



# Ab initio DMFT: introduction to DFTTools, solid\_dmft, and related tools

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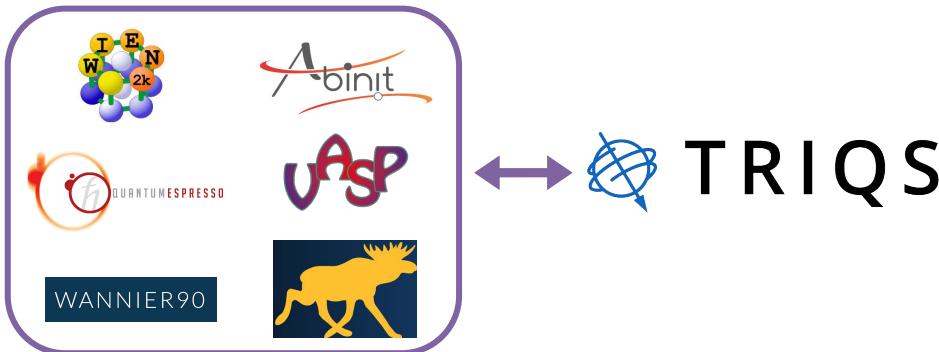
08/31/2023

# Outline

1. TRIQS/DFTTools: connection to ab initio codes
2. TRIQS/solid\_dmft: full DFT+DMFT wrapper
3. TRIQS: available impurity solvers
4. Analytic continuation with TRIQS
5. FermiSee: phenomenology & visualization
6. solid\_dmft tutorial

# 1. TRIQS/DFTTools: triqs.github.io/dft\_tools

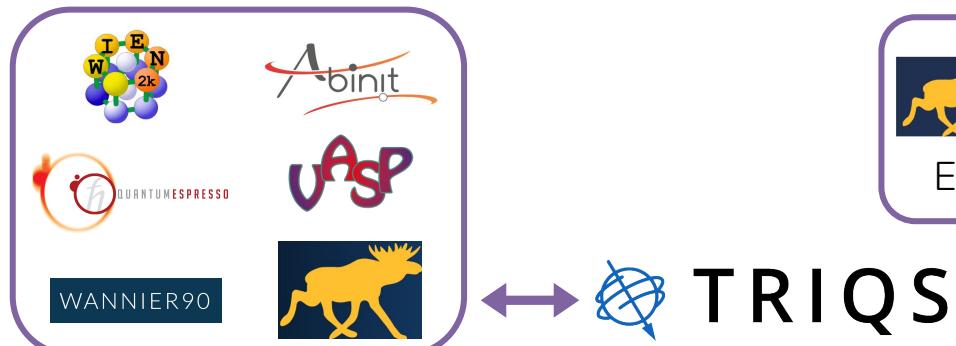
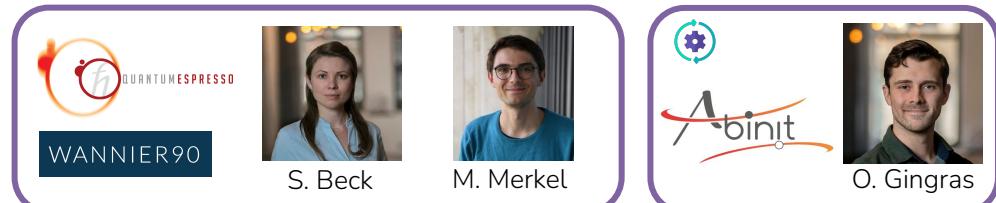
- same structure as TRIQS main project, i.e. automatic reference manual and tutorials
- issues and discussions on [github.com/triqs/dft\\_tools](https://github.com/triqs/dft_tools)
- M. Aichhorn et al. CPC '16  
~ 150 citations



The screenshot shows the homepage of the TRIQS DFTTools documentation at [triqs.github.io/dft\\_tools](https://triqs.github.io/dft_tools). The header includes the TRIQS logo and version 3.1.0. The main content area has a sidebar with links to Installation, Documentation (Basic notions, Construction of local orbitals from DFT, DFT+DMFT, Advanced Topics, Postprocessing), Reference manual (block\_structure, converters, sumk\_dft, sumk\_dft\_tools, symmetry, trans\_basis), FAQs, Tutorials, Reporting issues, Changelog, and About. The main content area discusses the application's purpose for correlated materials calculations, its interface to Wien2K and VASP packages, and its generic interface for DFT+DMFT calculations. It also mentions the built-in generic H(k) converter. A GitHub link is present in the sidebar.

