Activating many-body localization in solids by driving with light

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Can signatures of MBL be observed in solid state experiment?

Coupling to phonons = coupling to a thermalizing bath
Many-body localization transition

- Many-body generalization of Anderson localization
- Disordered interacting systems

\[ H_{\text{xxzV}} = \sum_j V_j S_j^z + J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y) + \Delta S_j^z S_{j+1}^z, \quad V_j \in [-h, h] \]

- In the presence of interactions exists critical disorder strength \( h_c \)

Transition even at infinite temperature
Many-body localization transition

- **Ergodic**: a few conservation laws $H, S^z$
- **MBL**: Macroscopically many local conservation laws $[\tau_i^z, H] = 0$

\[ \tau_i^z = S_i^z + \ldots, \]
\[ H = \sum_j V_j S_j^z + J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y) + \Delta S_j^z S_{j+1}^z \]

- $\tau_i^z$: localized in space $\rightarrow$ no transport
  - Vosk and Altman '13, Serbyn and Abanin '13, Ros et al. '15, ...
Many-body localization transition

- **Ergodic**: a few conservation laws $H, S^z$
- **MBL**: Macroscopically many local conservation laws $[\tau^z_i, H] = 0$

\[
\tau^z_i = S^z_i + \ldots,
\]

\[
H = \sum_j V_j S^z_j + J(S^x_j S^x_{j+1} + S^y_j S^y_{j+1}) + \Delta S^z_j S^z_{j+1}
\]

- $\tau^z_i$: **localized in space** $\rightarrow$ no transport
  - Vosk and Altman '13, Serbyn and Abanin '13, Ros et al. '15, ...
- **MBL lost when coupled to a bath.**
Numerical limitations in studying MBL transition

Entanglement entropy of eigenstates

- **Ergodic**: Volume-law
- **MBL**: Area-law

Transport properties

<table>
<thead>
<tr>
<th>Diffusive</th>
<th>Subdiffusive</th>
<th>MBL</th>
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Limited to exact diagonalization

- using spectral properties, properties of eigenstates
  → effect of level spacing

Need for numerical approaches beyond exact-diagonalization:

Experimental limitations in studying MBL transition

Cold atoms, Trapped ions:

Bloch, Monroe, Greiner group

- Closed systems only up to some time
- Can trap a small number of particles

Solid state materials:

- Easily performed experiments in TD limit
- Inevitable coupling to phonons → break localization
Distinct steady states

Ergodic
→ only H
→ \textbf{thermal} steady-state

MBL
→ more conservation laws $C_i$
→ \textbf{non-thermal} steady-state
Greenhouse: thermal state

- Temperature from rate Eq.
  \[ \partial_t \langle H \rangle = \epsilon_d \text{ (gain)} + \epsilon_p \text{ (loss)} = 0 \]

- Driven setup, still can use temperature
  \[ \rho \sim e^{-\beta H} \]

- Nonlinear response
  \[ \langle H \rangle \sim \text{tr} [H e^{-\beta H}], \quad \beta \approx \beta \left( \frac{\epsilon_d}{\epsilon_p} \right) \]
Generalized greenhouse: non-thermal state

- More conservation law, $[H, C_i] = 0$
  \[ \partial_t \langle C_i \rangle = \epsilon_d \text{ (gain)} + \epsilon_p \text{ (loss)} = 0 \]

- Need additional parameters $\lambda_i$,
  \[ \rho \sim e^{-\sum_j \lambda_j C_j} \]

- Nonlinear response in $C_i$
  \[ \langle C_i \rangle \sim \text{tr} \left[ C_i e^{-\sum_j \lambda_j C_j} \right], \quad \lambda_i \approx \lambda_i \left( \frac{\epsilon_d}{\epsilon_p} \right) \]
Ergodic $\rightarrow$ only H $\rightarrow$ thermal steady-state

MBL $\rightarrow$ more conservation laws $C_i$ $\rightarrow$ non-thermal steady-state

Look at local temperatures $T_i$
Dominant unitary evolution under disordered $H_f$,

$$H_f = \sum_i \tilde{t}(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) + V_i n_i + Un_i n_{i+1}, \quad V_i \in [-h, h]$$
• **Dominant unitary evolution** under disordered $H_f$,

$$H_f = \sum_i \tilde{t}(c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) + V_i n_i + Un_i n_{i+1}, \quad V_i \in [-h, h]$$

