Activating many-body localization in solids by driving with light

Zala Lenarčič

University of California, Berkeley, USA

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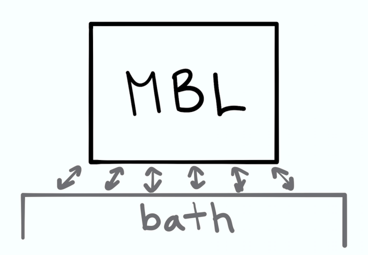
Can signatures of MBL be observed in solid state experiment?

Coupling to phonons = coupling to a thermalizing bath

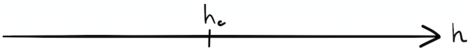


Many-body localization transition

- Many-body generalization of Anderson localization
- Disordered interacting systems

$$H_{xxzV} = \sum_j V_j S_j^z + J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y) + \Delta S_j^z S_{j+1}^z, \;\; V_j \in [-h, h]$$

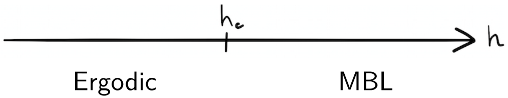
- In the presence of interactions exists critical disorder strength h_c



Ergodic MBL

- Transition even at infinite temperature

Many-body localization transition



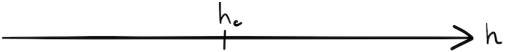
- **Ergodic**: a few conservation laws H, S^z
- **MBL**: Macroscopically many local conservation laws $[\tau_i^z, H] = 0$

$$\tau_i^z = S_i^z + \ldots,$$
$$H = \sum_j V_j S_j^z + J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y) + \Delta S_j^z S_{j+1}^z$$

  - τ_i^z: **localized in space** $\to$ no transport

    Vosk and Altman '13, Serbyn and Abanin '13, Ros et al. '15, ...

Many-body localization transition



Ergodic MBL

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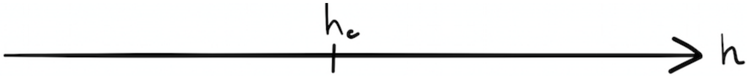
    Vosk and Altman '13, Serbyn and Abanin '13, Ros et al. '15, ...
  - **MBL lost when coupled to a bath.**

Numerical limitations in studying MBL transition

Entanglement entropy of eigenstates

- **Ergodic**: Volume-law
- **MBL**: Area-law

Transport properties



Diffusive Subdiffusive MBL

Limited to exact diagonalization

- using spectral properties, properties of eigenstates
  $\rightarrow$ effect of level spacing

Need for numerical approaches beyond exact-diagonalization:

- arXiv:1905.06345, arXiv:1911.04501, arXiv:1911.07882 (2019)

Experimental limitations in studying MBL transition

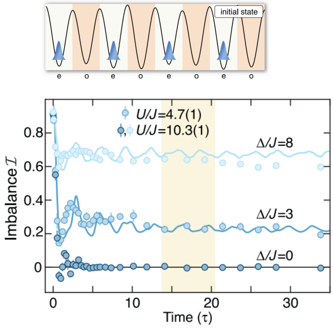
Cold atoms, Trapped ions:

Bloch, Monroe, Greiner group

- Closed systems only up to some time
- Can trap a small number of particles

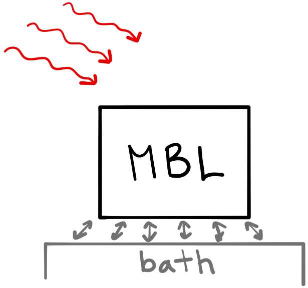
Solid state materials:

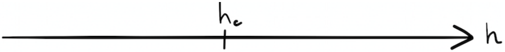
- Easily performed experiments in TD limit
- Inevitable coupling to phonons
  $\rightarrow$ break localization



Bloch group, Science 349.6250 (2015)

Distinct steady states





Ergodic	MBL
$\rightarrow$ only H	$\rightarrow$ more conservation laws C_i
$\rightarrow$ **thermal** steady-state	$\rightarrow$ **non-thermal** steady-state

Greenhouse: thermal state

- Temperature from rate Eq.

$$\partial_t \langle H \rangle = \epsilon_d \ (\text{gain}) + \epsilon_p \ (\text{loss}) = 0$$

- Driven setup, still can use temperature

$$\rho \sim e^{-\beta H}$$

- Nonlinear response

$$\langle H \rangle \sim \ \mathrm{tr}\left[H e^{-\beta H} \right], \quad \beta \approx \beta\left(\frac{\epsilon_d}{\epsilon_p}\right)$$

