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Fonds de recherche
Nature et
technologies
Québec




Bad metals and more

André-Marie Tremblay

Reza Nourafkan, Alexis Reymbaut, Charles-David
Hébert, Simon Bergeron, C. Walsh, P. Sémon,
D. Poulin, G. Sordi

**Quantum Matter:
Computation Meets Experiments**
Aspen, Colorado, 8 – 13 March 2020

Bad metals



Reza Nourafkan



Alexis Reymbaut



Charles-David Hébert



Simon Bergeron

Peter T. Brown, et al.

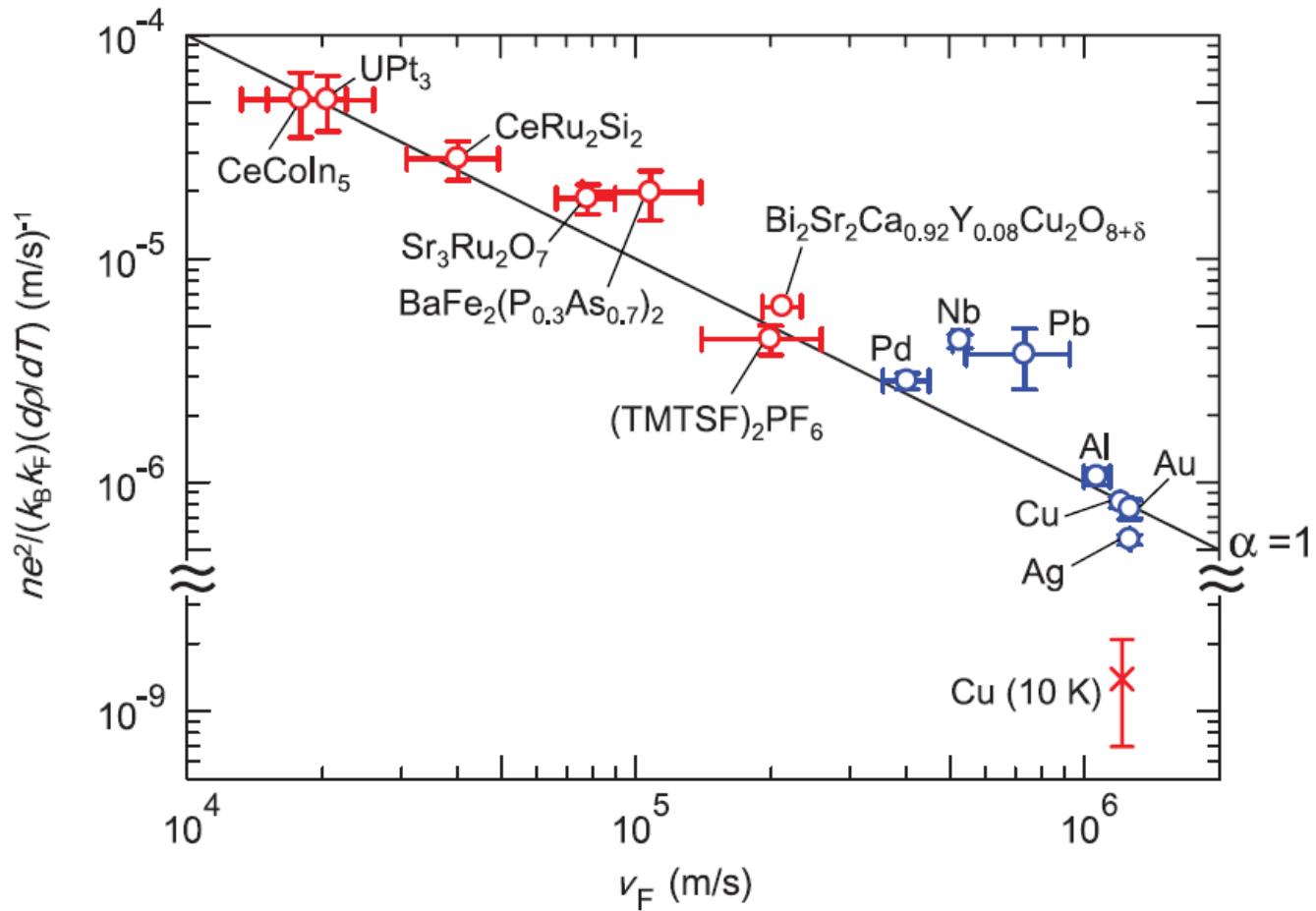
Bad metallic transport in a cold atom Fermi-Hubbard system

Science [10.1126/science.aat4134](https://doi.org/10.1126/science.aat4134) (2018).

Motivation

$$\hbar/\tau = k_B T$$

$$(T\tau)^{-1} \sim \alpha k_B / \hbar$$



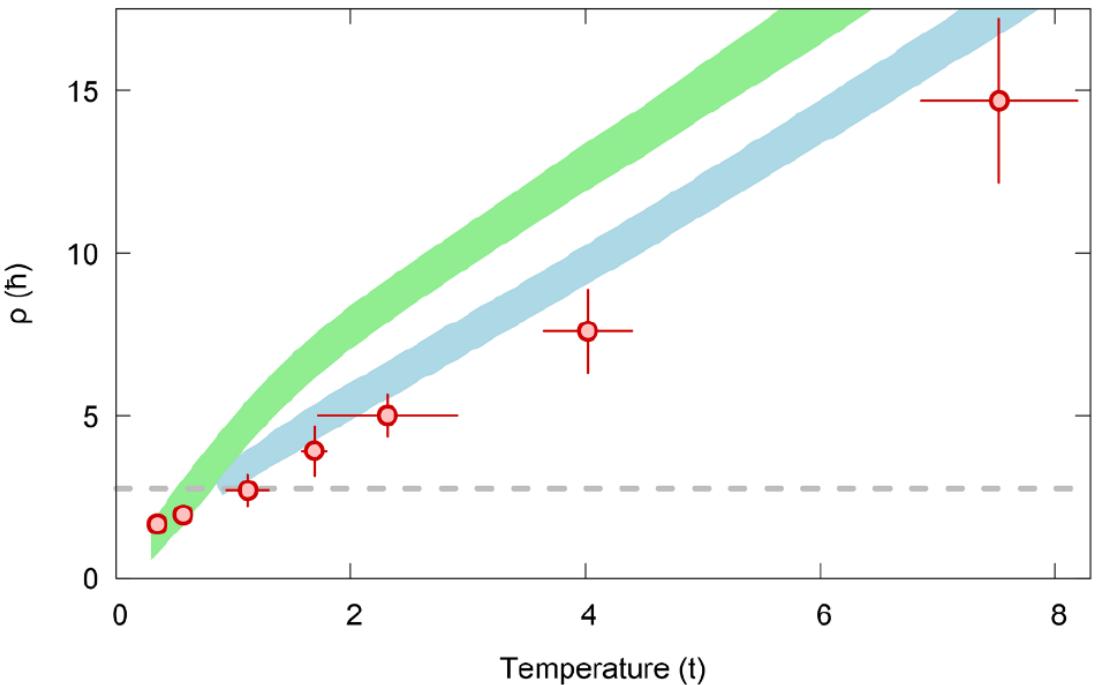
J. A. N. Bruin, H. Sakai, R. S. Perry, A. P. Mackenzie SCIENCE 339, 804 (2013)

Cold atoms, high temperature

Linear resistivity at high temperature

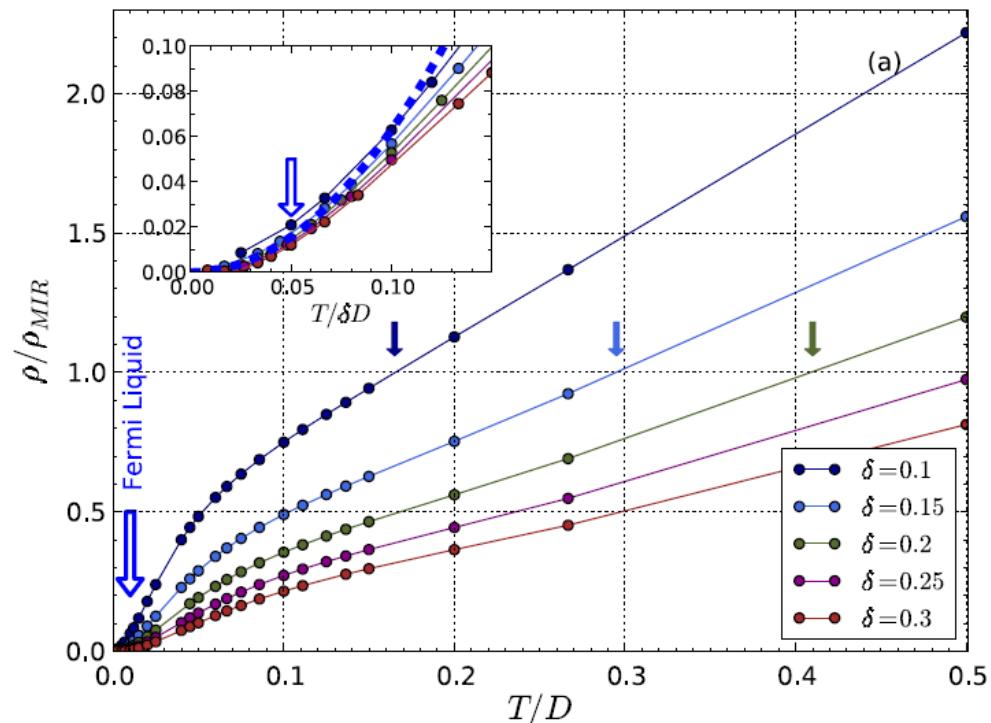
10

DMFT (Green), FTLM (Blue)



$$U = 7.5t, n = 0.825, t' = 0$$

Deng et al. PRL 110, 086401 (2013)



$$U = 4D = 16t$$

H. Terletska, et al. Phys. Rev. Lett. **107**, 026401 (2011).

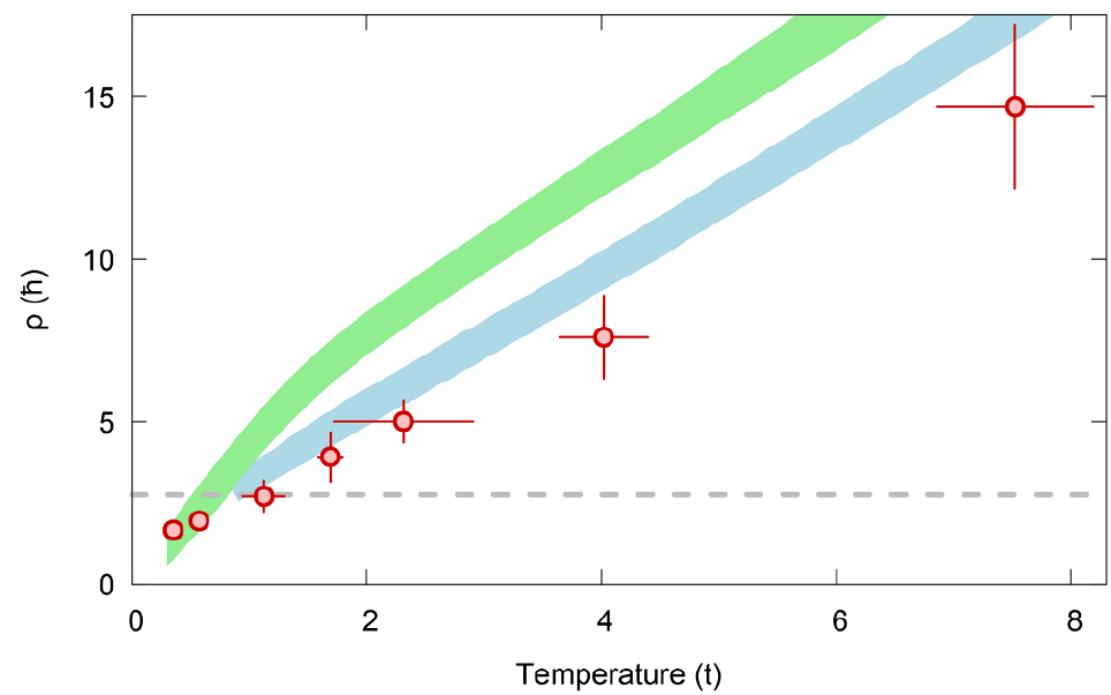
X. Deng, et al. Phys. Rev. Lett. **110**, 086401 (2013).

