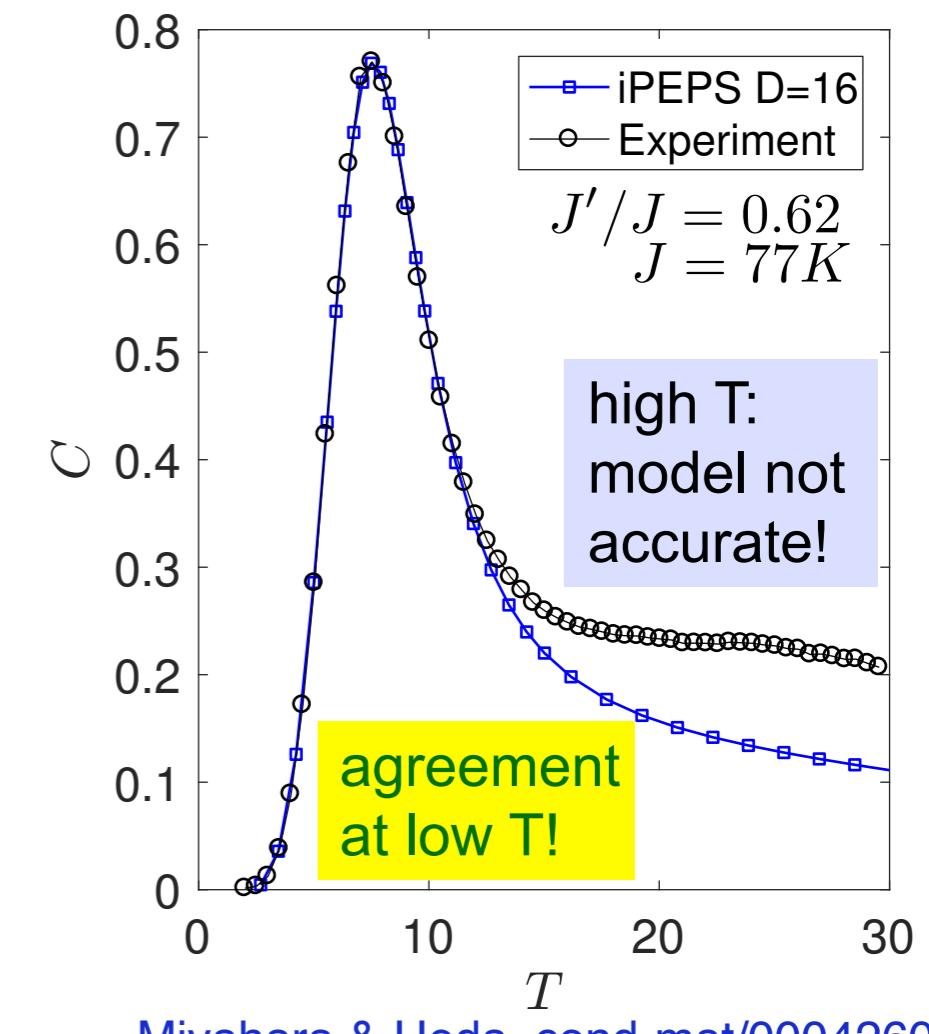
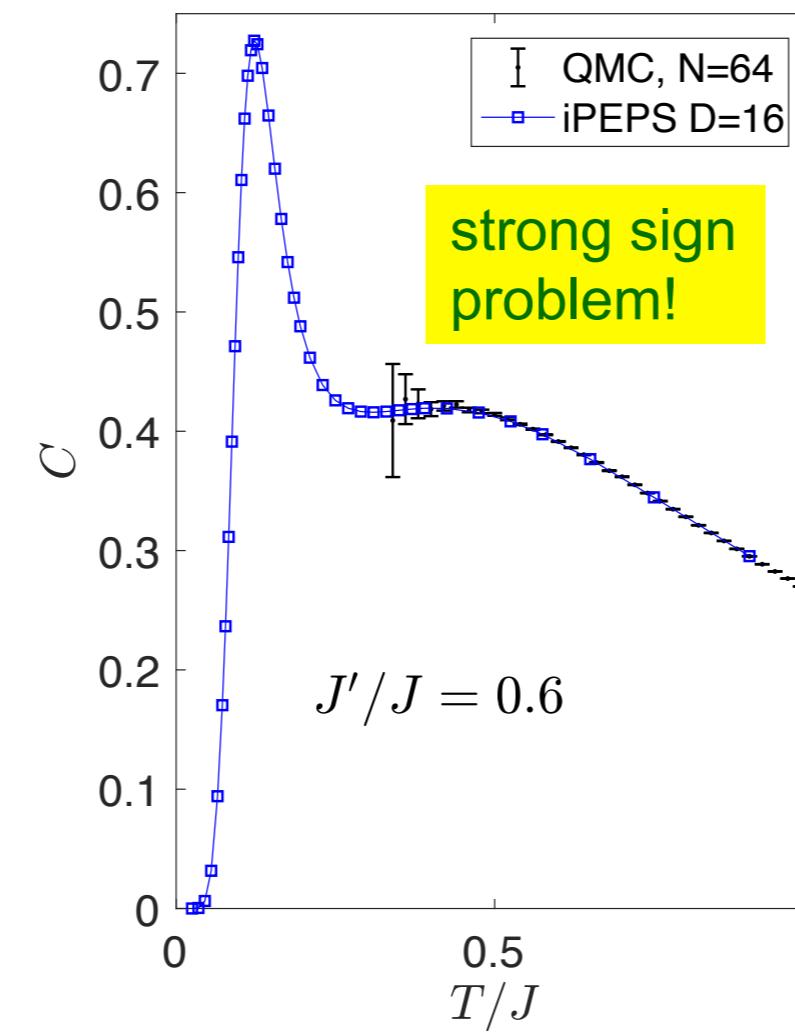
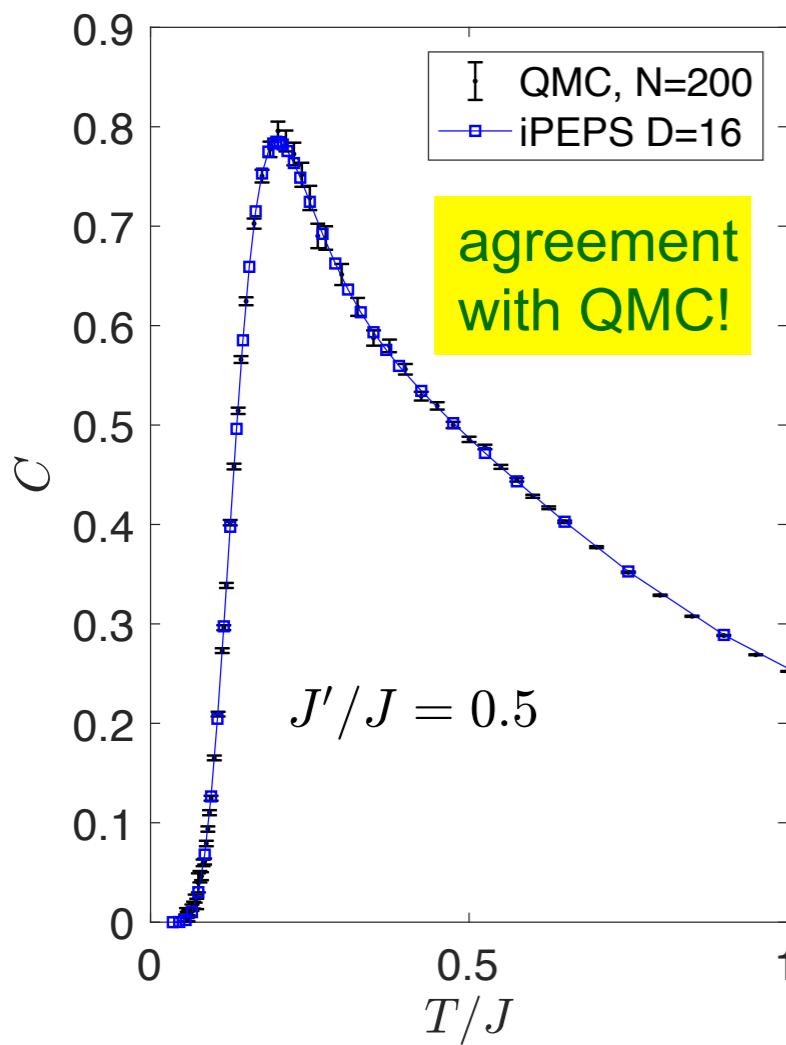
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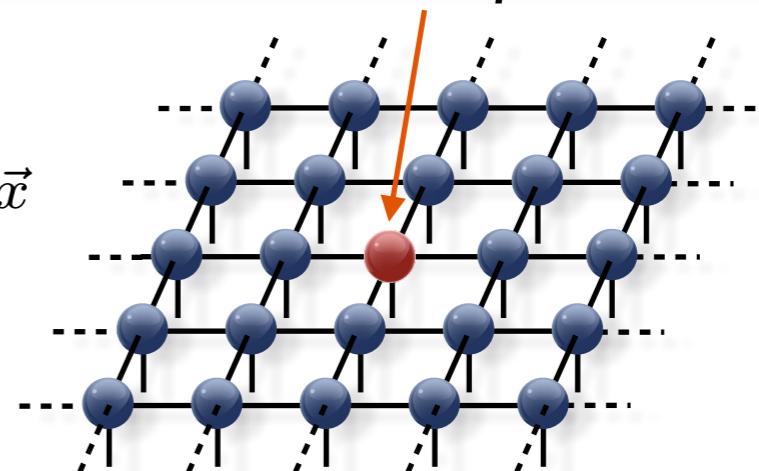