Generalized greenhouse: non-thermal state

- More conservation law, $[H, C_i] = 0$

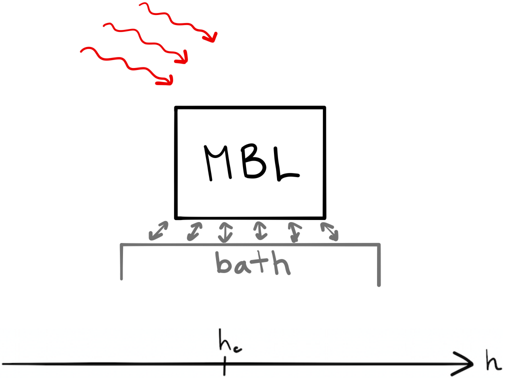
$$\partial_t \langle C_i \rangle = \epsilon_d \ (\text{gain}) + \epsilon_p \ (\text{loss}) = 0$$

- Need additional parameters λ_i,

$$\rho \sim e^{-\sum_j \lambda_j C_j}$$

- Nonlinear response in C_i

$$\langle C_i \rangle \sim \operatorname{tr}\left[C_i e^{-\sum_j \lambda_j C_j}\right], \quad \lambda_i \approx \lambda_i \left(\frac{\epsilon_d}{\epsilon_p}\right)$$



Ergodic

$\rightarrow$ only H

$\rightarrow$ **thermal** steady-state

MBL

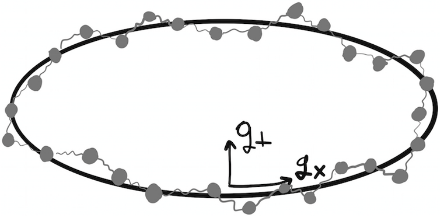
$\rightarrow$ more conservation laws C_i

$\rightarrow$ **non-thermal** steady-state

Look at **local temperatures** T_i

- **Dominant unitary evolution** under disordered H_f,

$$H_f = \sum_i \tilde{t}(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) + V_i\, n_i + U n_i n_{i+1}, \quad V_i \in [-h, h]$$



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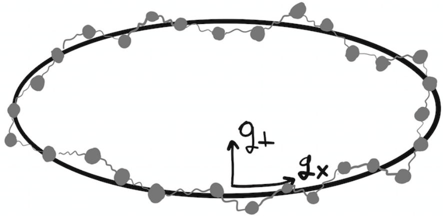
- **Coupling to 3D acoustic phonons at T_p**

$$H_{fp} = \epsilon_p \sum_{q_x} \int \frac{dq_\perp^2}{(2\pi)^2} (a_{\boldsymbol{q}} + a_{-\boldsymbol{q}}^\dagger) \frac{iq_x}{\sqrt{2\omega_{\boldsymbol{q}}}}\, H_{q_x}, \quad H_p = \sum_{\boldsymbol{q}} \omega_{\boldsymbol{q}} a_{\boldsymbol{q}}^\dagger a_{\boldsymbol{q}}$$

$$H_{q_x} = \frac{1}{\sqrt{N}} \sum_j \tilde{t} e^{iq_x j} (c_{j+1}^\dagger c_j + c_j^\dagger c_{j+1})$$



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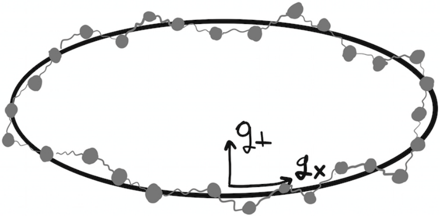
$$H_{q_x} = \frac{1}{\sqrt{N}} \sum_j \tilde{t} e^{iq_x j} (c_{j+1}^\dagger c_j + c_j^\dagger c_{j+1})$$

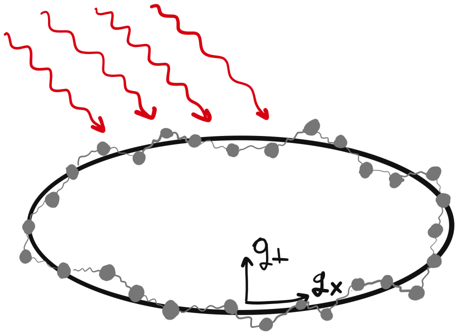
- **Thermal response**

$$\boxed{\langle \dot{C}_i \rangle = -\Gamma_i^{phonons}(\langle C_i \rangle - \langle C_i \rangle_{th})}$$