• **Coupling to 3D acoustic phonons at** $T_p$

$$H_{fp} = \epsilon_p \sum_{q_x} \int \frac{dq^2_\perp}{(2\pi)^2} \left( a_q + a_{-q}^\dagger \right) \frac{iq_x}{\sqrt{2\omega_q}} H_{q_x}, \quad H_p = \sum_q \omega_q a_q^\dagger a_q$$

$$H_{q_x} = \frac{1}{\sqrt{N}} \sum_j \tilde{t} e^{iq_x j} (c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j)$$
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• **Thermal response**

$$\langle \dot{C}_i \rangle = - \Gamma_i^{\text{phonons}} (\langle C_i \rangle - \langle C_i \rangle_{\text{th}})$$
Driving due to coupling to a light bulb

\[ H_d = \epsilon_d A(t) \sum_i \tilde{\tau} (c^\dagger_{i+1} c_i - c^\dagger_i c_{i+1}), \quad \langle A(t)A(t') \rangle = 2\pi \delta(t - t') \]
• **Driving** due to coupling to a **light bulb**

\[ H_d = \epsilon_d A(t) \sum_i \tilde{t} (c_{i+1}^{\dagger} c_i - c_i^{\dagger} c_{i+1}), \quad \langle A(t)A(t') \rangle = 2\pi \delta(t - t') \]

• **Decay of local conservation laws can be compensated!**

\[
\langle \dot{C}_i \rangle = -\Gamma_{i}^{\text{phonons}} (\langle C_i \rangle - \langle C_i \rangle_{\text{th}}) + \Gamma_{i}^{\text{drive}} (\langle C_i \rangle) \quad \forall i
\]
Steady state density matrix

Phase transition for infinitezimal coupling only

\[ \rho_0 = \lim_{\epsilon_d, \epsilon_p \to 0} \lim_{t \to \infty} \rho(t) = \sum_n p_n |n\rangle \langle n|, \quad H_f |n\rangle = E_n |n\rangle \]

\( p_n \) from rate equation

\[ \frac{d}{dt} p_n = \sum_m \Gamma_{nm} p_m - \Gamma_{mn} p_n, \quad \Gamma_{mn} = \Gamma_{mn}^p + \Gamma_{mn}^d \]

where \( \Gamma_{mn}^p \sim \epsilon_p^2 \), \( \Gamma_{mn}^d \sim \epsilon_d^2 \), so the important parameter is \( \epsilon_d^2 / \epsilon_p^2 \).
Measure local temperatures

Order parameter:

\[
\frac{\delta T}{\bar{T}} = \frac{\sqrt{\langle \text{Var}(T_i) \rangle}}{\langle \text{E}(T_i) \rangle}.
\]
Local thermometer

- Theoretically:
  - local coupling to external thermal bosons,

$$H_j = \epsilon_l A_j (a + a^\dagger), \quad A_j = c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j$$

$$T_j: \text{temperature of bosons at which } \langle \dot{H}_f \rangle_j = 0$$

- Experimentally: local Raman spectroscopy
  - Compare Stokes and anti-Stokes peaks
Results: temperature variations

Order parameter:

$$\frac{\delta T}{\bar{T}} = \frac{\sqrt{\langle \text{Var}(T_i) \rangle}}{\langle \text{E}(T_i) \rangle}.$$ 

In thermodynamic limit:

$$\lim_{\epsilon \to 0} \lim_{t \to \infty} \lim_{N \to \infty} \rho$$

On finite system size:

On finite system size:

$$\frac{\delta T}{\bar{T}}$$

$$h_c$$

$$h$$

$$N=6$$

$$N=8$$

$$N=10$$

$$N=12$$
Results: temperature variations

In the **ergodic phase exponential** drop of $\delta T$,

$$\delta T \sim e^{-L/\xi(h)}$$
Critical exponent for $h_c = 7$ (from Luitz et al PRB 2015, $\nu \sim 1$):

$$\xi(h) \sim \frac{1}{(h_c - h)^\nu}, \quad \nu = 2.6$$

Lenarčič, Altman, Rosch, PRL 121, 267603 (2018)
Finite coupling to environment and drive