# iPEPS excitation ansatz

- Excitation on top of ground state with momentum  $k$

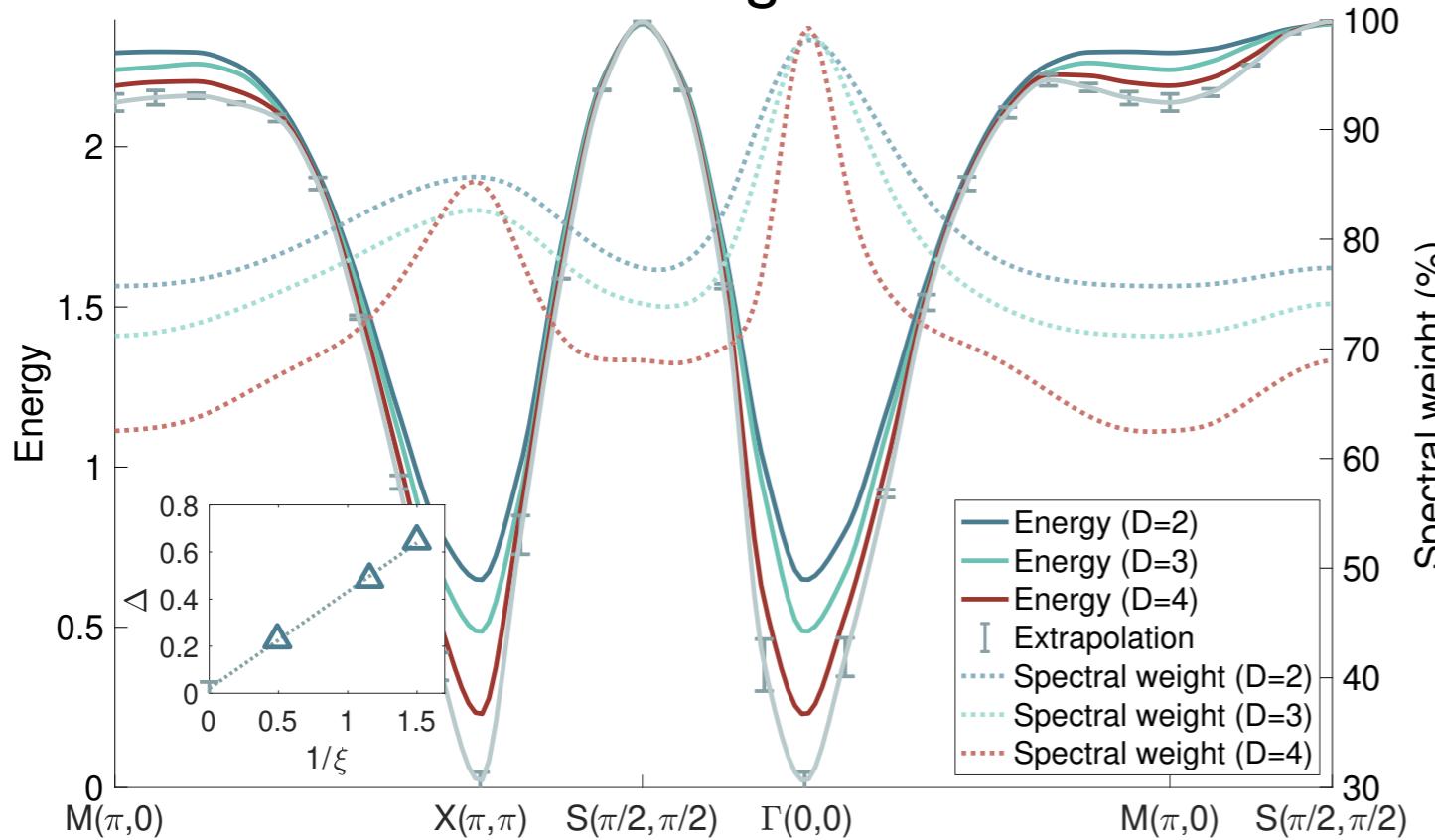
$$|\Phi_{\vec{k}}(B)\rangle \approx \sum_{\vec{x}} e^{i\vec{k}\vec{x}}$$