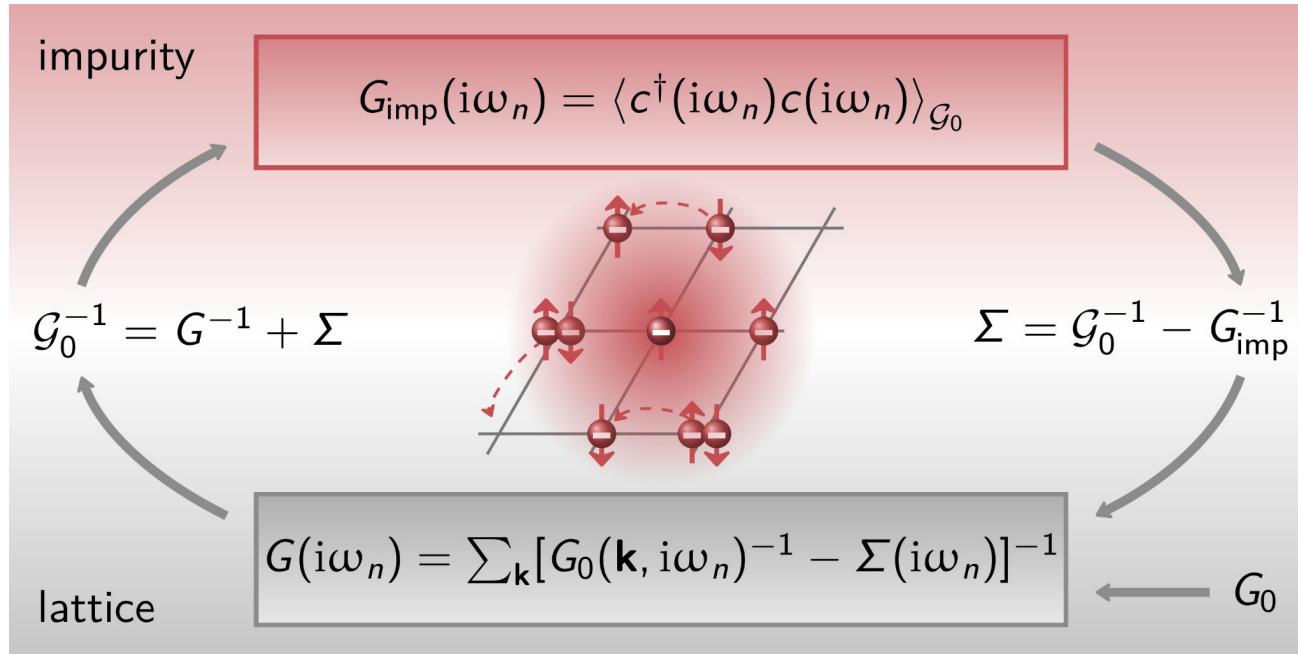
# 1. TRIQS/DFTTools: triqs.github.io/dft\_tools

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# 1. TRIQS/DFTTools: electronic structure interface

$$\hat{G}(\mathbf{k}, i\omega_n) = \sum_{\nu\nu'} \left[ i\omega_n + \mu - \hat{\epsilon}(\mathbf{k}) - \Delta \hat{\Sigma}(\mathbf{k}, i\omega_n) \right]_{\nu\nu'}^{-1} |\phi_{\nu\mathbf{k}}\rangle \langle \phi_{\nu'\mathbf{k}}|$$



Adapted from S.Beck

# 1. TRIQS/DFTTools: core functionality

- `lattice_gf()`       $G(\mathbf{k}, \omega)$
- `extract_G_loc()`     $G(\omega) = \sum_k G(\mathbf{k}, \omega)$
- downfold / upfold     $P_{m\nu R}(\mathbf{k})$
- `calc_mu()`             $\mu$
- `calc_dc()`             $\Sigma^{\text{DC}}$
- blockstructure class:  
`analyse_block_structure_from_gf()`