- **Driving** due to coupling to a **light bulb**

$$H_d = \epsilon_d A(t) \sum_i i\tilde{t}\,(c^\dagger_{i+1} c_i - c^\dagger_i c_{i+1}), \quad \langle A(t)A(t')\rangle = 2\pi\delta(t-t')$$



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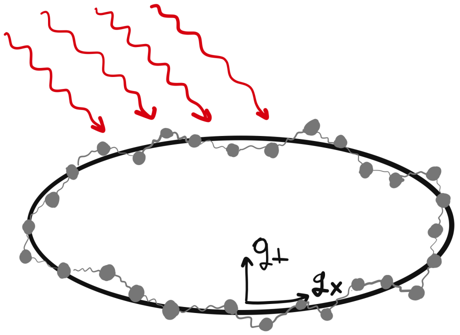
- **Decay of local conservation laws can be compensated!**

$$\boxed{\langle \dot{C}_i \rangle = \; -\Gamma_i^{phonons}(\langle C_i\rangle - \langle C_i\rangle_{th}) + \Gamma_i^{drive}(\langle C_i \rangle) \quad \forall i}$$



Steady state density matrix

Phase transition for infinitezimal coupling only

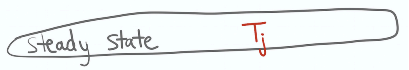
$$\rho_0 = \lim_{\epsilon_d,\epsilon_p \to 0} \lim_{t\to\infty} \rho(t) = \sum_n p_n |n\rangle\langle n|, \quad H_f|n\rangle = E_n|n\rangle$$

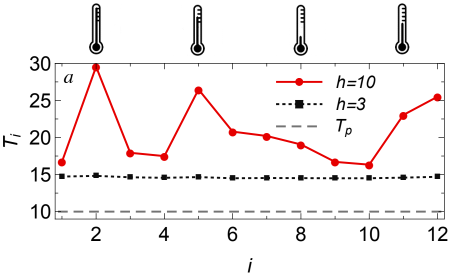
p_n from rate equation

$$\frac{d}{dt} p_n = \sum_m \Gamma_{nm} p_m - \Gamma_{mn} p_n, \quad \Gamma_{mn} = \Gamma^p_{mn} + \Gamma^d_{mn}$$

where $\Gamma^p_{mn} \sim \epsilon_p^2, \quad \Gamma^d_{mn} \sim \epsilon_d^2$, so the important parameter is $\epsilon_d^2/\epsilon_p^2$

Measure local temperatures

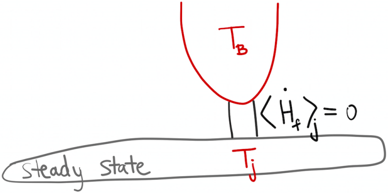




Order parameter:

$$\frac{\delta T}{\bar{T}} = \frac{\sqrt{\langle\langle \mathrm{Var}(T_i) \rangle\rangle}}{\langle\langle \mathbb{E}(T_i) \rangle\rangle}.$$

Local thermometer



- Theoretically:
  - local coupling to external thermal bosons,

$$H_j = \epsilon_I A_j (a + a^\dagger), \quad A_j = c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j$$

T_j: temperature of bosons at which $\langle \dot{H}_f \rangle_j = 0$

- **Experimentally: local Raman spectroscopy**
  - Compare Stokes and anti-Stokes peaks

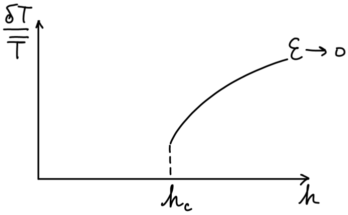
Results: temperature variations

Order parameter:

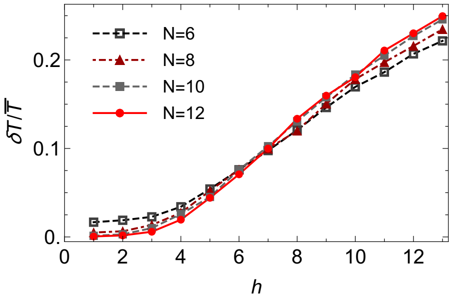
$$\frac{\delta T}{\bar{T}} = \frac{\sqrt{\langle\!\langle \mathrm{Var}(T_i) \rangle\!\rangle}}{\langle\!\langle \mathbb{E}(T_i) \rangle\!\rangle}.$$