W. Xu, et al. Phys. Rev. Lett. **111**, 036401 (2013).

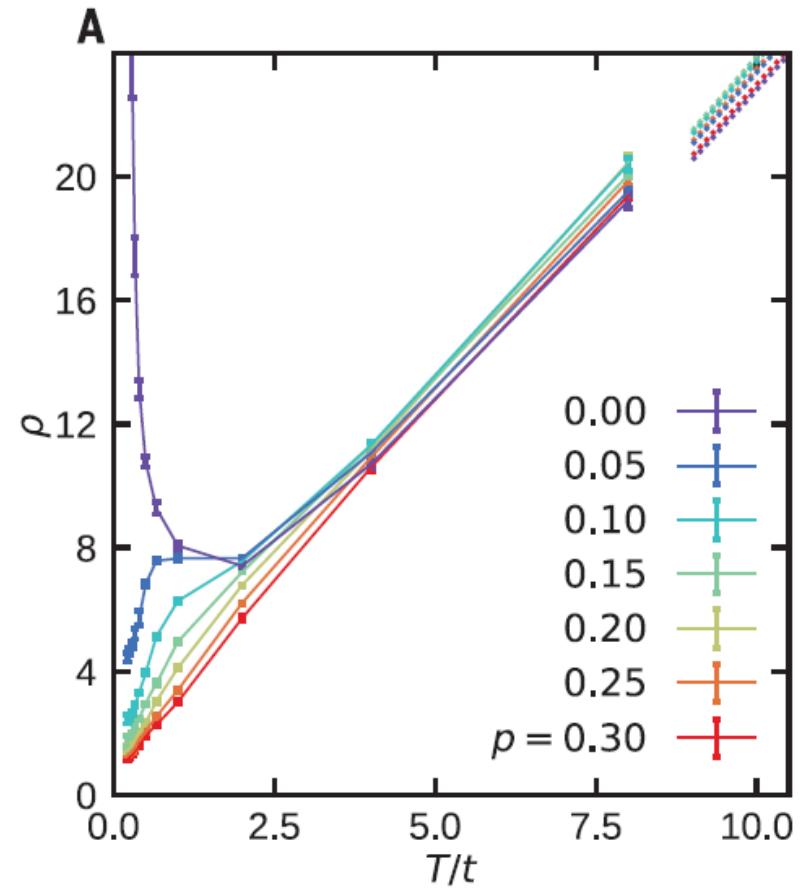
J. Vučičević, Phys. Rev. Lett. **114**, 246402 (2015).

Linear resistivity at high temperature

DMFT (Green), FTLM (Blue)



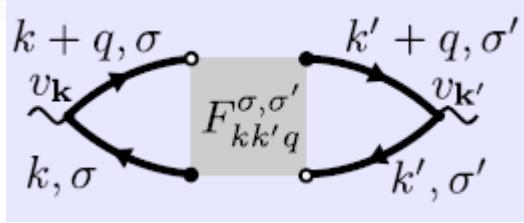
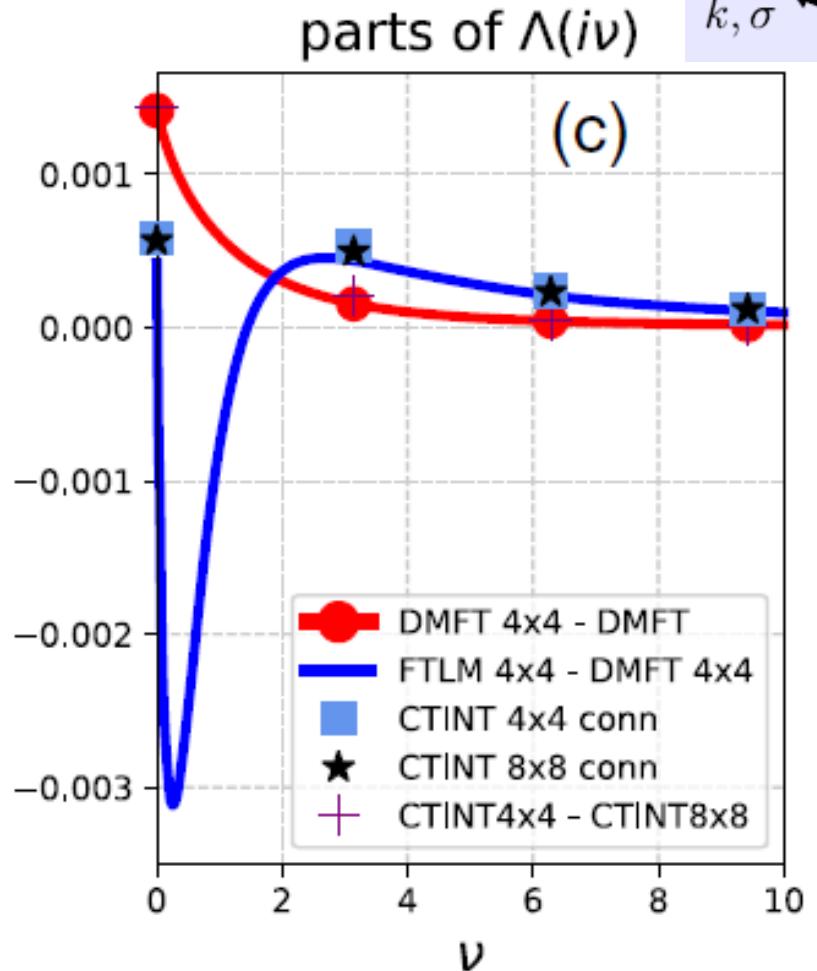
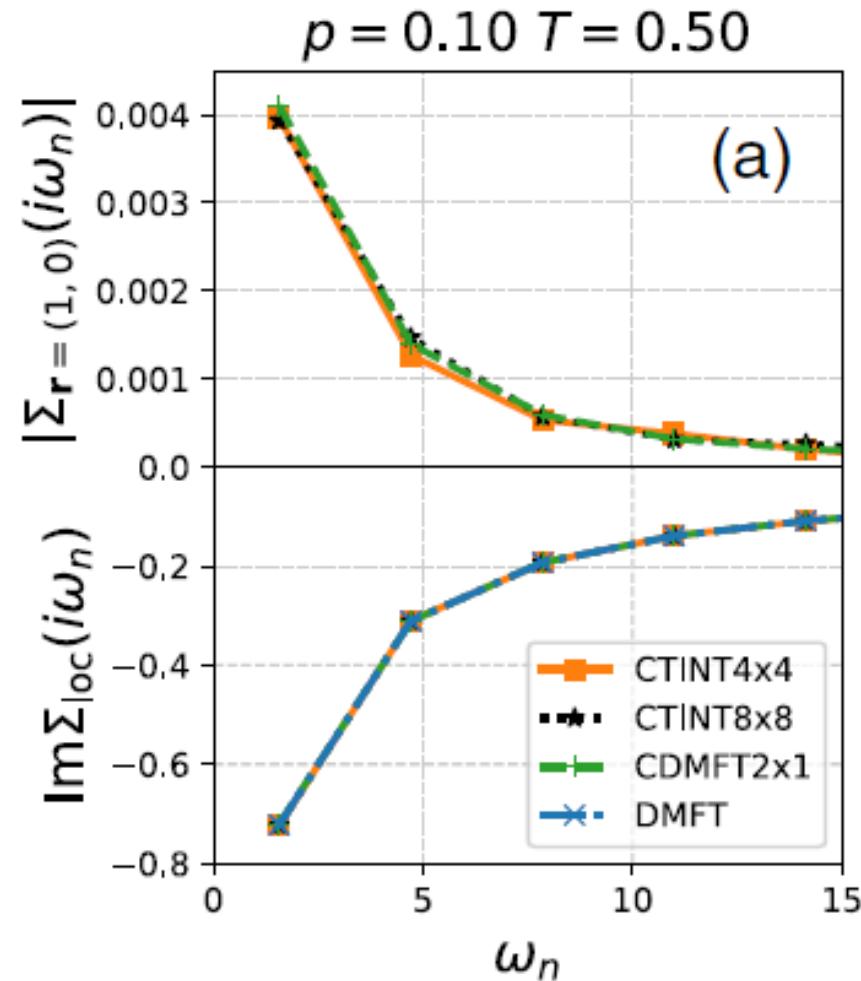
$$U = 7.5t, \quad n = 0.825, \quad t' = 0$$