- Phase transition $\rightarrow$ crossover
- Experimentally relevant
  - $\epsilon$ controlled by light power and phonon temperature
Finite coupling to environment and drive

- **Finite** $\epsilon \leftrightarrow$ **finite temperature** in $T = 0$ quantum phase transition
- Numerically: need full density matrix: $\rho_\infty = \rho_0 + \delta \rho(\epsilon)$
  - **Tensor-network approach**
  - bond dimension controlled by coupling to the environment
Temperature fluctuation $\delta T$ on the ergodic side

- Ergodic phase: $T(r, \epsilon) = \bar{T} + \delta T(r, \epsilon)$.
- **Hydrodynamic theory** based on approximate energy conservation.
  
  $$\partial_t e - \nabla(\kappa(r) \nabla T(r)) = -\epsilon g_p(r)(T(r) - T_p) + \epsilon g_d(r)$$

  *disorder*  *sink*  *source*

- Fluctuations in $d$-dimension
  
  $$\delta T(r) \sim \epsilon^{d/4}$$
Temperature fluctuation on the ergodic side

Subdiffusive transport precedes MBL

- Fractional derivative: $\nabla^2 \rightarrow \nabla^z$, $z$-dynamical exponent
  - $z = 2$: diffusion
  - $z > 2$: subdiffusion
- Fluctuations in $d$-dimension

$\delta T(r) \sim \epsilon^{d/2z}$
Subdiffusion - Griffiths regions

In 1D subdiffusion due to insulating regions: bottlenecks for transport
In 1D subdiffusion due to insulating regions: bottlenecks for transport

• Long insulating regions are exponentially rare $\ell$,

$$P(\ell) = \frac{1}{N} e^{-\ell/\xi}, \quad \xi \sim \text{correlation length}$$

• Conductance of insulating region

$$\Gamma = \Gamma_0 e^{-\ell/a},$$

• Distributions leads to different regimes

$$P(\Gamma) \sim \left( \frac{\Gamma}{\Gamma_0} \right)^{-1+\alpha}, \quad \alpha = \frac{a}{\xi}$$

$\alpha > 1$: diffusive, $0 < \alpha < 1$: subdiffusive
Higher dimensions

• Distribution of insulator lengths
  \[ P(\ell) \sim e^{-\left(\frac{\ell}{\xi}\right)^d} \]

• Distribution of conductances
  \[ P(\Gamma) \sim \frac{1}{\Gamma} e^{-\alpha^d \left(\ln\left(\frac{\Gamma_0}{\Gamma}\right)\right)^d} \quad \text{for} \quad \Gamma \gg \epsilon \frac{\kappa_0}{a^{2-d}}. \]
Coupling to environment $\rightarrow$ ohmic term

- Conductances in the presence of coupling to environment
  \[
  \Gamma = \epsilon \frac{\kappa_0}{\ell^{2-d}} + \Gamma_0 e^{-\ell/a}, \quad \alpha = \frac{a}{\xi}
  \]

- Energy continuity equation
  \[
  \partial_t e_i - (\Gamma_{i,i+1}(T_{i+1} - T_i) - \Gamma_{i-1,i}(T_i - T_{i-1})) = -\epsilon g_{p,i}(T_i - T_p) + \epsilon g_{d,i}
  \]
$\delta T$ detects dynamical exponent $z$

One dimension:

$\delta T \sim \epsilon^{1/2z(\alpha)}$

diffusive $z = 2$, $\alpha > 1$

subdiff. $z \sim \xi$, $\alpha < 1$

Two dimensions:

$\delta T \sim \epsilon^{1/z}$

always diffusive, $z = 2$. 
Other experimental signatures

- Charge continuity equation
  \[ (\Gamma_{i,i+1}(\mu_{i+1} - \mu_i) - \Gamma_{i-1,i}(\mu_i - \mu_{i-1})) = 0, \quad \Gamma = \epsilon \frac{\sigma_0}{l} + \Gamma_0 e^{-l/a} \]

- Average resistance for subdiffusive regime
  \[ \bar{\rho}(\epsilon) \sim \left( \frac{\epsilon}{\ln \epsilon^{-1}} \right)^{1/z} \]