In thermodynamic limit:

$$\lim_{\epsilon\to 0} \lim_{t\to\infty} \lim_{N\to\infty} \rho$$



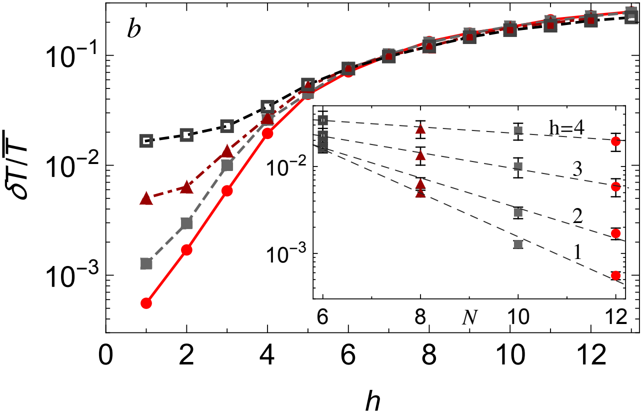
On finite system size:



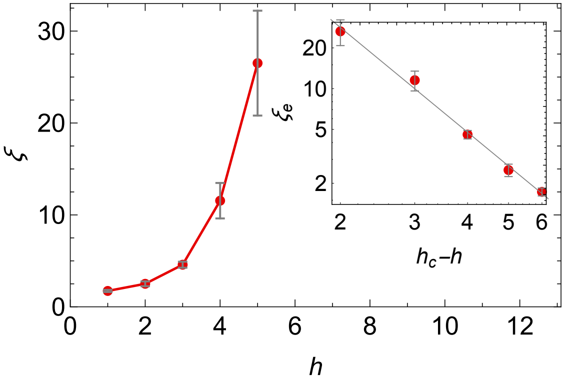
Results: temperature variations

In the **ergodic phase exponential** drop of δT,

$$\boxed{\delta T \sim e^{-L/\xi(h)}}$$



Correlation length

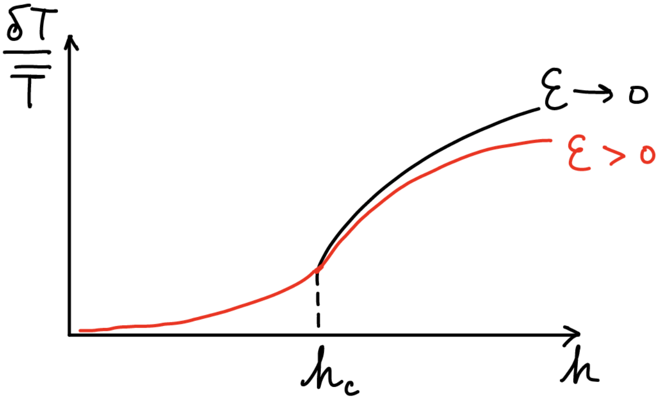


Critical exponent for $h_c = 7$ (from Luitz et al PRB 2015, $\nu \sim 1$):

$$\xi(h) \sim \frac{1}{(h_c - h)^{\nu}}, \quad \nu = 2.6$$

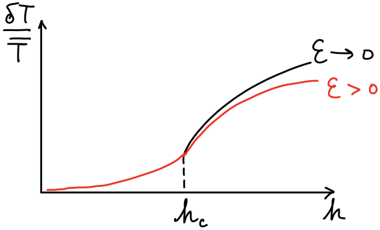
Lenarčič, Altman, Rosch, PRL 121, 267603 (2018)

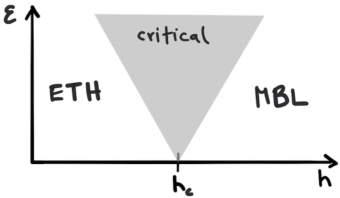
Finite coupling to environment and drive



- Phase transition $\to$ crossover
- Experimentally relevant
  - ϵ controlled by light power and phonon temperature

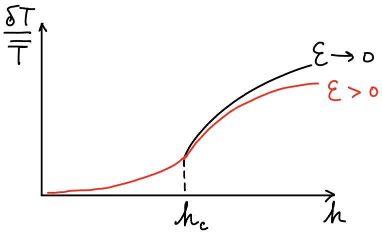
Finite coupling to environment and drive





- **Finite** $\epsilon \leftrightarrow$ **finite temperature** in $T = 0$ quantum phase transition
- Numerically: need full density matrix: $\rho_\infty = \rho_0 + \delta\rho(\epsilon)$
  - **Tensor-network approach**
  - bond dimension controlled by coupling to the environment
  - Lenarčič, Alberton, Rosch, Altman, arXiv:1910.01548 (2019)

Temperature fluctuation δT on the ergodic side



- Ergodic phase: $T(\boldsymbol{r}, \epsilon) = \bar{T} + \delta T(\boldsymbol{r}, \epsilon)$.
- **Hydrodynamic theory** based on approximate energy conservation.