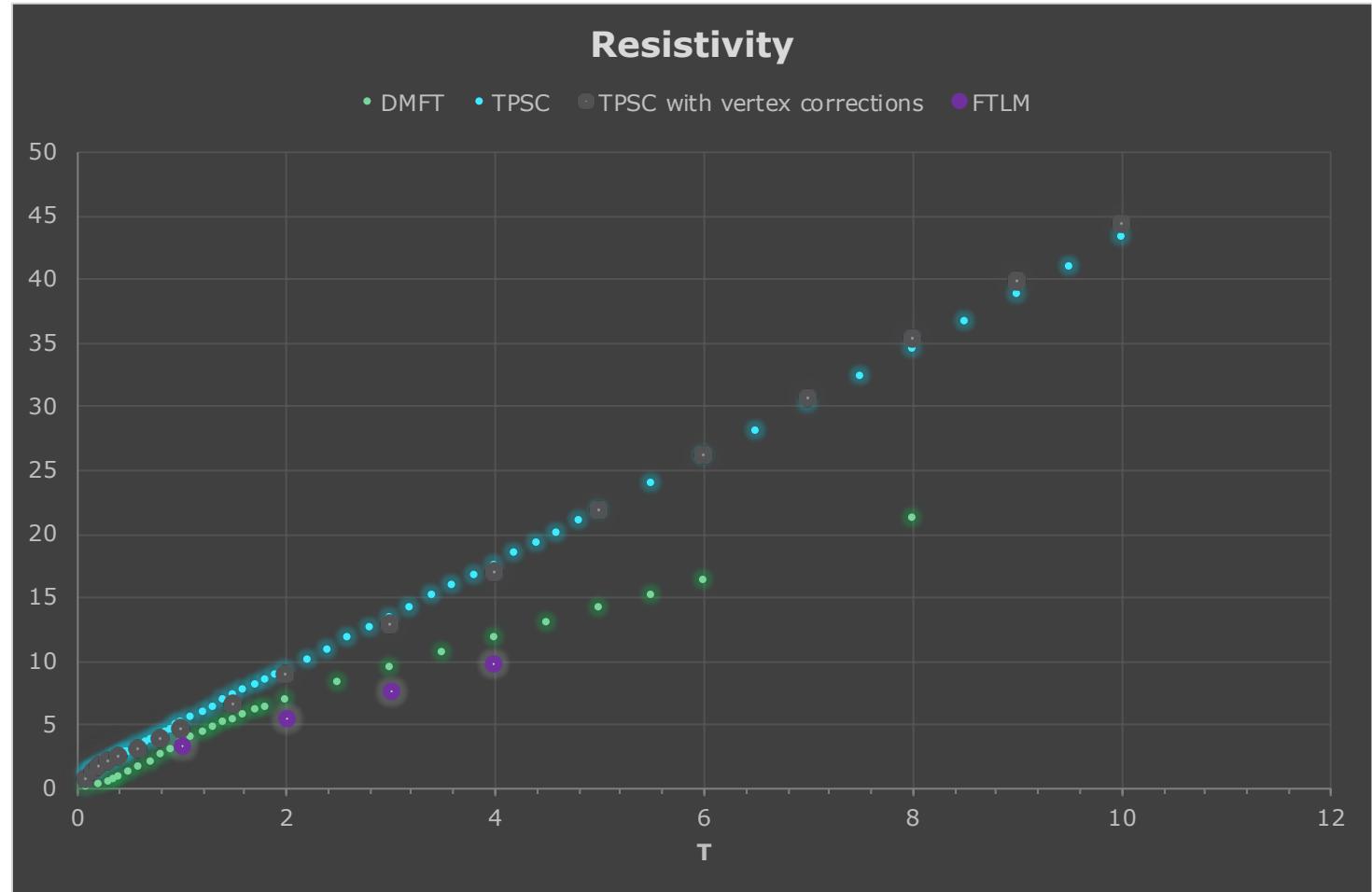
Huang et al. Science 366, 987 (2019)

$$U = 6t, \quad t' = -0.25$$

Linear resistivity at high temperature : vertex corrections

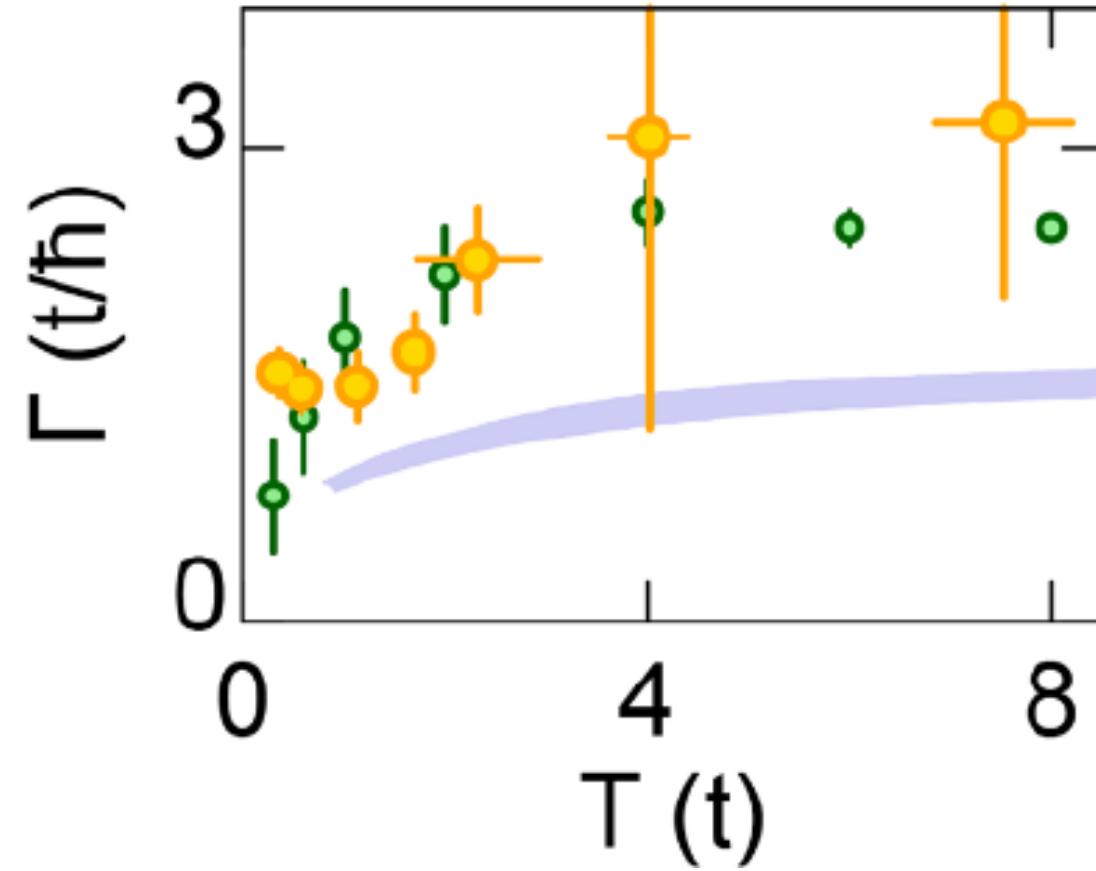


Another view of vertex corrections



Dominic Bergeron

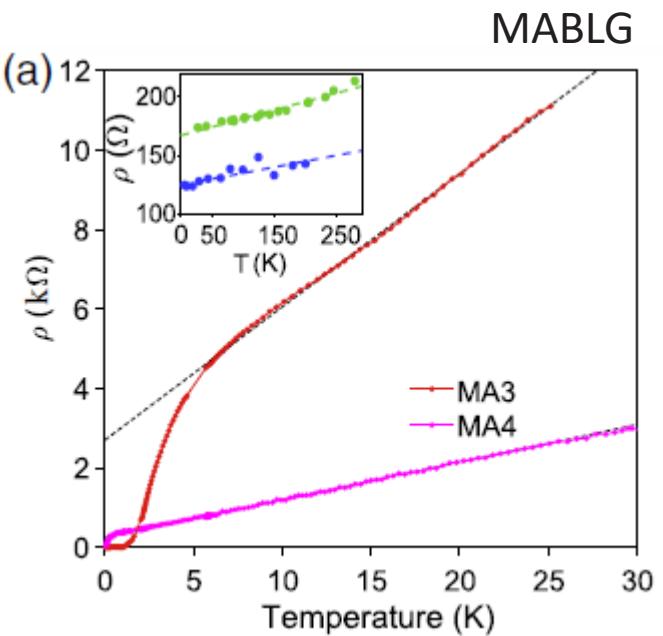
Unexplained : Width of the Drude Peak



Perspective

$$\rho = (m^*/ne^2) (1/\tau)$$

$$\hbar/\tau = k_B T$$

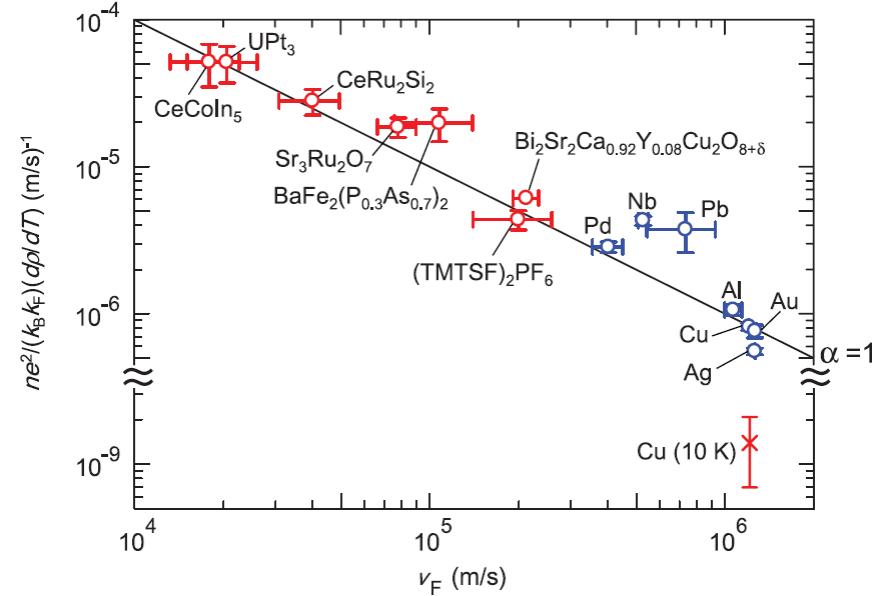


Cao *et al.* PRL 124, 076801 (2020)

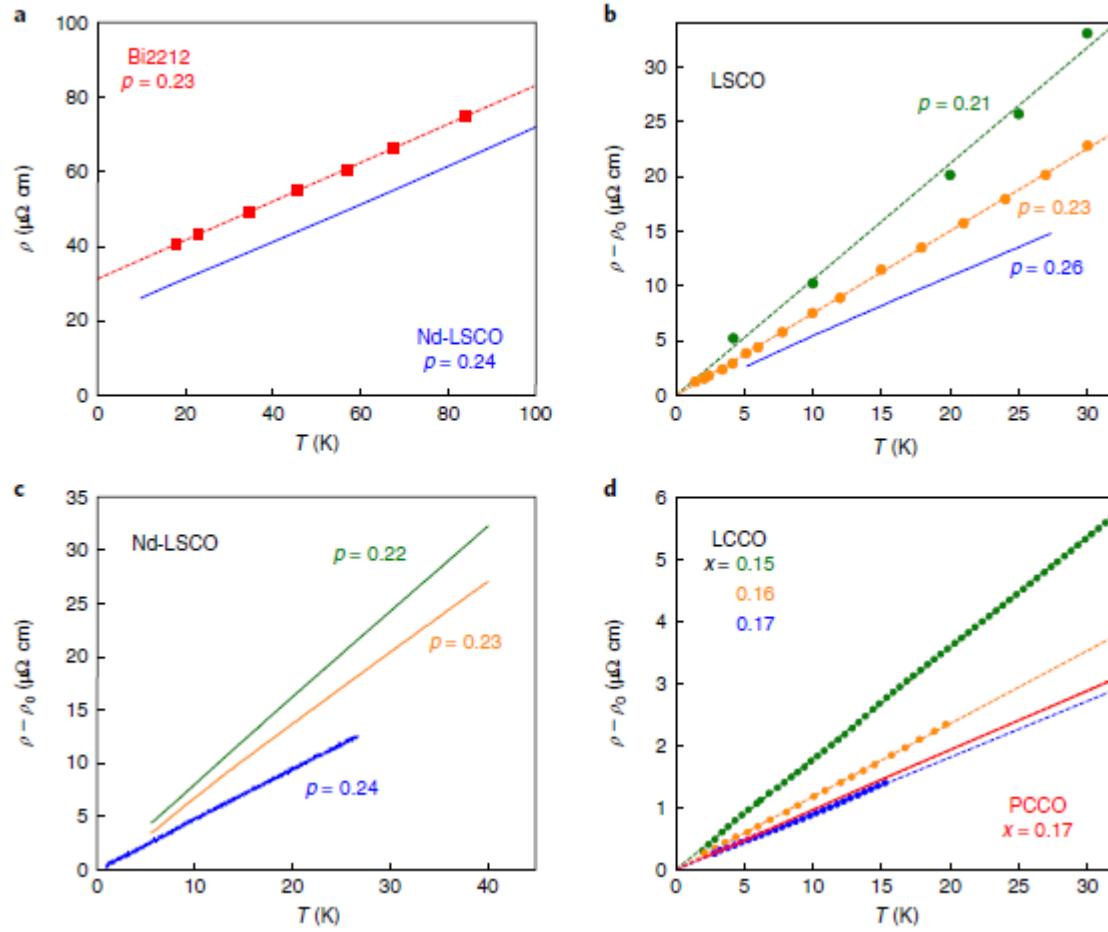
Low temperature ($k_B T \ll E_F$) experiments