# 1. DFTTools example: Wannier90 converter + basics

```
from triqs_dft_tools.sumk_dft import SumkDFT
from triqs_dft_tools.converters import Wannier90Converter
from triqs.gf import *

Converter = Wannier90Converter(seedname='svo_t2g')
Converter.convert_dft_input()

mesh = MeshImFreq(beta=40, S='Fermion', n_iw=1025)
sumk = SumkDFT(hdf_file='svo_t2g.h5', mesh=mesh)

sumk.calc_mu()

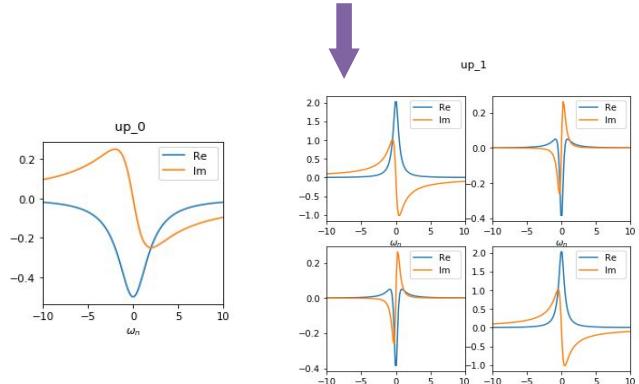
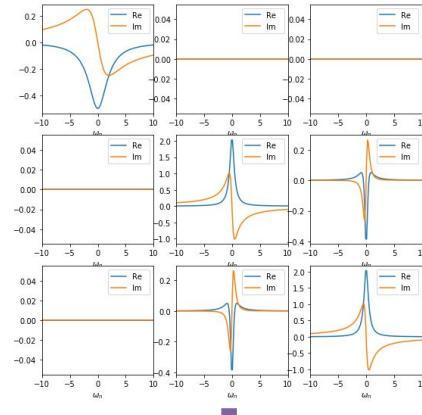
Gloc_iw = sumk.extract_G_loc()

sumk.analyse_block_structure_from_gf(Gloc_iw)

sumk.set_Sigma(Sigma)
```

# 1. DFTTools functionality: SumkDFT

- tutorials for DMFT calculation with Wien2k, Elk, Vasp, and QE / Wannier90
- post-processing:
  - spectral function: `density_of_states()`, `spaghettis()`
  - Fermi surfaces: `spectral_contours()`
  - transport / optical conductivity  $\sigma(\Omega)$
- manipulate orbital structure with `blockstructure` class (right)
- charge self-consistency updates of  $\rho$



## 2. solid\_dmft: DFT+DMFT calculations

🚢 TRIQS flagship implementation of DFT+DMFT

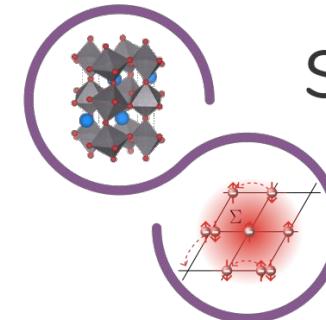
📈 Scalability with scriptable config file

🔗 interface to Vasp and Quantum Espresso for CSC calculations [2]

🔥 Reproducibility: versioning, h5 storage, convergence metrics

⚛️ Flexible solver choice: cthyb, ctseg, ctint, FTPS, HubbardI, Hartree, ...

📋 Online documentation & tutorials:  
[triqs.github.io/solid\\_dmft](https://triqs.github.io/solid_dmft)



# solid\_dmft

A versatile python wrapper to perform DFT + DMFT calculations utilizing the TRIQS software library.