$$\partial_t e - \nabla(\kappa(\boldsymbol{r})\nabla T(\boldsymbol{r})) = -\epsilon g_p(\boldsymbol{r})(T(\boldsymbol{r}) - T_p) + \epsilon g_d(\boldsymbol{r})$$

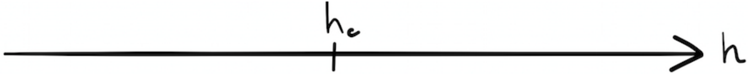
disorder sink source

- Fluctuations in d-dimension

$$\boxed{\delta T(\boldsymbol{r}) \sim \epsilon^{d/4}}$$

Temperature fluctuation on the ergodic side

Subdiffusive transport precedes MBL



Diffusive Subdiffusive MBL

- Fractional derivative: $\nabla^2 \to \nabla^z$, z-dynamical exponent
  - $z = 2$: diffusion
  - $z > 2$: subdiffusion
- Fluctuations in d-dimension

$$\boxed{\delta T(\boldsymbol{r}) \sim \epsilon^{d/2z}}$$

Subdiffusion - Griffiths regions

In 1D subdiffusion due to insulating regions: bottlenecks for transport

Subdiffusion - Griffiths regions

In 1D subdiffusion due to insulating regions: bottlenecks for transport

- Long insulating regions are exponentially rare ℓ,

$$P(\ell) = \frac{1}{\mathcal{N}} e^{-\ell/\xi}, \quad \xi \sim \text{correlation length}$$

- Conductance of insulating region

$$\Gamma = \Gamma_0 e^{-\ell/a},$$

- Distributions leads to different regimes

$$P(\Gamma) \sim \left(\frac{\Gamma}{\Gamma_0}\right)^{-1+\alpha}, \quad \alpha = \frac{a}{\xi}$$

$\alpha > 1$: diffusive, $0 < \alpha < 1$: subdiffusive

Higher dimensions

- Distribution of insulator lengths

$$P(\ell) \sim e^{-(\ell/\xi)^d}$$

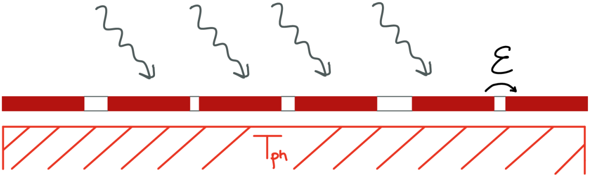
- Distribution of conductances

$$P(\Gamma) \sim \frac{1}{\Gamma} e^{-\alpha^d \left(\ln\left(\frac{\Gamma_0}{\Gamma}\right)\right)^d} \qquad \text{for} \;\; \Gamma \gg \epsilon \frac{\kappa_0}{a^{2-d}}.$$

Coupling to environment $\to$ ohmic term

- Conductances in the presence of coupling to environment

$$\Gamma = \epsilon \frac{\kappa_0}{\ell^{2-d}} + \Gamma_0 e^{-\ell/a}, \quad \alpha = \frac{a}{\xi}$$



- Energy continuity equation

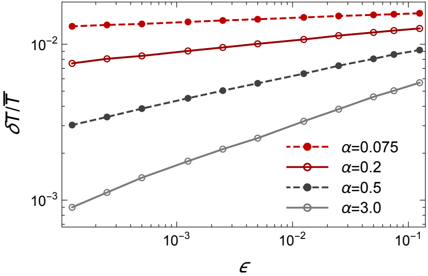
$$\partial_t e_i - \left(\Gamma_{i,i+1}(T_{i+1} - T_i) - \Gamma_{i-1,i}(T_i - T_{i-1})\right) = -\epsilon g_{p,i}(T_i - T_p) + \epsilon g_{d,i}$$

δT detects dynamical exponent z

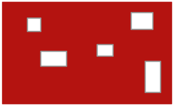
One dimension:

$$\delta T \sim \epsilon^{1/2z(\alpha)}$$

$$\text{diffusive } z = 2, \quad \alpha > 1$$
$$\text{subdiff. } z \sim \xi, \quad \alpha < 1$$

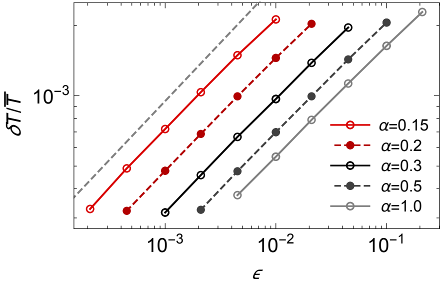


Two dimensions:

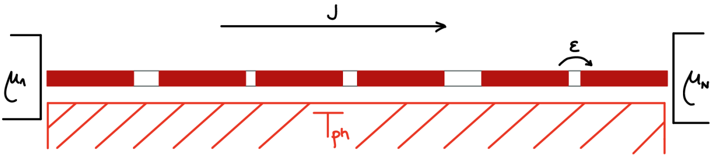


$$\delta T \sim \epsilon^{1/z}$$

always diffusive, $z = 2$.