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Cuprates



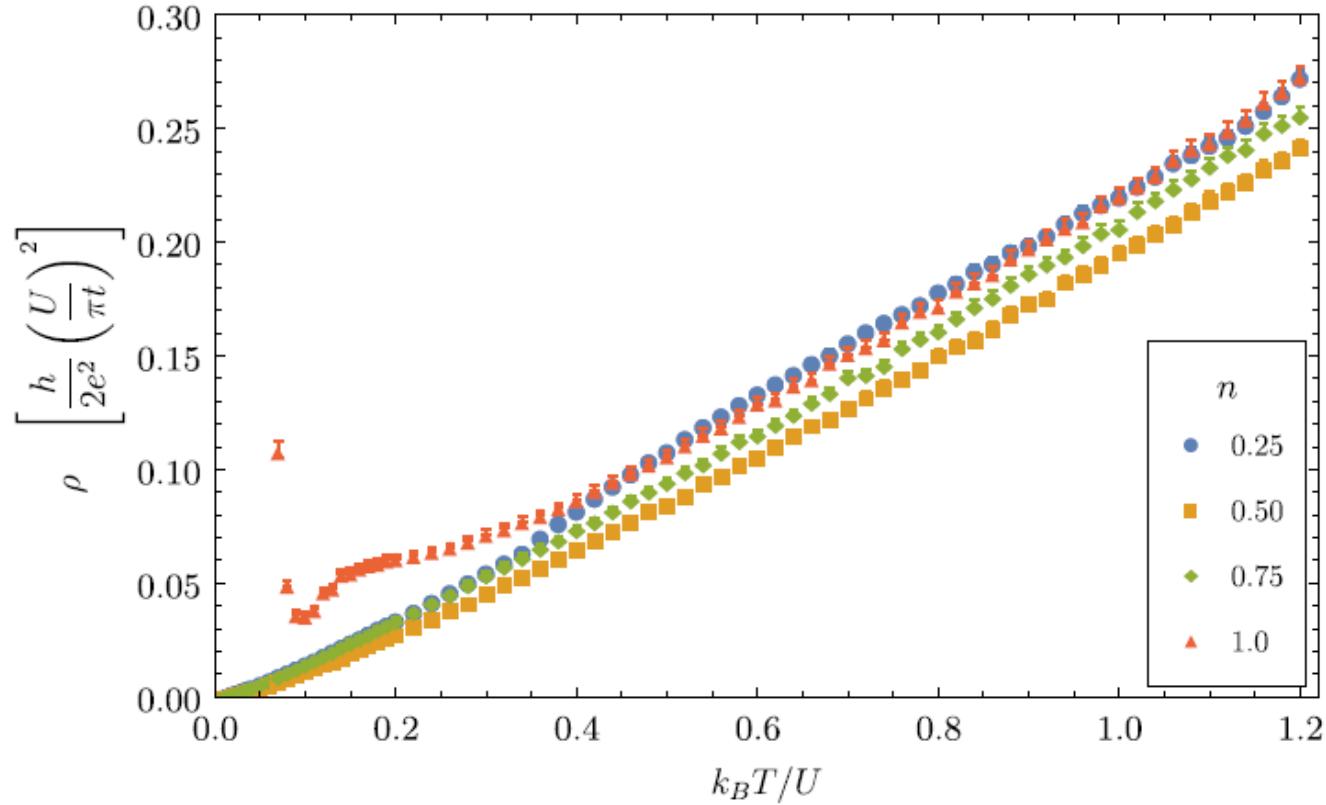
J. A. N. Bruin, H. Sakai, R. S. Perry, A. P. Mackenzie SCIENCE 339, 804 (2013)



A. Legros *et al.* Nature Physics (2018)

Modified Hubbard model

$$H = t \sum_{\langle ij \rangle, s} c_{is}^\dagger c_{js} + U \sum_i n_{i\uparrow} n_{i\downarrow} + \frac{V}{2} \sum_{i \neq j} e^{-|\vec{x}_i - \vec{x}_j|/\ell} n_i n_j.$$

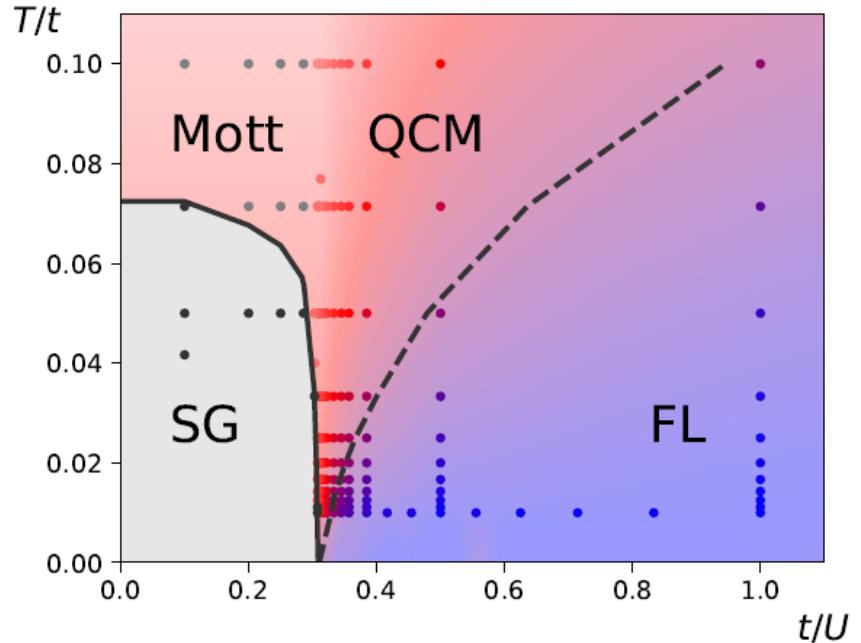


Mousatov *et al.* PRL **122**, 186601 (2019)

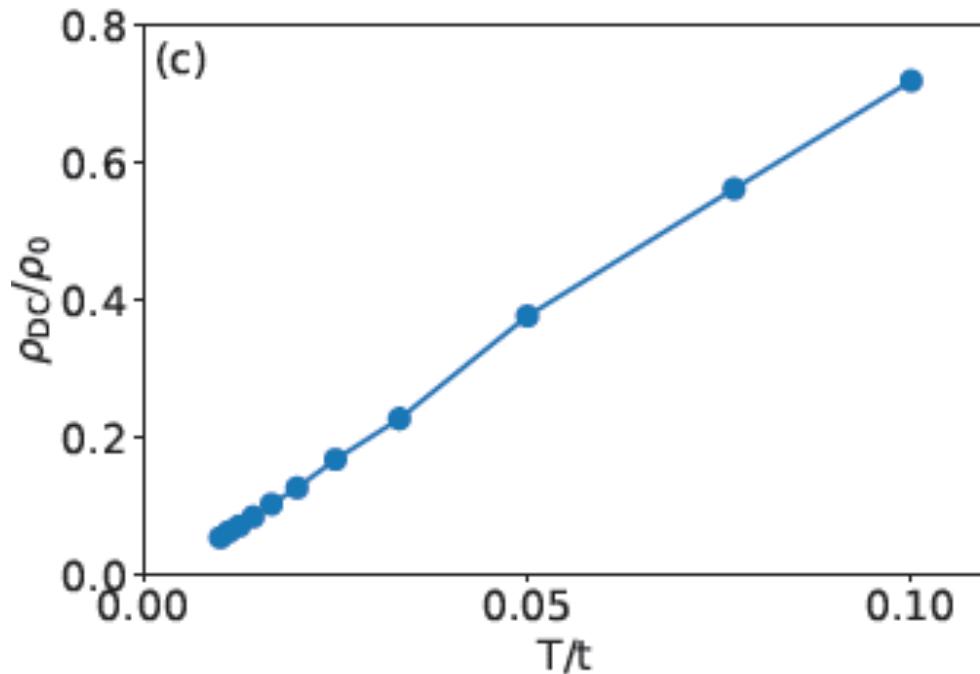
$$\begin{aligned}\dot{V} &= 0.1U \\ t \ll k_B T &\lesssim U\end{aligned}$$

SYK like models (similar to DMFT at high T)

$$H = - \sum_{\langle ij \rangle, s=\uparrow,\downarrow} t_{ij} c_{is}^\dagger c_{js} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \sum_{i < j} \frac{J_{ij}}{\sqrt{N}} \vec{S}_i \cdot \vec{S}_j$$



P. Cha *et al.* arXiv:2002.07181

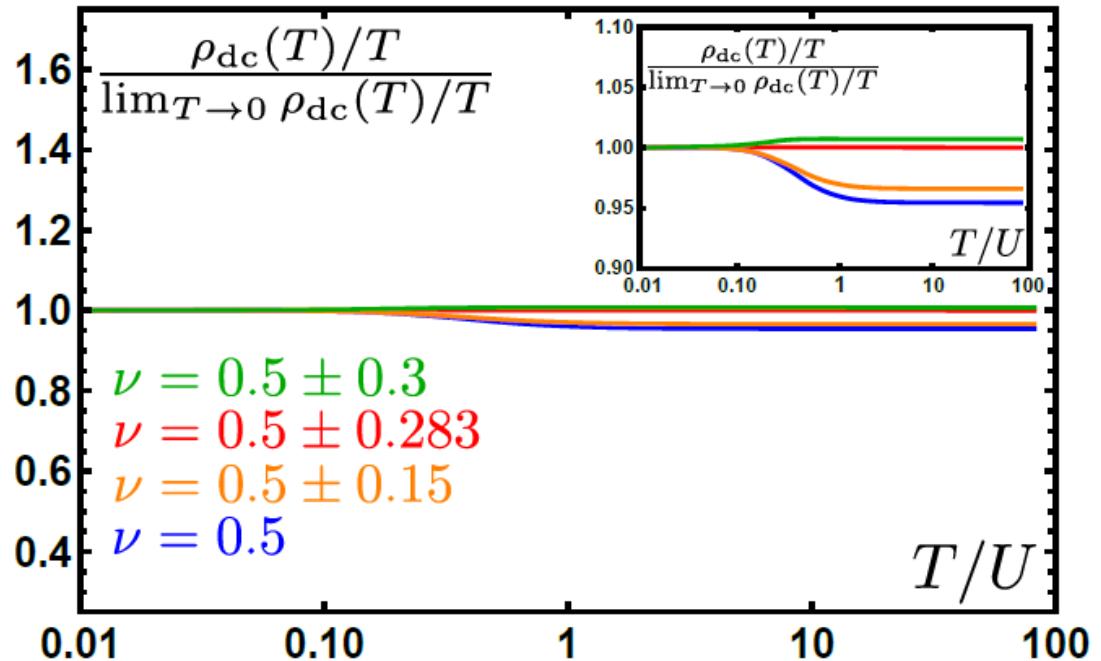


P. Cha *et al.* arXiv:2002.07181

- X.-Y. Song, *et al.* Phys. Rev. Lett. **119**, 216601 (2017).
- A. A. Patel, *et al.* Phys. Rev. X **8**, 021049 (2018).
- D. Chowdhury, *et al.* Phys. Rev. X **8**, 031024 (2018).
- S. A. Hartnoll, A. Lucas, and S. Sachdev, *Holographic Quantum Matter* (MIT Press, Cambridge, MA, 2018).