Tensor  $B$  at position  $\vec{x}$



Haegeman, et al, PRB 85 (2012); Haegeman, et al, PRL 111 (2013).  
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Benchmark: 2D Heisenberg model:

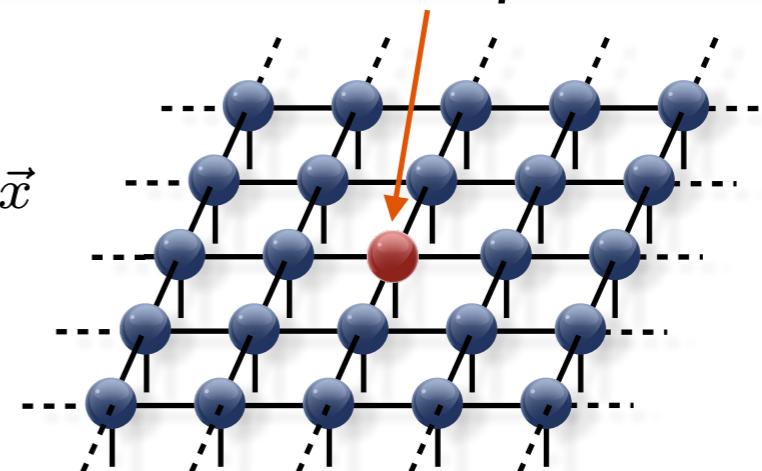


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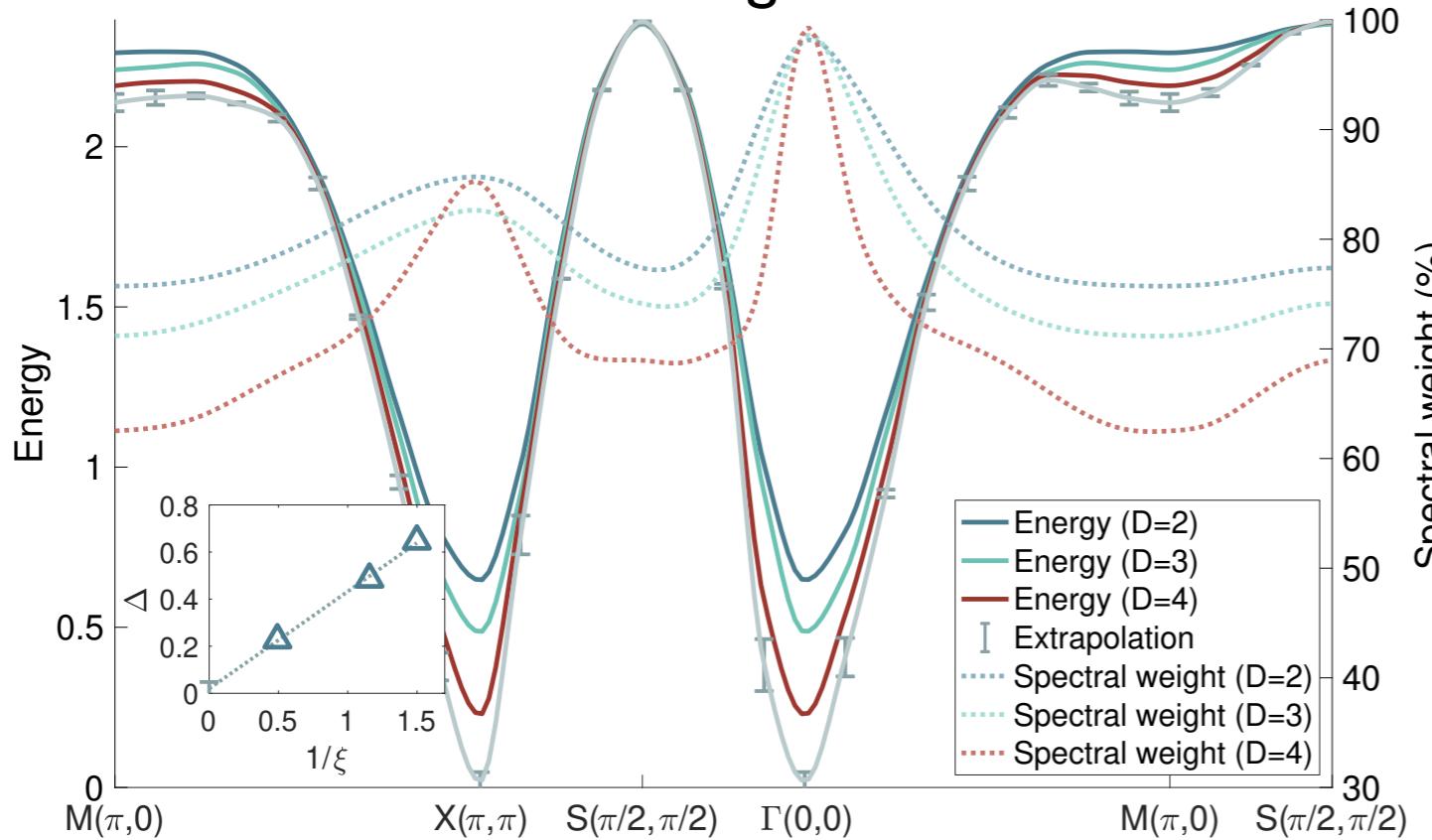
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Study of excitations in  
EPP / FPP phase  
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# Summary & outlook