- Convert \( \bar{\rho}(\epsilon) \) to \( \bar{\rho}(T_{ph}) \)
  \[ \rightarrow \text{tune } T_{ph} \text{ to detect critical behaviour of the MBL transition} \]
  \[ \rightarrow \text{get dynamical exponent } z(h) \]
TEBD calculations for coupling to Markovian baths

Steady state

\[ \hat{\mathcal{L}} \rho_\infty = -i[H, \rho_\infty] + \epsilon \hat{D} \rho_\infty = 0 \]

Dominant Hamiltonian part

\[ H = \sum_i S_i \cdot S_{i+1} + h(\alpha_i^z S_i^z + \alpha_i^x S_i^x), \quad \alpha_i^{x,z} \in [-1, 1] \]

Phonons and driving replaced by coupling to Markovian baths, \( \mathcal{D} \)

- openness
- driving

Calculate steady state with TEBD evolution of \( \hat{\mathcal{L}} \rightarrow \rho_{NESS} \)
TEBD calculations for coupling to Markovian baths

Steady state
\[ \hat{\mathcal{L}} \rho_\infty = -i[H, \rho_\infty] + \epsilon \hat{\mathcal{D}} \rho_\infty = 0 \]

Dominant Hamiltonian part
\[ H = \sum_i S_i \cdot S_{i+1} + h(\alpha_i^x S_i^x + \alpha_i^z S_i^z), \quad \alpha_i^x,^z \in [-1, 1] \]

Coupling to Markovian baths
\[ \hat{\mathcal{D}} = \sum_\alpha \hat{\mathcal{D}}^{(\alpha)}, \quad \hat{\mathcal{D}}^{(\alpha)} \rho = \sum_i L_i^{(\alpha)} \rho (L_i^{(\alpha)})^\dagger - \frac{1}{2} \{ (L_i^{(\alpha)})^\dagger L_i^{(\alpha)}, \rho \} \]

Lindblad operators
\[ L_i^{(1a)} = S_i^+ \left( \frac{1}{2} \mathbb{1}_{i+1} - S_{i+1}^z \right), \quad L_i^{(1b)} = \left( \frac{1}{2} \mathbb{1}_i - S_i^z \right) S_{i+1}^+, \]
\[ L_i^{(2a)} = S_i^- \left( \frac{1}{2} \mathbb{1}_{i+1} + S_{i+1}^z \right), \quad L_i^{(2b)} = \left( \frac{1}{2} \mathbb{1}_i + S_i^z \right) S_{i+1}^-, \]
\[ L_i^{(3)} = S_i^z \]
Condition for local temperature $T_i$

Minimize Frobenium norm with respect to $T_i$

$$F[T_i] = \text{tr} \left[ \left( \rho^{(i,i+1)}(\infty) - \rho^{(i,i+1)}(T_i) \right)^2 \right]$$
Detect the underlying transition from $\epsilon$ dependence

- Two regimes

$$\frac{\delta \beta}{\beta}(\epsilon) \sim \begin{cases} 
\epsilon^{1/2z}, & \text{ergodic : } h < h_c, \\
\left. \frac{\delta \beta}{\beta} \right|_{\epsilon \to 0} - b \epsilon + O(\epsilon^2), & \text{MBL : } h \geq h_c
\end{cases}$$
Critical disorder strength

- New criterion for transition

\[
\frac{\delta \beta}{\beta}(\epsilon) \sim \begin{cases} 
\epsilon^{1/2z}, & h < h_c, \\
\frac{\delta \beta}{\beta}|_{\epsilon \to 0} - b \epsilon + O(\epsilon^2), & h \geq h_c
\end{cases}
\]

- Critical disorder strength: \( h_c = 8.75 \pm 0.5 \)
Divergence of dynamical exponent

\[ \frac{\delta \beta}{\bar{\beta}} \sim \epsilon^{1/2z} \rightarrow z \sim \xi \sim (h_c - h)^{-\nu}, \quad \nu = 4 \pm 0.9 \]

Direct bulk measurement of dynamical exponent \( z(h) \)
Comparison with ED

**Critical exponent** $\nu$

\[ z \sim \xi \sim (h_c - h)^{-\nu}, \quad \nu = 4 \pm 0.9 \]