SYK like models (similar to DMFT at high T)

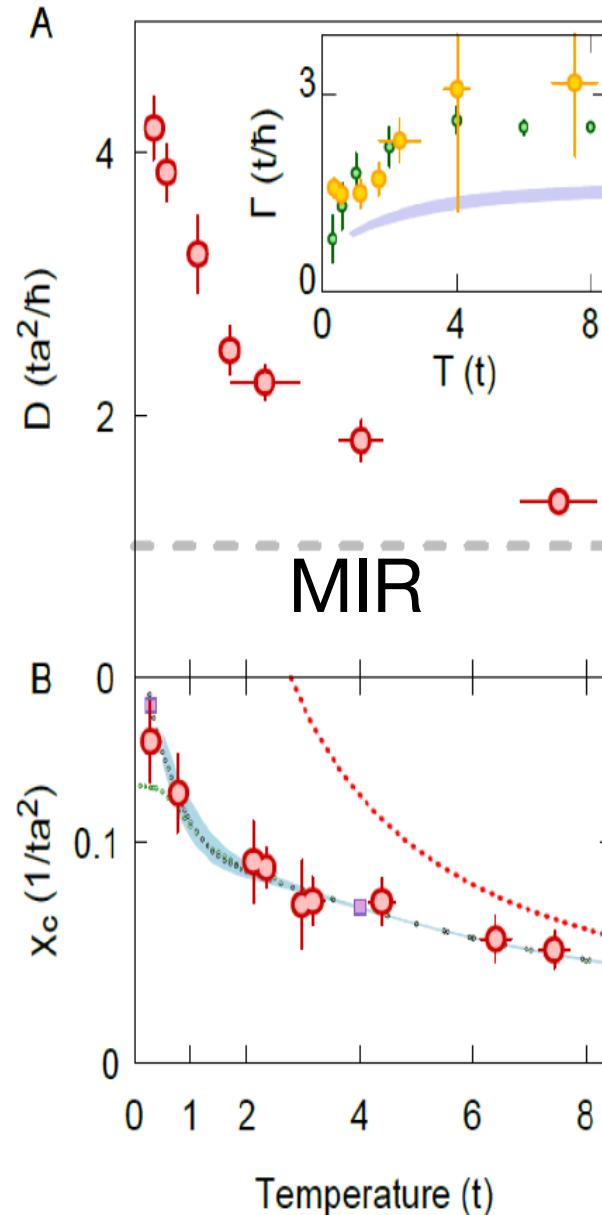
« Slope invariance »



P. Cha *et al.* arXiv:1910.07530

- X.-Y. Song, *et al.* Phys. Rev. Lett. **119**, 216601 (2017).
- A. A. Patel, *et al.* Phys. Rev. X **8**, 021049 (2018).
- D. Chowdhury, *et al.* Phys. Rev. X **8**, 031024 (2018).

Linear at high temperature, but not because of scattering



$$\sigma = \chi_c D$$

J. Kokalj, Phys. Rev. B 95, 041110(R) (2017)

In DMFT, bounds violated

$$D \gtrsim \hbar v_F^2 / (k_B T)$$

$$D_s \sim 1.3\hbar/m$$

N. Pakhira *et al.*, PRB 91, 075124 (2015).

$$\chi_c \sim n(1 - n/2)/T,$$

E. Perepelitsky, *et al.* Phys. Rev. B 94, 235115 (2016).

General Case

$$\begin{array}{ccccccccc}
 xx & 0 & \frac{E_n}{k_B T} & n, m & \frac{e}{Z} & n | j_x | m & m | j_x | n & E_m & E_n
 \end{array}$$

Mukerjee *et al.* PRB **73**, 035113 (2006)

P. Cha *et al.* arXiv:1910.07530

E. Perepelitsky, *et al.* Phys. Rev. B **94**, 235115 (2016).

Entanglement entropy and mutual information near the Mott transition



Caitlin Walsh

Patrick Sémon

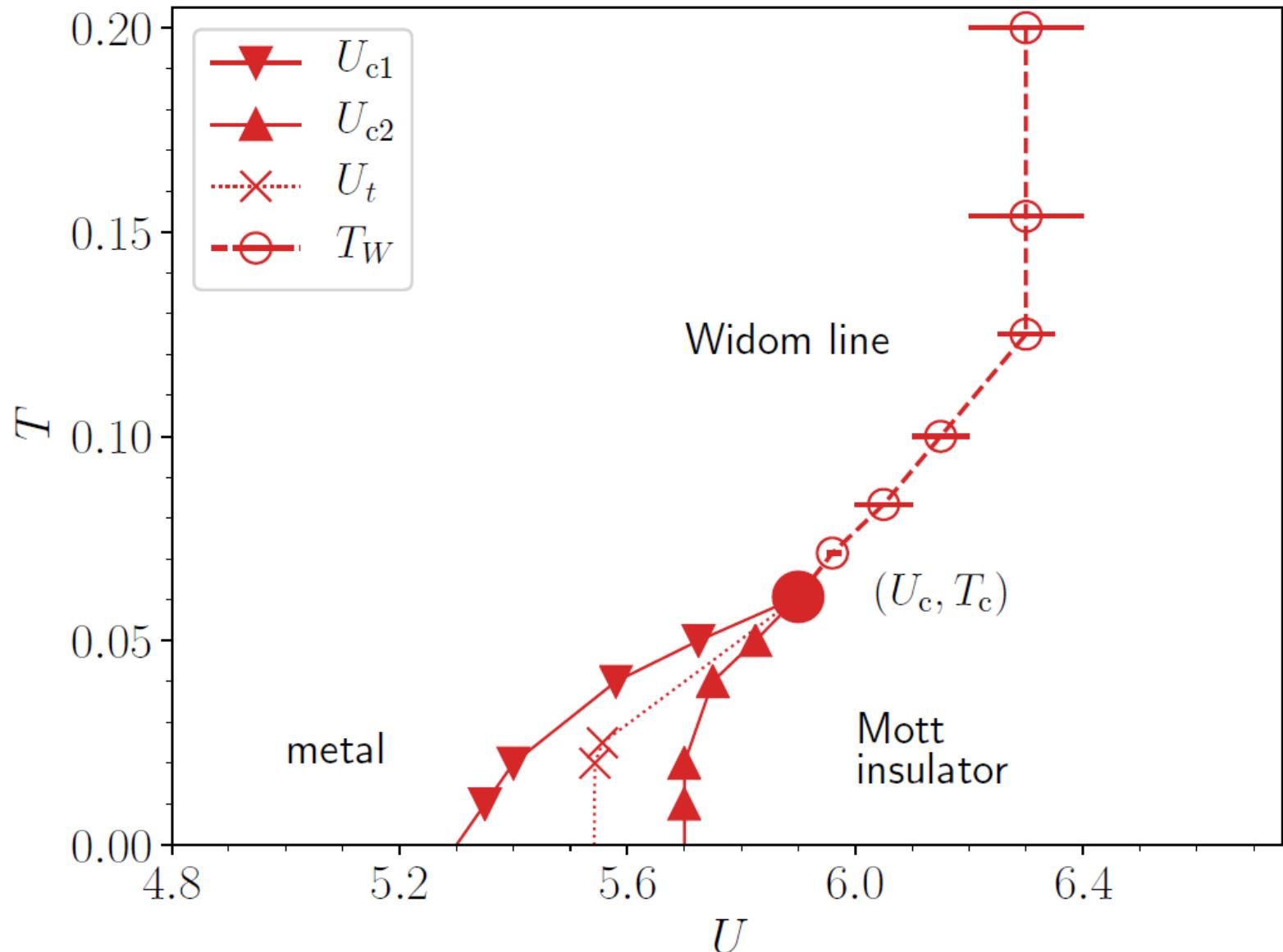
David Poulin

Giovanni Sordi

C. Walsh, *et al.*
Phys. Rev. Lett. **122**, 067203 (2019)
Phys. Rev. B **99**, 075122 (2019)

The Mott transition at half-filling

C. Walsh, et al. PRB **99**, 075122 (2019)
H. Park, et al. PRL **107**, 137007 (2011).



Motivation

PHYSICAL REVIEW X 7, 031025 (2017)

Measuring Entropy and Short-Range Correlations in the Two-Dimensional Hubbard Model

E. Cocchi,^{1,2} L. A. Miller,^{1,2} J. H. Drewes,¹ C. F. Chan,¹ D. Pertot,¹ F. Brennecke,¹ and M. Köhl¹

First-order nature of the transition,
universality class of the end point,
crossovers emanating from the end point.

For quantum critical or finite temperature critical points

- A. Anfossi *et al.* Phys. Rev. Lett. **95**, 056402 (2005).
- L. Amico *et al.* Europhys. Lett. **77**, 17001 (2007).
- L. Amico *et al.* Rev. Mod. Phys. **80**, 517 (2008).
- D. Larsson *et al.* Phys. Rev. A **73**, 042320 (2006).
- D. Larsson *et al.* Phys. Rev. Lett. **95**, 196406 (2005).

Single-site entanglement entropy



What is measured (Using CDMFT CT-HYB on plaquette)

- Single site entanglement entropy for fermions [1]

$$\rho_A = \text{Tr}_B[\rho_{AB}] \quad s_A = -\text{Tr}_A[\rho_A \ln \rho_A]$$

$$\rho = \text{diag}(p_0, p_\uparrow, p_\downarrow, p_{\uparrow\downarrow}) \quad s_1 = -\sum_i p_i \ln(p_i)$$

$$p_{\uparrow\downarrow} = \langle n_{i\uparrow} n_{i\downarrow} \rangle \quad p_\uparrow = p_\downarrow = \langle n_{i\uparrow} - n_{i\uparrow} n_{i\downarrow} \rangle \quad p_0 = 1 - 2p_\uparrow - p_{\uparrow\downarrow}$$

[1] P. Zanardi *et al.* Phys. Rev. A **65**, 042101 (2002).