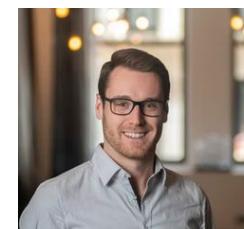
S. Beck



M. Merkel (ETH)



A. Carta (ETH)

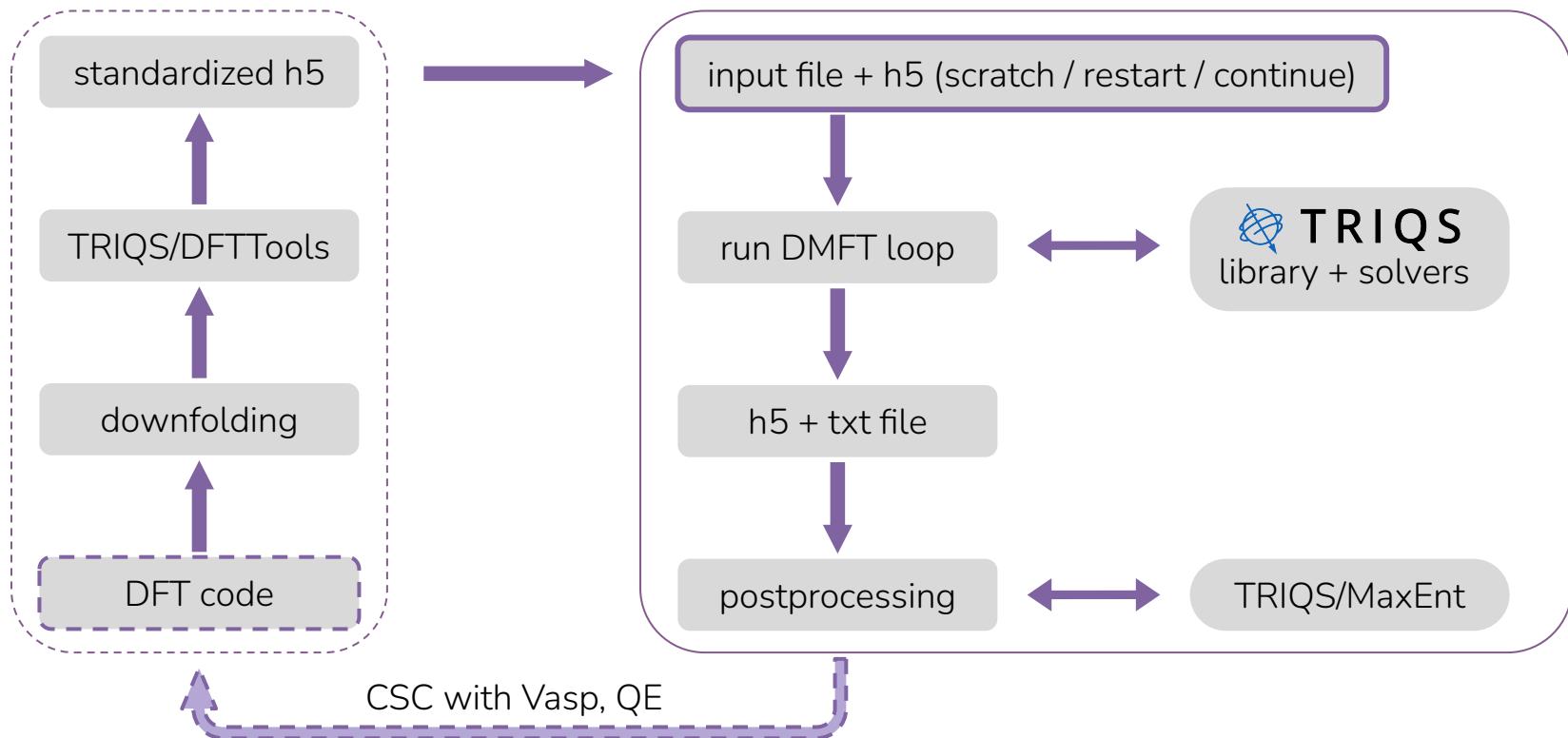


A. Hampel

[1] M. Merkel, A. Carta, S. Beck, AH, JOSS, doi.org/10.21105/joss.04623 (2022)

[2] S. Beck, AH, O. Parcollet, C. Ederer, and A. Georges, JoP: Condensed Matter, 34 (2022)