- ✓ iPEPS: many new insights into the physics of  $\text{SrCu}_2(\text{BO}_3)_2$ 
  - ★ New understanding of the magnetization process at low magnetic fields
  - ★ Supersolid phases at high magnetic fields
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  - ★ Competition of plaquette phases in the extended Shastry-Sutherland model
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## Acknowledgements:

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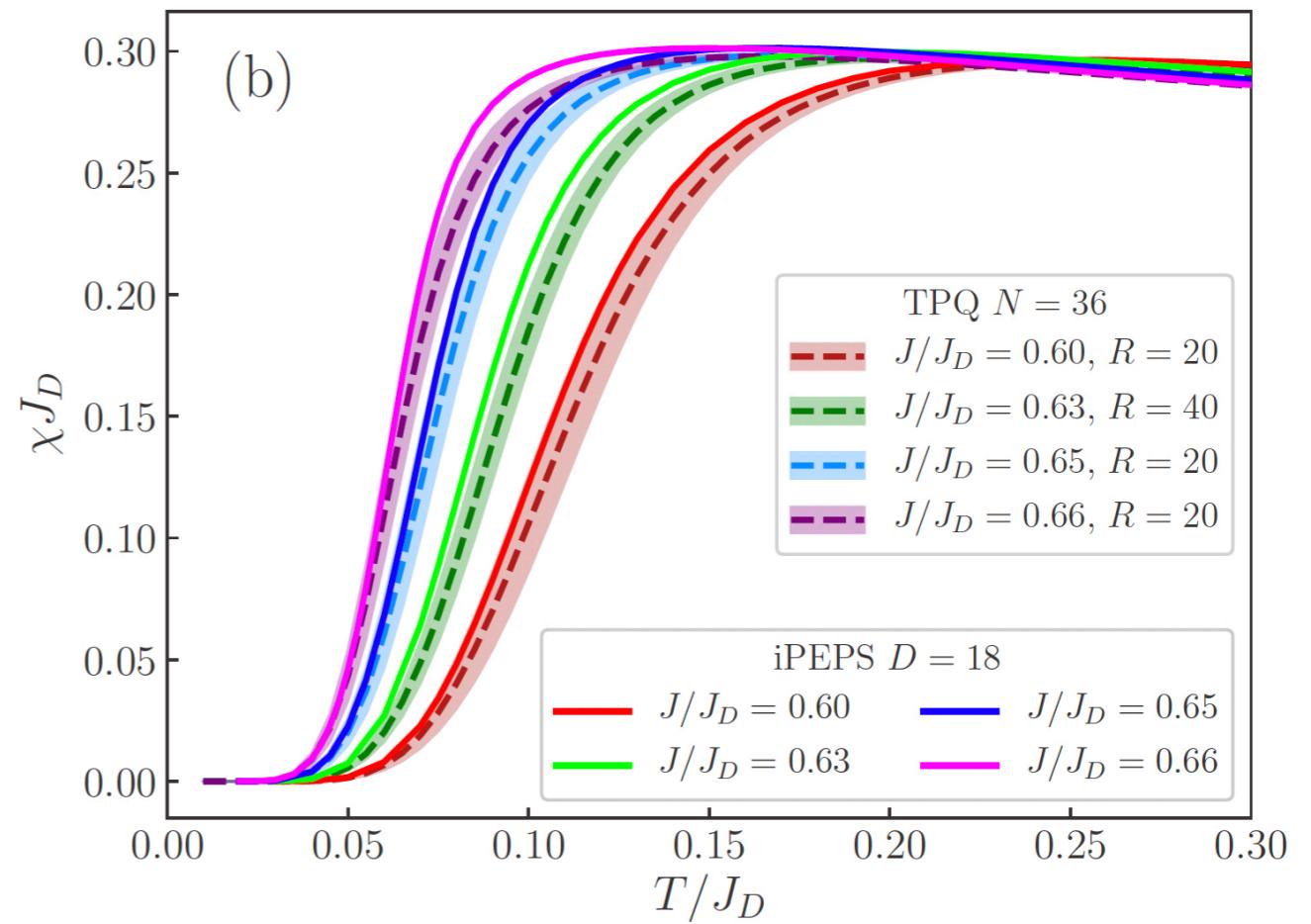
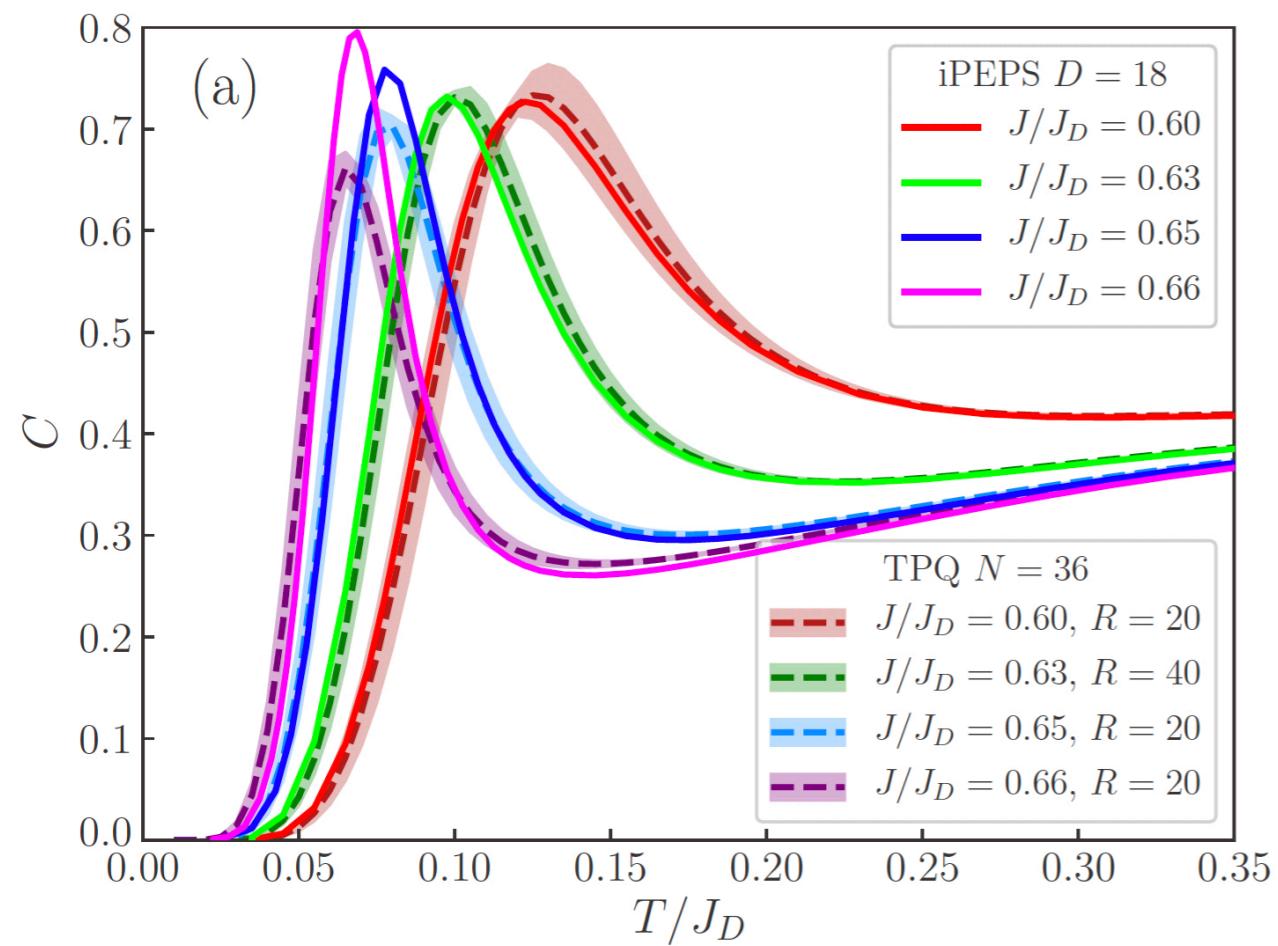
Thank you for your attention!