What is measured

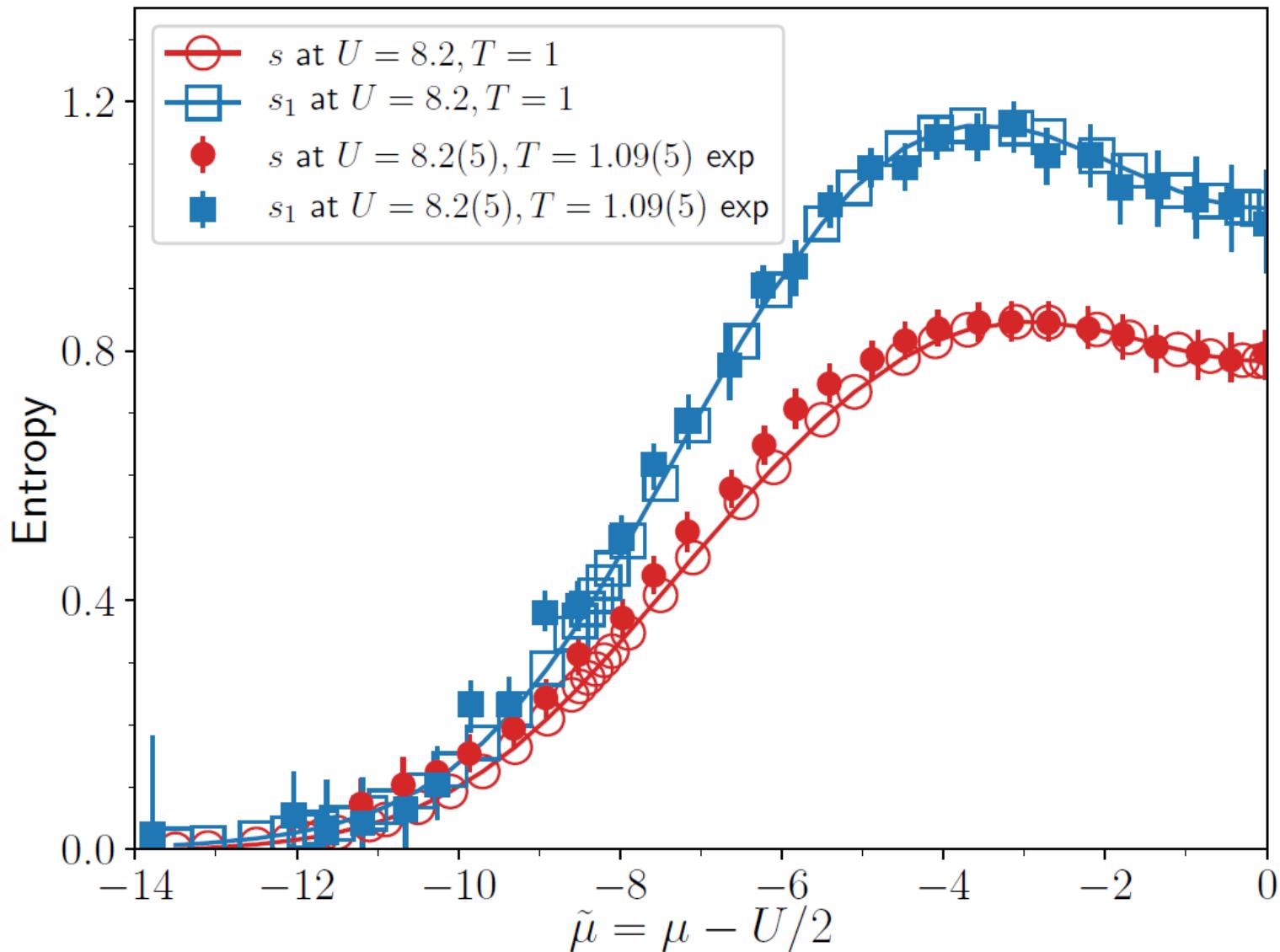
- Entropy

$$sdT - adP + nd\mu = 0$$

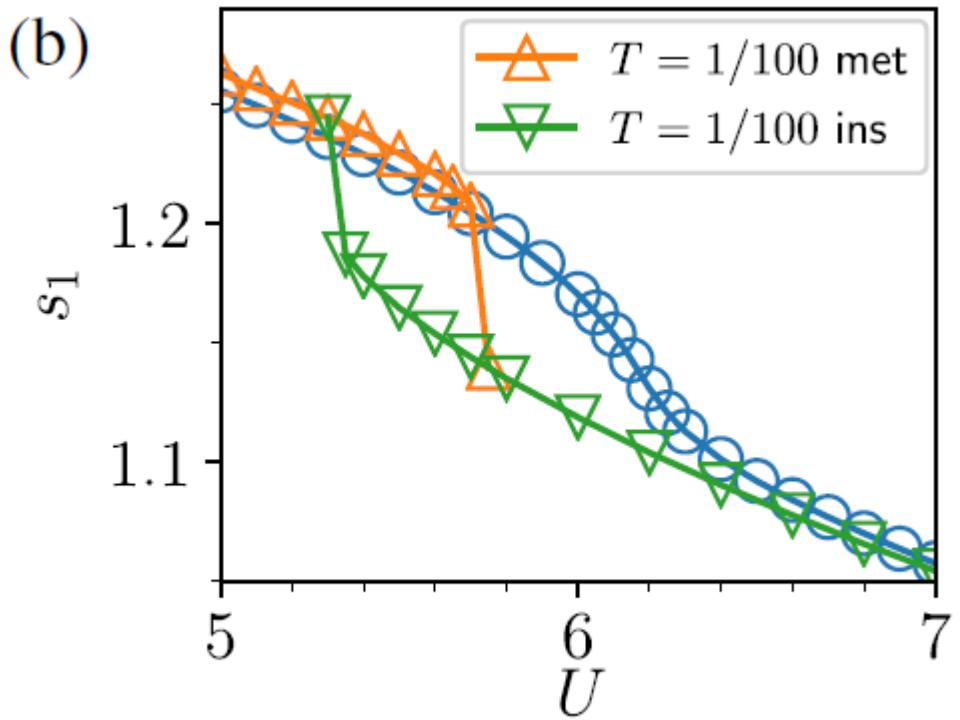
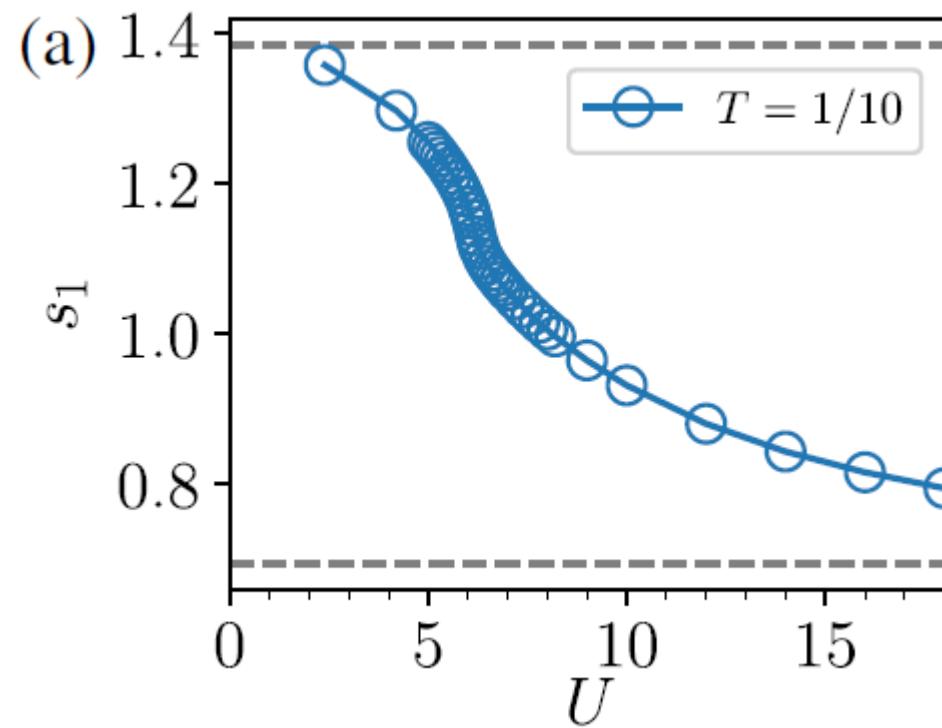
$$P(T)_U = \frac{1}{a} \int_{-\infty}^{U/2} n(\mu, T) d\mu$$