## 2. solid\_dmft: workflow

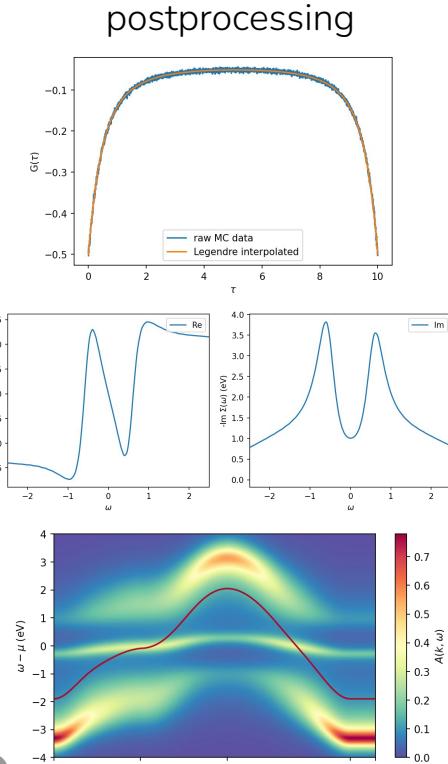
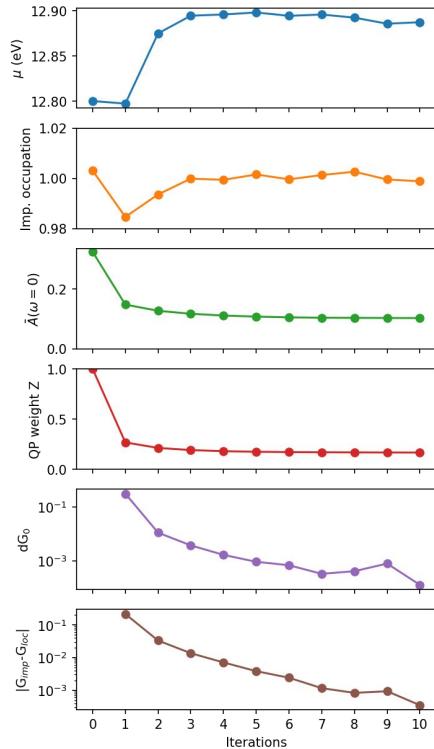


# 2. solid\_dmft: example & tutorials

example config file:

```
1 [general]
2 seedname = SVO
3 beta = 10
4 prec_mu = 0.001
5
6 solver_type = cthyb
7 n_l = 35
8
9 h_int_type = kanamori
10 U = 8.0
11 J = 0.65
12
13 n_iter_dmft = 10
14 g0_mix_type = linear
15 g0_mix = 0.9
16
17 dc_type = 1
18 dc = True
19 dc_dmft = False
20
21 [solver]
22 length_cycle = 120
23 n_warmup_cycles = 1e+4
24 n_cycles_tot = 1e+8
25 measure_G_l = True
```

mpirun solid\_dmft



[github.com/TRIQS/tutorials/AbinitioDMFT](https://github.com/TRIQS/tutorials/AbinitioDMFT)  
more tutorials: [triqs.github.io/solid\\_dmft/tutorials](https://triqs.github.io/solid_dmft/tutorials)

## 2. solid\_dmft: input

- divided into 4 sections:
  - general
  - solver
  - dft
  - advanced
- extensive tutorials (including charge self-consistency) on [triqs.github.io/solid\\_dmft/tutorials](https://triqs.github.io/solid_dmft/tutorials)