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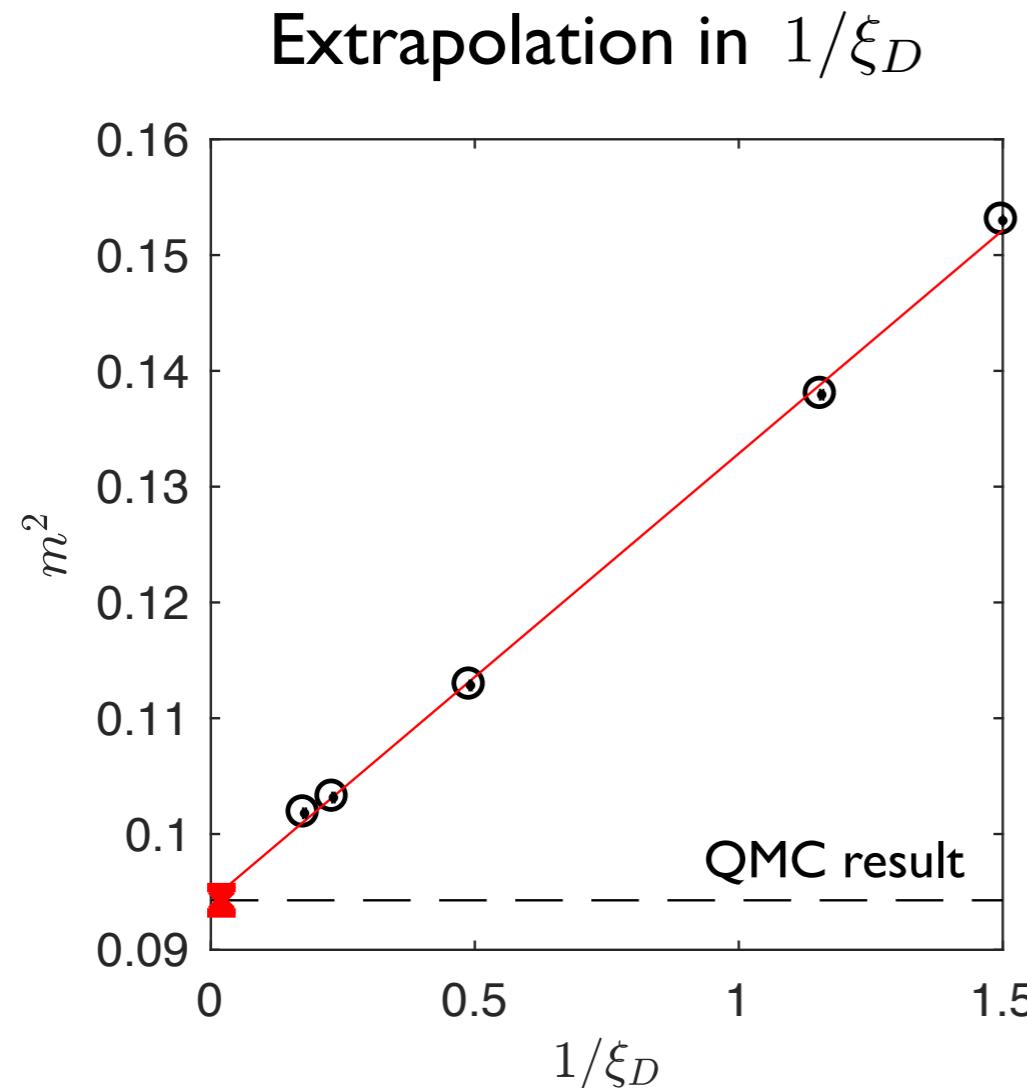