$$s = a(dP/dT)_\mu$$

Agreement with experiment

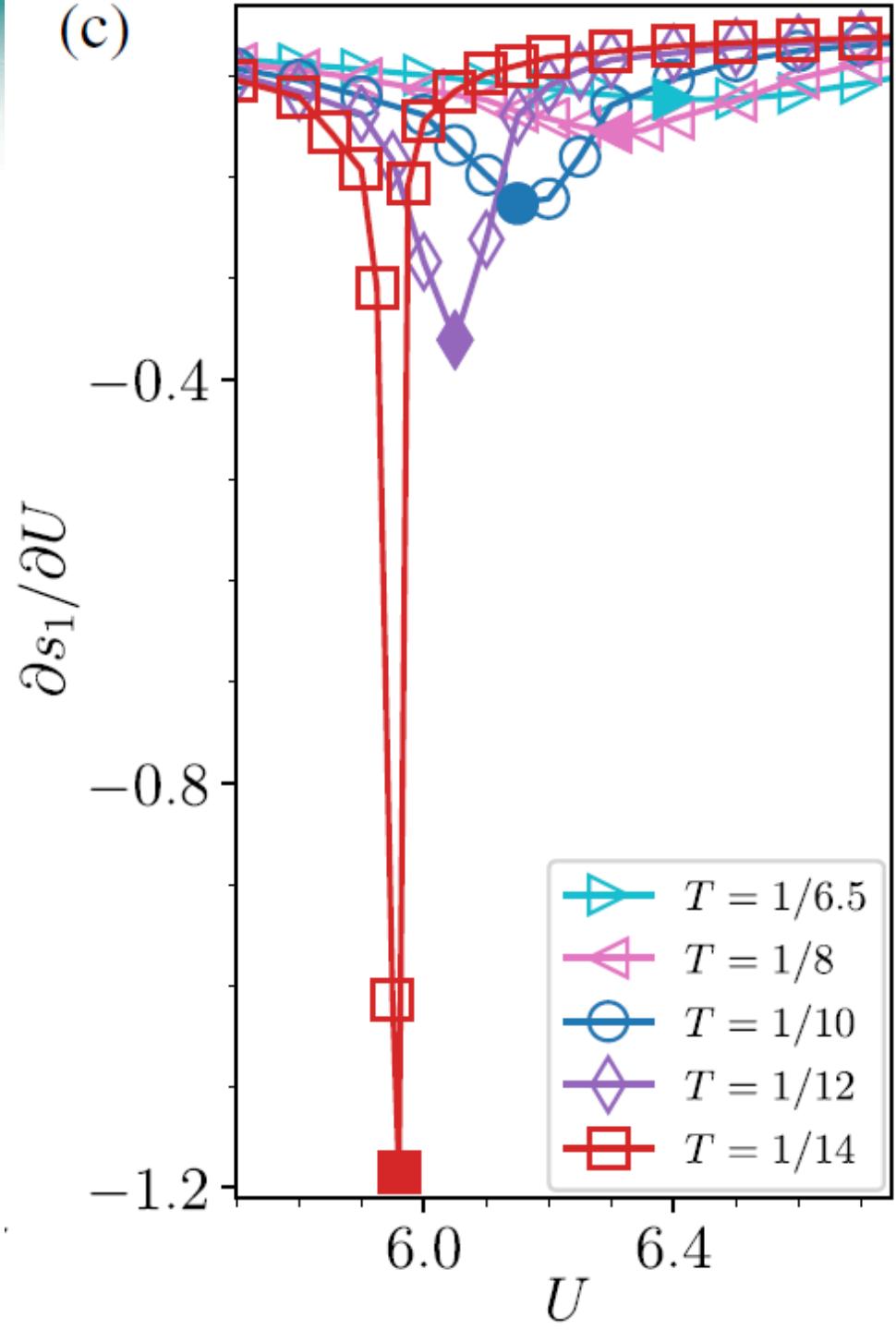


Results



$$\partial s_1 / \partial T < 0$$

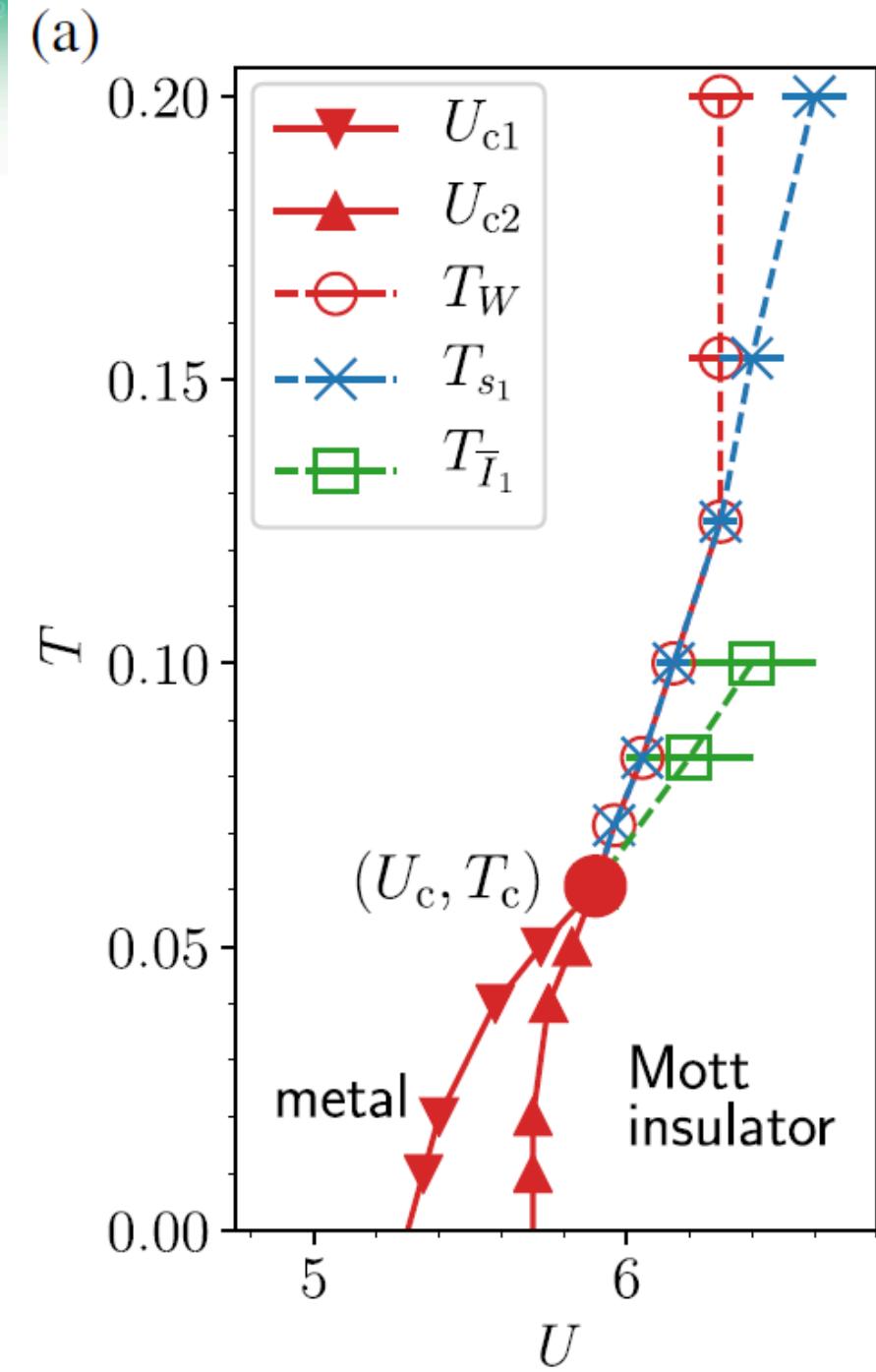
Results



$$(\partial s_1 / \partial D)(\partial D / \partial U)$$

$$\partial D / \partial U \sim -|U - U_c|^{-1+1/\delta}$$

Transition and crossovers



From single-site entanglement entropy

- The Mott transition,
- Critical exponent (not usually the case)
- Associated high-temperature crossovers,
 - Without knowledge of the order parameter of the transition

Mutual information

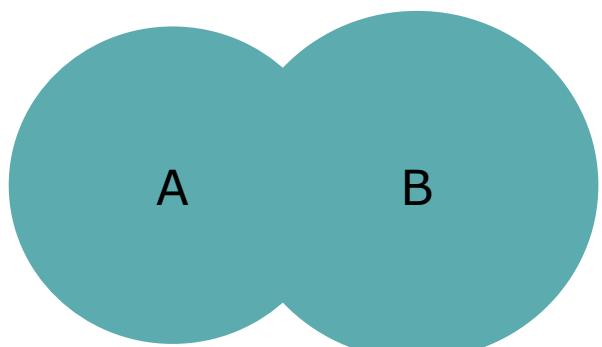
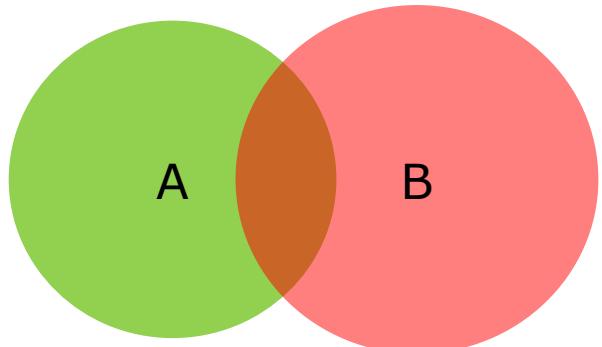
Mutual information

$$I(A:B) = s_A + s_B - s_{AB}$$

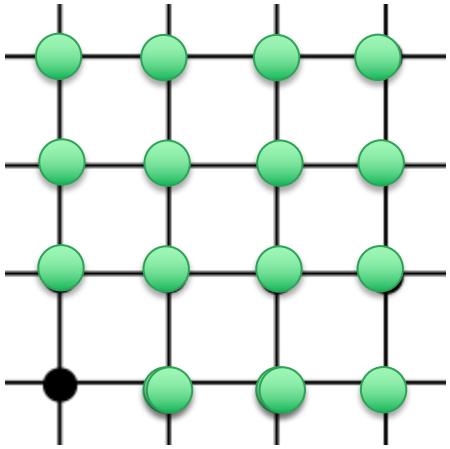
Here we are *not* looking at the area law