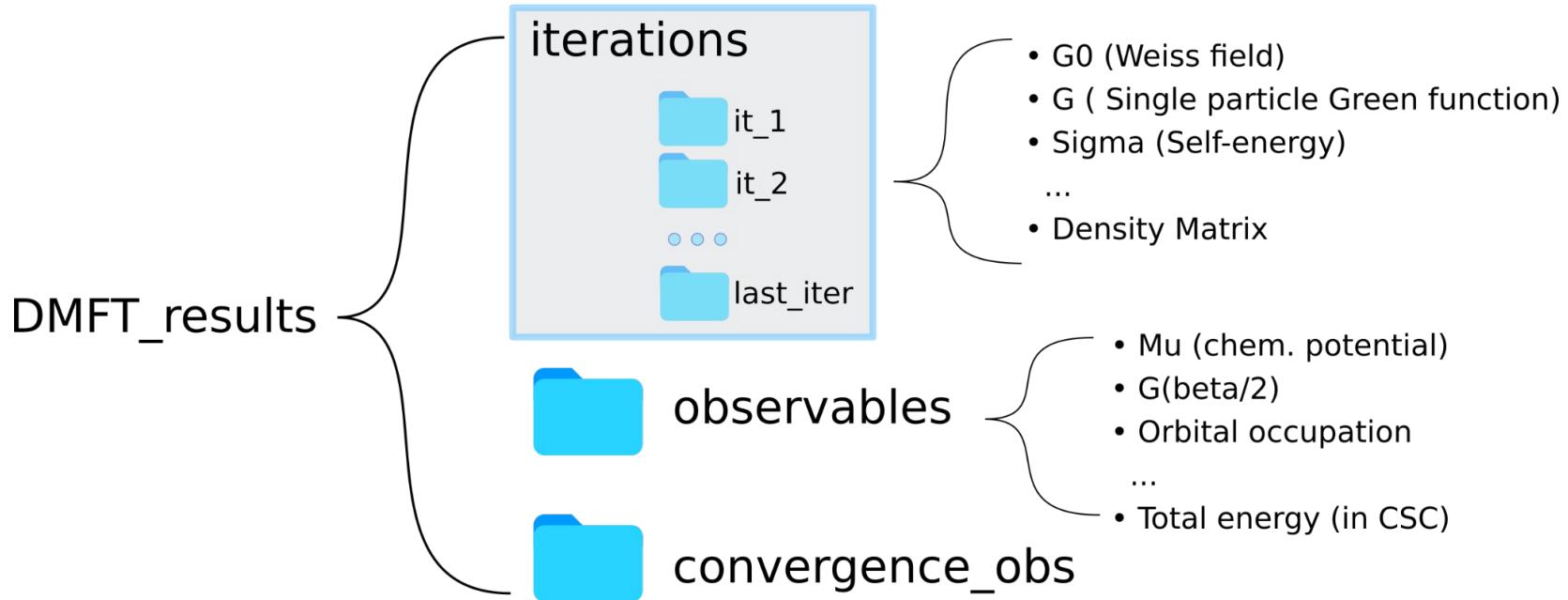
```
[general]
seedname = vasp
csc = True
solver_type = cthyb
beta = 10
n_iter_dmft_first = 5
n_iter_dmft = 12

h_int_type = kanamori
U = 6.5
J = 0.65
dc_type = 1
dc_dmft = True
calc_energies = True

[solver]
length_cycle = 120
n_warmup_cycles = 8000
n_cycles_tot = 1e+6
measure_density_matrix = True

[dft]
dft_code = vasp
dft_exec = vasp_std
n_cores = 1
```

## 2. solid\_dmft: standardized output for reproducibility



standardized output to continue previous calculations or just load self-energy

## 2. solid\_dmft utils: cRPA reader

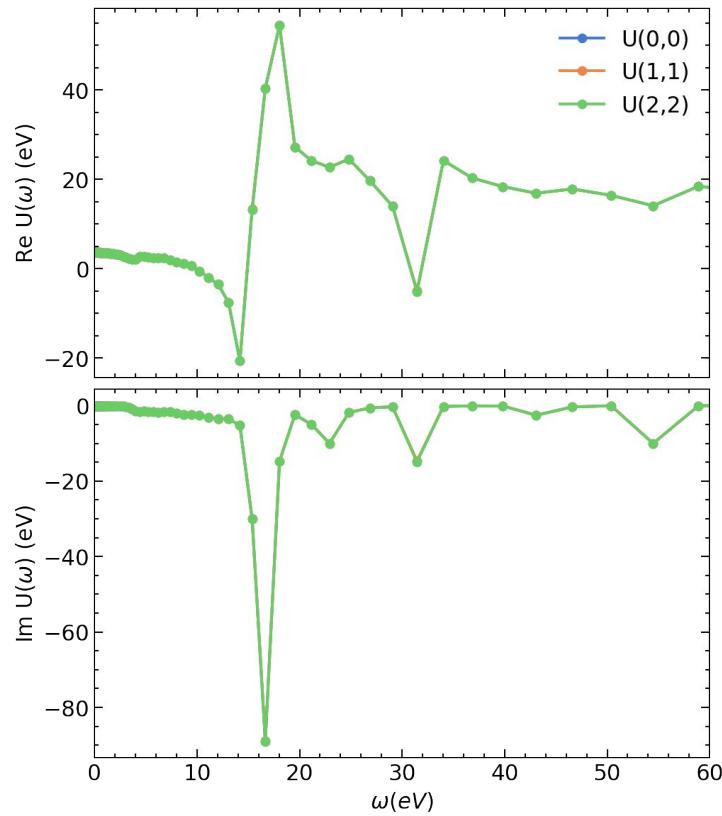
- RESPACK results reader

```
from solid_dmft.postprocessing.eval_U_cRPA_RESPACK
import read_interaction

RP = read_interaction(seed='svo',
                      path='./cRPA')

for key, value in RP.__dict__.items():
    print(key)
```