# Extrapolation of order parameter: 2D Heisenberg model

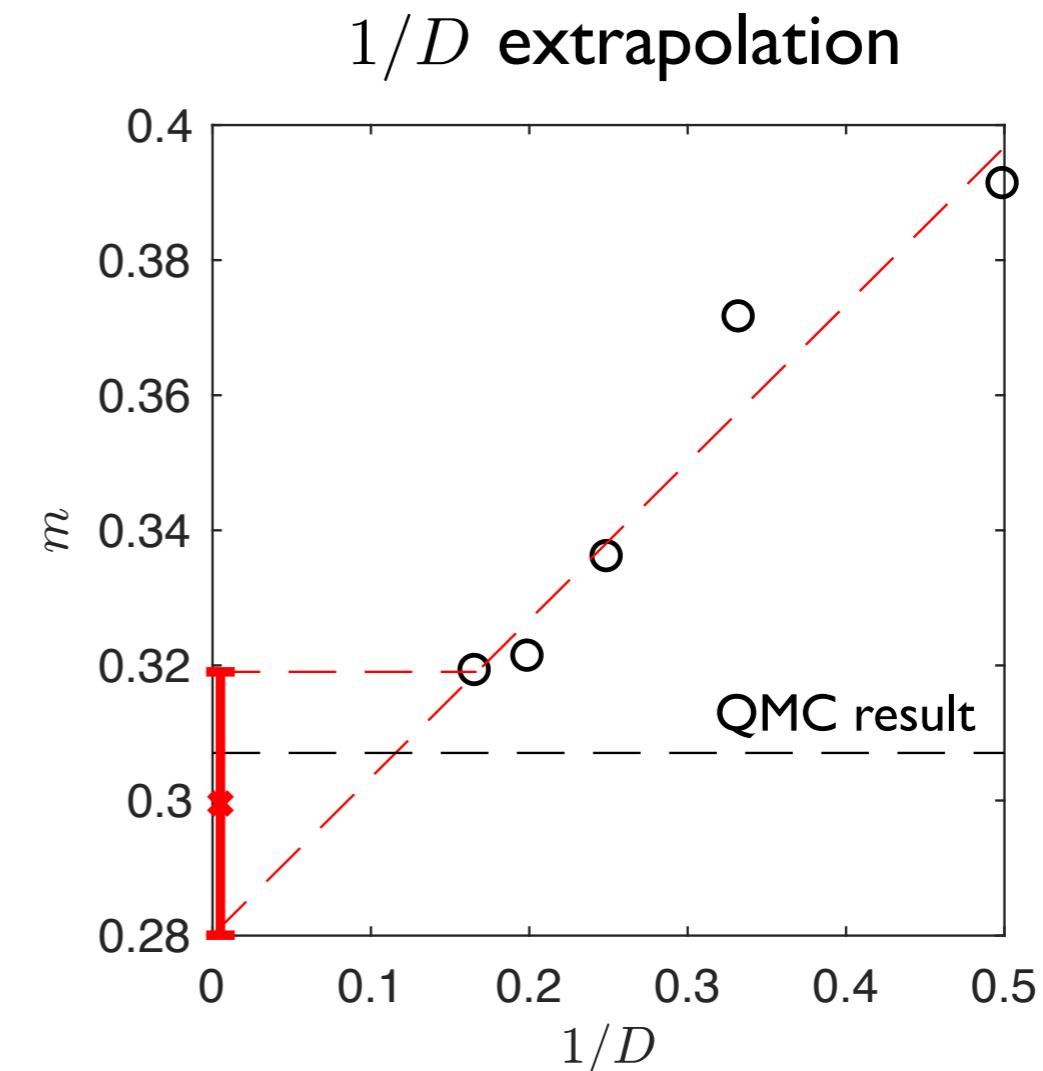
- Use FCL scaling to extrapolate the order parameter in gapless system



iPEPS:  $m = 0.307 \pm 0.002$

QMC:  $m = 0.30743(1)$

Sandvik & Evertz (2010)



Strong improvement  
compared to “naive”  
I/D extrapolation!

# Finite temperature simulations with iPEPS ( $J'/J=0.63$ )