Other experimental signatures



- Charge continuity equation

$$\left(\Gamma_{i,i+1}(\mu_{i+1}-\mu_i)-\Gamma_{i-1,i}(\mu_i-\mu_{i-1})\right)=0, \quad \Gamma=\epsilon\frac{\sigma_0}{l}+\Gamma_0 e^{-l/a}$$

- Average resistance for subdiffusive regime

$$\bar{\rho}(\epsilon)\sim\left(\frac{\epsilon}{\ln\epsilon^{-1}}\right)^{1/z}$$

- Convert $\bar{\rho}(\epsilon)$ to $\bar{\rho}(T_{ph})$

  $\rightarrow$ tune T_{ph} to detect critical behaviour of the MBL transition

  $\rightarrow$ get dinamical exponent $z(h)$

TEBD calculations for coupling to Markovian baths

Steady state

$$\hat{\mathcal{L}}\rho_\infty = -i[H, \rho_\infty] + \epsilon\hat{\mathcal{D}}\rho_\infty = 0$$

Dominant Hamiltonian part

$$H = \sum_i S_i \cdot S_{i+1} + h(\alpha_i^z S_i^z + \alpha_i^x S_i^x), \quad \alpha_i^{x,z} \in [-1, 1]$$

Phonons and driving replaced by coupling to Markovian baths, $\mathcal{D}$

- openness
- driving

Calculate steady state with TEBD evolution of $\hat{\mathcal{L}} \to \rho_{NESS}$

TEBD calculations for coupling to Markovian baths

Steady state

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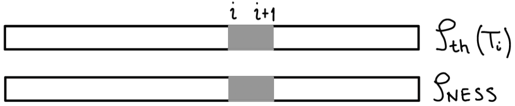
Coupling to Markovian baths

$$\hat{\mathcal{D}} = \sum_\alpha \hat{\mathcal{D}}^{(\alpha)}, \quad \hat{\mathcal{D}}^{(\alpha)}\rho = \sum_i L_i^{(\alpha)}\rho\,(L_i^{(\alpha)})^\dagger - \frac{1}{2}\{(L_i^{(\alpha)})^\dagger L_i^{(\alpha)},\rho\}$$

Lindblad operators

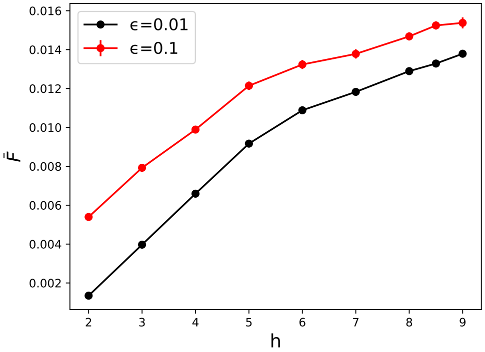
$$L_i^{(1a)} = S_i^+\left(\frac{1}{2}\mathbb{1}_{i+1} - S_{i+1}^z\right),\ L_i^{(1b)} = \left(\frac{1}{2}\mathbb{1}_i - S_i^z\right)S_{i+1}^+,$$
$$L_i^{(2a)} = S_i^-\left(\frac{1}{2}\mathbb{1}_{i+1} + S_{i+1}^z\right),\ L_i^{(2b)} = \left(\frac{1}{2}\mathbb{1}_i + S_i^z\right)S_{i+1}^-,$$
$$L_i^{(3)} = S_i^z$$

Condition for local temperature T_i



Minimize Frobenium norm with respect to T_i

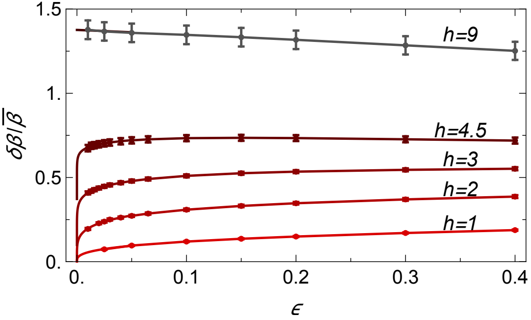
$$F[T_i] = \mathrm{tr}\Big[\Big(\rho_{\infty}^{(i,i+1)} - \rho_{\mathrm{th}}^{(i,i+1)}(T_i)\Big)^2\Big]$$