- gives access to numpy arrays of:
  - $U_R, V_R, J_R, X_R$
  - $U_{ijkl}, V_{ijkl}$
  - $U_{ij\omega}, J_{ij\omega}$
- same for Vasp cRPA
- results can be used in DMFT run\*

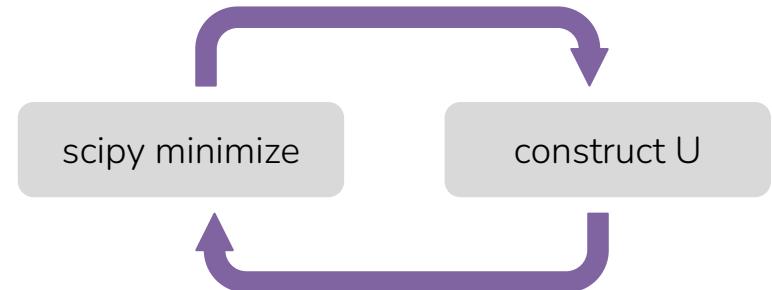


## 2. solid\_dmft utils: parameterize Coulomb tensor

- module  
`eval_U_cRPA_Vasp.py`:

$$\hat{H}_{\text{int}} = \frac{1}{2} \sum_{\sigma\sigma'} \sum_{mm'm''m'''} U_{mm'm''m'''} c_{m\sigma}^\dagger c_{m'\sigma'}^\dagger c_{m''' \sigma'} c_{m'' \sigma}$$

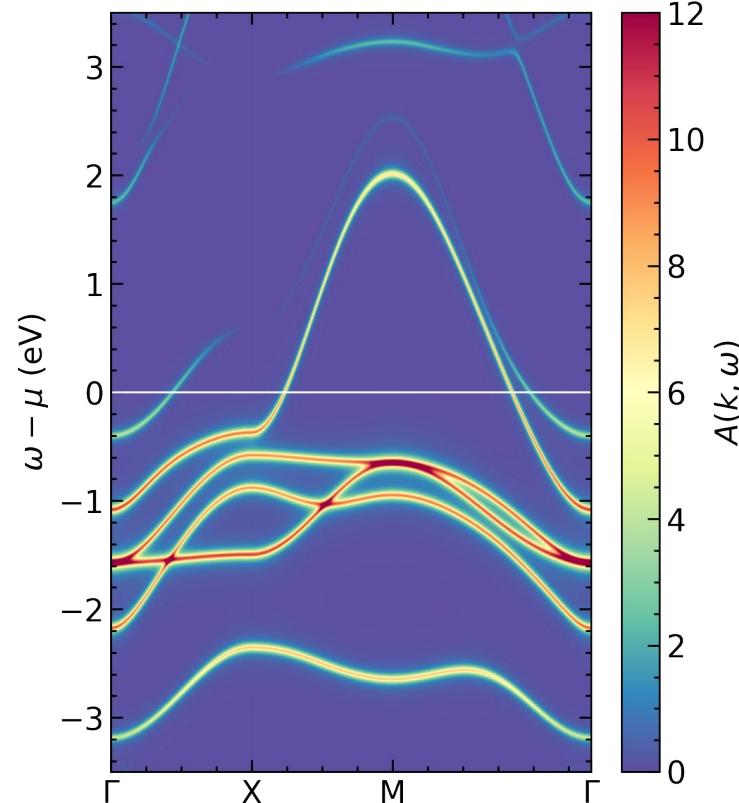
- `fit_kanamori(uijkl, n_orb, fit_2, fit_3, fit_4)`
- `fit_slater_fulld(u_ijij, u_ijji, U_init, J_init, fixed_F4_F2)`