- Obeys Harris bound: $\nu > 2/d$
- Exact diagonalization: $\nu \approx 1$
  
  Luitz, Laflorecie, Alet, PRB '16

- Close to RG results: $\nu \sim 3.3$
  
  Vosk et al, Potter at al, 2xPRX '15

- Cannot be distinguished from KT

\[ z \sim e^{c/\sqrt{h_c-h}} \]

Goremykina, Vasseur, Serbyn, PRL '19
Dumitrescu et al, PRB '19
Comparison with ED

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  Goremykina, Vasseur, Serbyn, PRL '19
  Dumitrescu et al, PRB '19

Critical disorder strength

$h_c \geq 8.75$

- Larger then ED, $h_c \in [2, 7]$
  Geraedts et el, New J. Phys. '17

\[ \frac{\delta \beta}{\beta} \]

\[ h=9 \quad h=4.5 \quad h=3 \quad h=2 \quad h=1 \]

\[ \epsilon \]

0.0 0.1 0.2 0.3 0.4
Limitations of our method

- TEBD: convergence time $\epsilon^{-1}$
- Variational approach?
- Perturbative approaches: is bond dimension really a problem?
  - MBL side: low bond dimension
  - ergodic side: $\rho = \frac{e^{-\beta H}}{Z} + \delta \rho(\epsilon)$
Useful non-thermal steady states?

- Pump into heat and spin current in Heisenberg spin chains
- Large current expectation value $\langle J_h/s \rangle \sim \text{tr} \left[ J_h/s \epsilon_j C_j \right] \sim O(1)$,
  $\lambda_h/s (\epsilon_d \epsilon_p) \sim O(1)$

Nat. Commun. 8, 15767 (2017)
Useful non-thermal steady states?

- Pump into **heat and spin current** in Heisenberg spin chains
- Large current expectation value

\[
\langle J_{h/s} \rangle \sim \text{tr} \left[ J_{h/s} e^{-\sum_j \lambda_j c_j} \right] \sim \mathcal{O}(1),
\]

\[
\lambda_{h/s} \left( \frac{\epsilon_d}{\epsilon_p} \right) \sim \mathcal{O}(1)
\]

Nat. Commun. 8, 15767 (2017)
Pumping into heat and spin current in Heisenberg chain

- Heisenberg spin 1/2 chain in magnetic field:

\[ H_0 = \sum_j \frac{J}{2} (S_j^+ S_{j+1}^- + S_j^- S_{j+1}^+) + \Delta S_j^z S_{j+1}^z + \mu B \sum_i S_i^z \]

- Coupling to thermal phonon bath

- Weak periodic driving

\[ H_d = \epsilon_d \sum_i (\Delta J (-1)^i \mathbf{S}_i \cdot \mathbf{S}_{i+1} e^{i \omega t} + \mu \Delta B (-1)^i S_i^z e^{i \omega t + \phi} ) + \text{H.c.} \]

- Need material that shows:
  - staggering of exchange coupling upon application of electric field
  - staggered g-tensor
Conclusions

- New, experimentally relevant order parameter $\frac{\delta T}{\bar{T}}$.

- Measurement of dynamical exponent $z(h)$ and critical disorder

\[
\frac{\delta T}{\bar{T}}(\epsilon) \sim \begin{cases} 
\epsilon^{1/2z}, & h < h_c, \\
\delta T \bigg|_{\epsilon \to 0} - b\epsilon + O(\epsilon^2), & h \geq h_c
\end{cases}
\]

- New scalable numerical approach, which goes beyond limitation of ED

$\epsilon \to 0$: Lenarčič, Altman, Rosch, PRL 121, 267603 (2018),
Results: temperature variations

Critical exponent $\nu$, 

$$\xi_e(h) \sim \frac{1}{(h_c - h)^\nu}$$

for $h_c = 7$: $\nu = 2.6, \beta = 0$.

Pale&Huse, PRB ’10, Luitz et al, PRB ’15

For $h_c \in [5.5, 7.5]$:

$$\nu \sim 2.5 \pm 0.5, \quad \beta = 0.08 \pm 0.08$$

Obeys Harris-Chayes bound: 

$\nu > 2/d$

Harris ’74, Chayes ’89, Chandran et al ’15

Exact diagonalization: $\nu \approx 1$

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Close to RG results: $\nu \sim 3.3$

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