What is measured experimentally

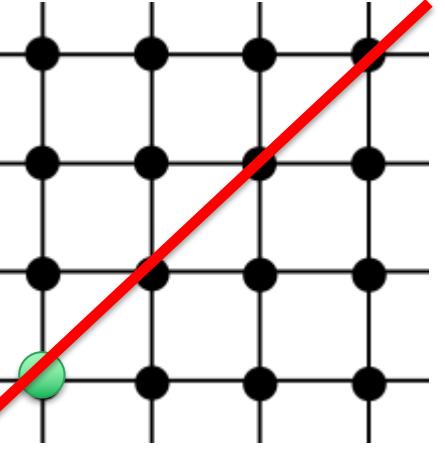
$$\bar{I}_1 = s_1 - s.$$



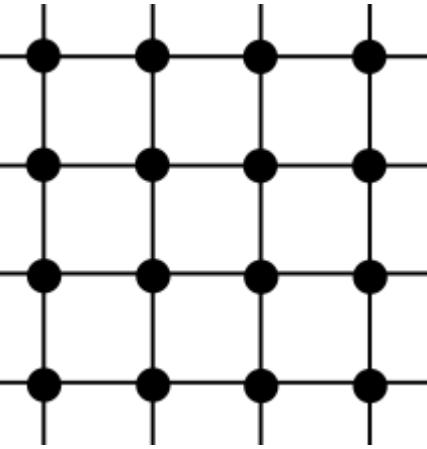
Total mutual information



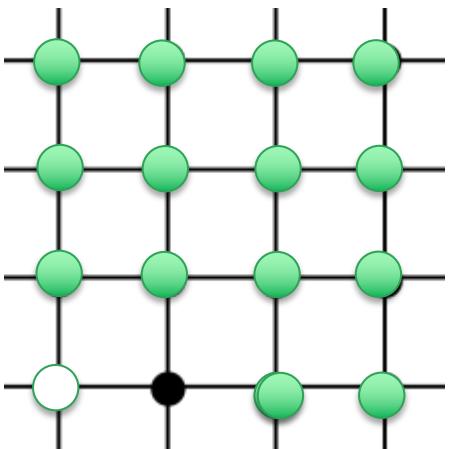
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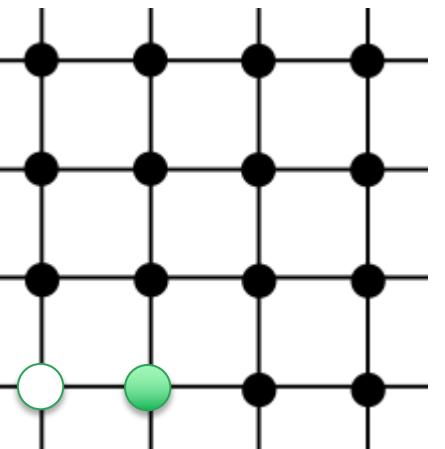
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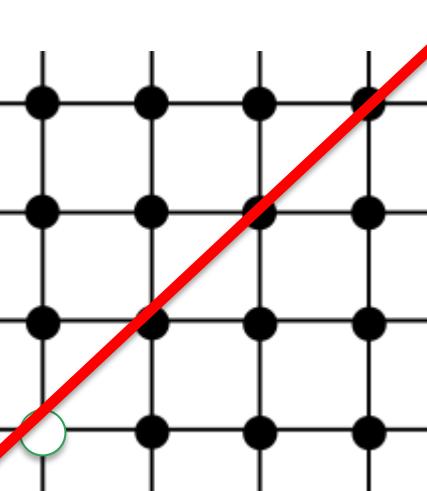
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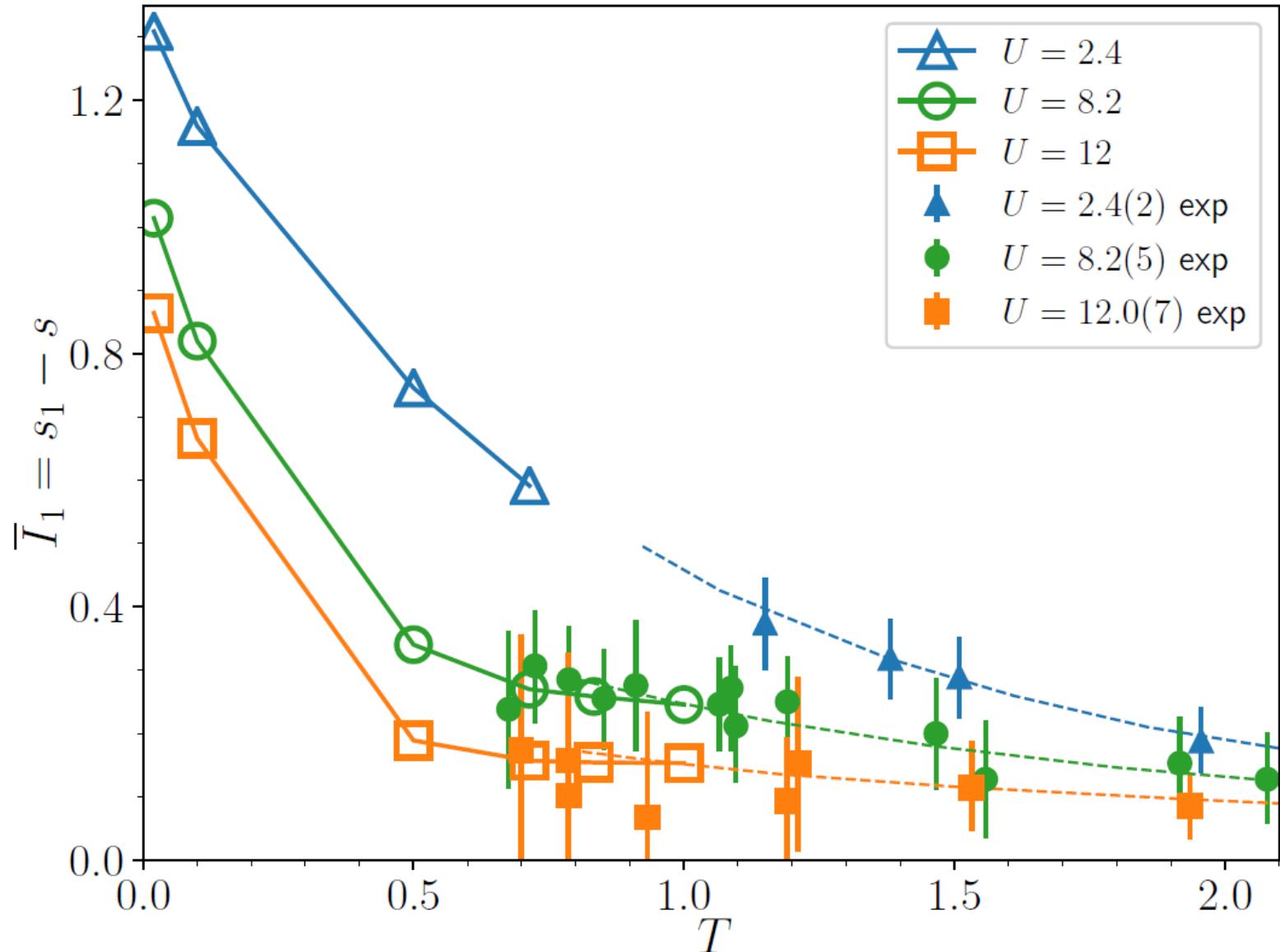
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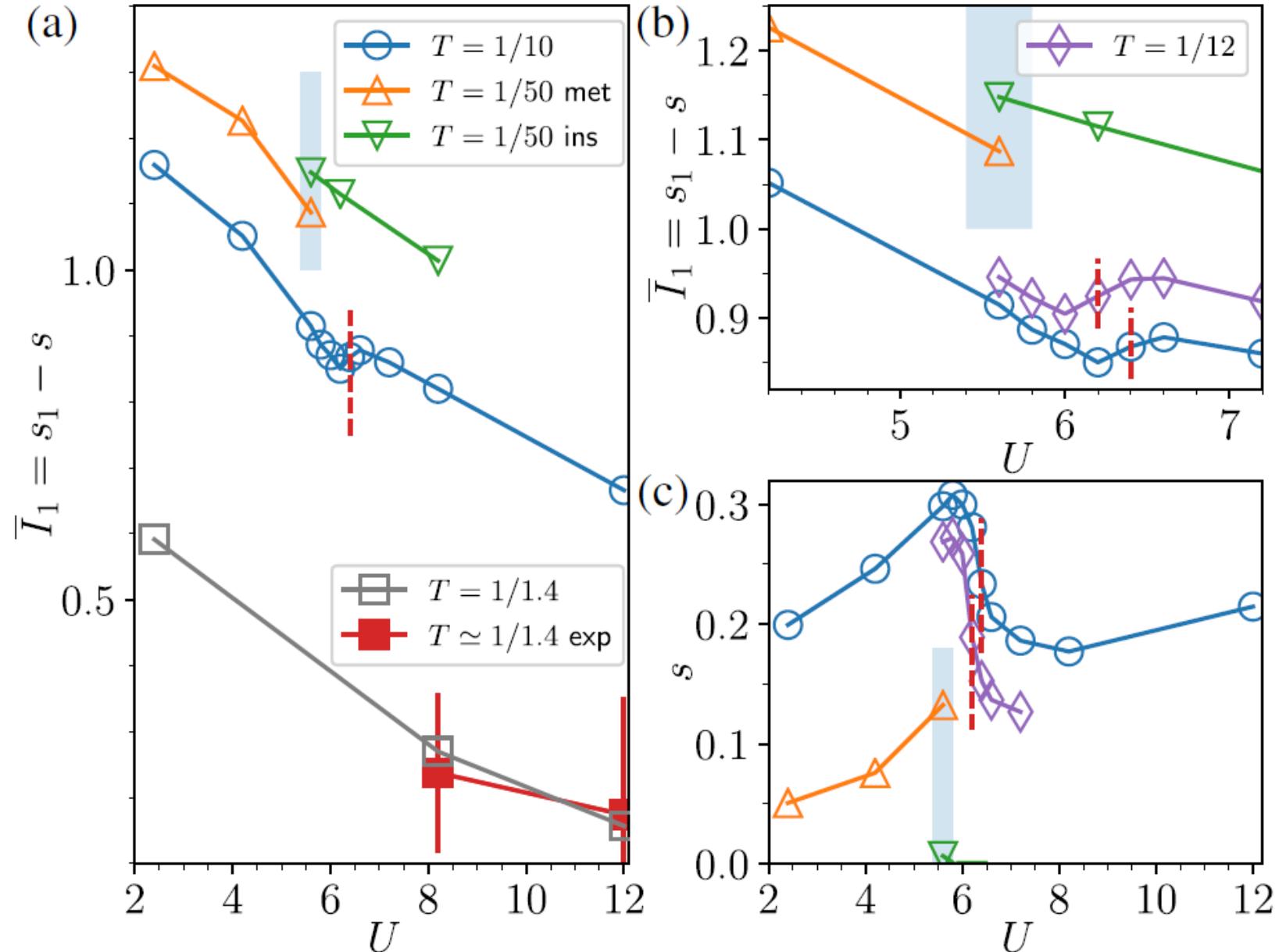
Agreement with experiment



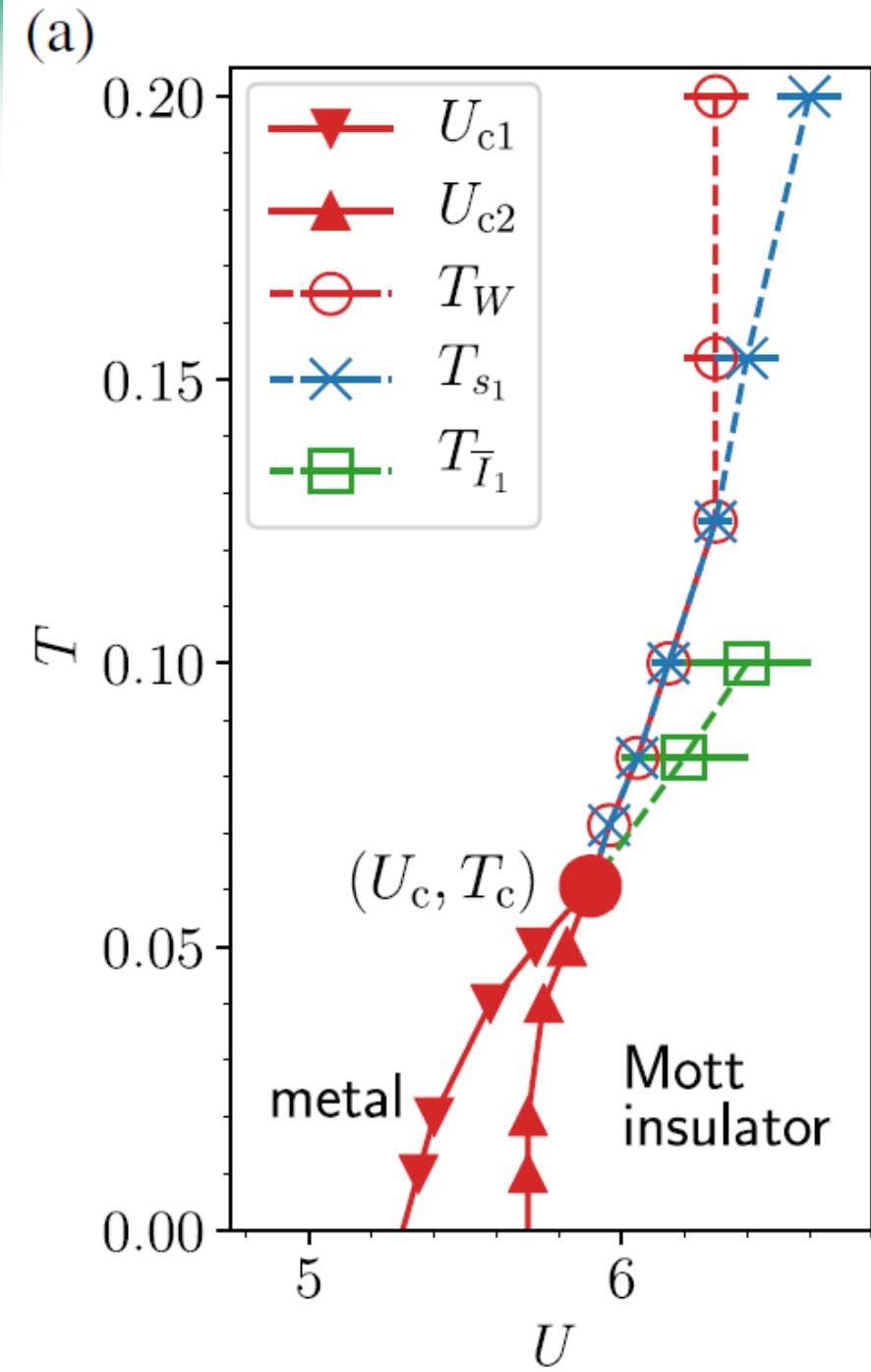
Results

$$\bar{I}_1 \sim \text{sgn}(U - U_c)|U - U_c|^{1/\delta}$$

10



Transition and crossovers



From average mutual information

- The Mott transition,
- Critical exponent (not usually the case)
- Associated high-temperature crossovers,
 - Without knowledge of the order parameter of the transition

Merci



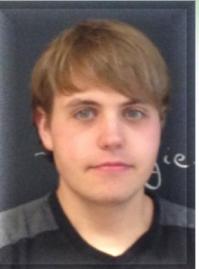
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Patrick Sémon



David Poulin



Giovanni Sordi