Detect the underlying transition from ϵ dependence

- Two regimes

$$\frac{\delta\beta}{\bar{\beta}}(\epsilon) \sim \begin{cases} \epsilon^{1/2z}, & \text{ergodic} : h < h_c, \\ \frac{\delta\beta}{\bar{\beta}}\Big|_{\epsilon\to 0} - b\,\epsilon + \mathcal{O}(\epsilon^2), & \text{MBL} : h \geq h_c \end{cases}$$

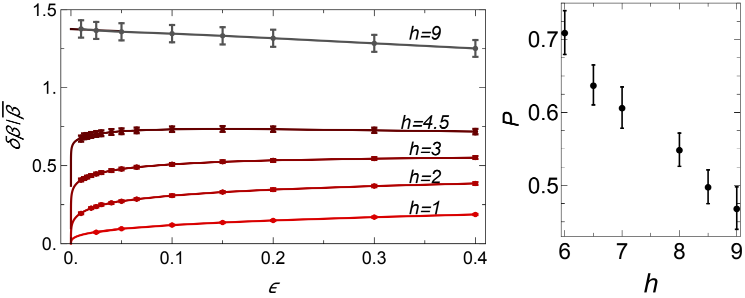


Critical disorder strength

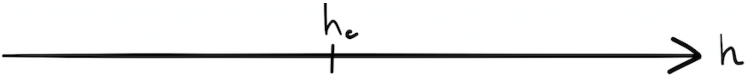
- New criterion for transition

$$\frac{\delta\beta}{\bar{\beta}}(\epsilon) \sim \begin{cases} \epsilon^{1/2z}, & h < h_c, \\ \frac{\delta\beta}{\bar{\beta}}\Big|_{\epsilon\to 0} - b\,\epsilon + \mathcal{O}(\epsilon^2), & h \geq h_c \end{cases}$$

- Critical disorder strength: $h_c = 8.75 \pm 0.5$



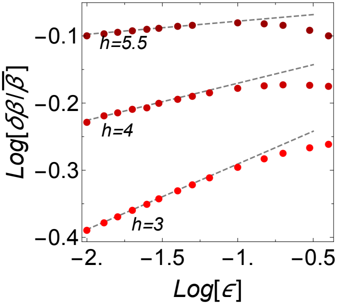
Divergence of dynamical exponent

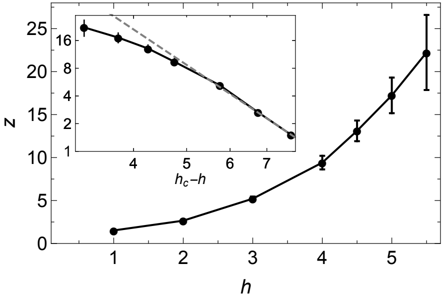


Diffusive Subdiffusive MBL

Direct bulk measurement of dynamical exponent $z(h)$

$$\frac{\delta\beta}{\bar{\beta}} \sim \epsilon^{1/2z} \quad \rightarrow \quad z \sim \xi \sim (h_c - h)^{-\nu}, \quad \nu = 4 \pm 0.9$$





Comparison with ED

Critical exponent ν

$$z \sim \xi \sim (h_c - h)^{-\nu}, \quad \nu = 4 \pm 0.9$$

- Obeys Harris bound: $\nu > 2/d$
- Exact diagonalization: $\nu \approx 1$
  Luitz, Laflorencie, Alet, PRB '16
- Close to RG results: $\nu \sim 3.3$
  Vosk et al, Potter at al, 2xPRX '15
- Cannot be distringuished from KT

$$z \sim e^{c/\sqrt{h_c - h}}$$

Goremykina, Vasseur, Serbyn, PRL '19
Dumitrescu et al, PRB '19

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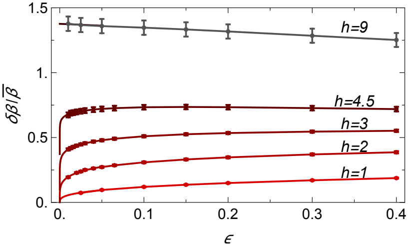
Dumitrescu et al, PRB '19

Critical disorder strength

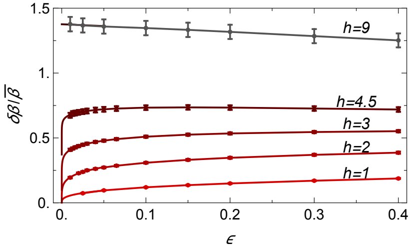
$$h_c >= 8.75$$

- Larger then ED, $h_c \in [2, 7]$
  Geraedts et el, New J. Phys. '17



Limitations of our method

- TEBD: convergence time ϵ^{-1}
- Variational approach?
- Perturbative approaches: is bond dimension really a problem?
  - MBL side: low bond dimension
  - ergodic side: $\rho = \frac{e^{-\beta H}}{Z} + \delta\rho(\epsilon)$



Useful non-thermal steady states?

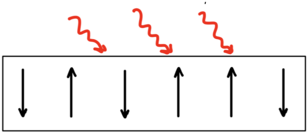
Useful non-thermal steady states?

- Pump into **heat and spin current** in Heisenberg spin chains
- Large current expectation value

$$\langle J_{h/s} \rangle \sim \operatorname{tr}\left[J_{h/s} e^{-\sum_j \lambda_j C_j} \right] \sim \mathcal{O}(1),$$

$$\lambda_{h/s}\left(\frac{\epsilon_d}{\epsilon_p}\right) \sim \mathcal{O}(1)$$

Nat. Commun. 8, 15767 (2017)



Pumping into heat and spin current in Heisenberg chain

- Heisenberg spin 1/2 chain in magnetic field:

$$H_0 = \sum_j \frac{J}{2}(S_j^+ S_{j+1}^- + S_j^- S_{j+1}^+) + \Delta S_j^z S_{j+1}^z + \mu B \sum_i S_i^z$$

- Coupling to **thermal phonon bath**
- **Weak periodic driving**

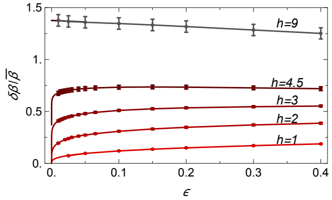
$$H_d = \epsilon_d \sum_i \left(\Delta J (-1)^i \; \mathbf{S}_i \cdot \mathbf{S}_{i+1} \; e^{i\omega t} + \mu \Delta B (-1)^i S_i^z \; e^{i\omega t + \phi}\right) + \text{H.c.}$$

- Need material that shows:
  - staggering of exchange coupling upon application of electric field
  - staggered g-tensor

Conclusions

- New, experimentally relevant order parameter $\frac{\delta T}{\bar{T}}$.
- Measurement of dynamical exponent $z(h)$ and critical disorder

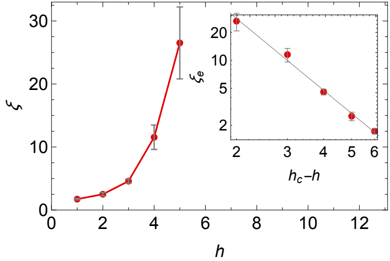
$$\frac{\delta T}{\bar{T}}(\epsilon) \sim \begin{cases} \epsilon^{1/2z}, & h < h_c, \\ \left.\frac{\delta T}{\bar{T}}\right|_{\epsilon\to 0} - b\,\epsilon + \mathcal{O}(\epsilon^2), & h \geq h_c \end{cases}$$

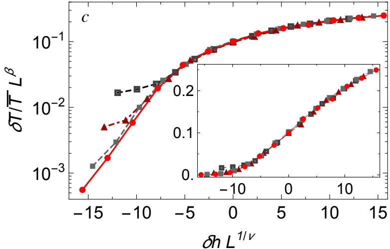


- New scalable numerical approach, which goes beyond limitation of ED

$\epsilon \to 0$: Lenarčič, Altman, Rosch, PRL 121, 267603 (2018),
$\epsilon > 0$: Lenarčič, Alberton, Rosch, Altman, arXiv:1910.01548 (2019)

Results: temperature variations





Critical exponent ν,

$$\xi_e(h) \sim \frac{1}{(h_c - h)^{\nu}}$$

for $h_c = 7$: $\nu = 2.6, \beta = 0.$

Pale&Huse, PRB '10, Luitz et al, PRB '15

For $h_c \in [5.5, 7.5]$:

$$\boxed{\nu \sim 2.5 \pm 0.5, \;\; \beta = 0.08 \pm 0.08}$$

Obeys Harris-Chayes bound:
$\nu > 2/d$

Harris '74, Chayes '89, Chandran et al '15

Exact diagonalization: $\nu \approx 1$

Luitz, Laflorencie, Alet, PRB '16

Close to RG results: $\nu \sim 3.3$