$$\begin{aligned}\hat{H}_{\text{kan}} = & \frac{1}{2} \sum_{\sigma} \sum_m \mathcal{U} \hat{n}_{m\sigma} \hat{n}_{m\bar{\sigma}} \\ & + \frac{1}{2} \sum_{\sigma} \sum_{m \neq m'} [\mathcal{U}' \hat{n}_{m\sigma} \hat{n}_{m'\bar{\sigma}} + (\mathcal{U}' - \mathcal{J}) \hat{n}_{m\sigma} \hat{n}_{m'\sigma}] \\ & + \frac{1}{2} \sum_{\sigma} \sum_{m \neq m'} [\mathcal{J} c_{m\sigma}^\dagger c_{m'\bar{\sigma}}^\dagger c_{m\bar{\sigma}} c_{m'\sigma} + \mathcal{J}_C c_{m\sigma}^\dagger c_{m\bar{\sigma}}^\dagger c_{m'\bar{\sigma}} c_{m'\sigma}]\end{aligned}$$

## 2. solid\_dmft post-processing with H( $R$ )

- leverage Wannier interpolation to calculate  $G(k, \omega)$
- no pre-computing of projectors on k-path necessary
- high precision mesh allows to find contours and QP dispersion
- loads automatically self-energy and other parameters from solid\_dmft h5



## 2. solid\_dmft post-processing with H(R)

```
from solid_dmft.postprocessing import plot_correlated_bands as pcb

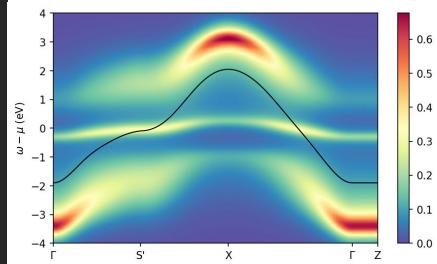
w90_dict = {'w90_path': './data/mlwf/', 'w90_seed': 'lco',
            'n_orb': 1, 'mu_tb': 12.7367}

tb_dict = {'bands_path': [('G', 'S'), ('S', 'X'), ('X', 'G'), ('G', 'Z')],
           'n_k': 50, 'G': [0.0, 0.0, 0.0],
           ...}

sigma_dict = {'dmft_path': 'b10-U3.6/lco_wannier.h5',
              'it': 'last_iter', 'spin': 'up', 'block' : 0 }

tb_bands, alatt_kw, freq = pcb.get_dmft_bands(with_sigma='calc',
                                                **w90_dict, **tb_dict, **sigma_dict)

fig, ax = plt.subplots(1)
pcb.plot_bands(fig, ax, alatt_kw, tb_bands, freq, n_orb=w90_dict['n_orb'],
               alatt=True, colorscheme_bands='Greys', colorscheme_alatt='Spectral_r')
```



### 3. TRIQS: available impurity solvers

solver name	method	# orb	measure	comments
cthyb	ct-qmc hyb expansion	~5	$G(\tau)$ , $G^{(2)}$ , $X_{AB}$ , $\rho_{imp}$	small $U/\Delta$ , off diag $\Delta \rightarrow$ sign
ctseg	ct-qmc hyb segment picture	~8	$G(\tau)$ , $G^{(2)}$ , $X_{AB}$ , $\rho_{imp}$	small $U/\Delta$ , nn int only, $U(\tau)$
ctint	ct-qmc interaction expansion	~80	$G(\tau)$ , $G^{(2)}$ , $X_{AB}$	small $\Delta/U$ , nn int only, $U(\tau)$
forkTPS	fork tensor product states	~5	$G(t)$ , $X_{AB}$	$\eta \sim 1e-2$ , Kanamori only
hartree_fock	Hartree / Hartree-Fock	~20	$\Sigma^{HF}$	no $\omega$ , HF only
hubbardl	ED of impurity problem	~7	$G(i\omega_n)$ , $G(\omega)$ , $G^{(2)}$ , $X_{AB}$	$\Delta > 0$
nrgljubljana_interface	NRG	~3	$G(\omega)$ , $A(\omega)$ , $X_{AB}$	log mesh around $\omega=0$
pomerol2triqs	ED of impurity problem	~3	$G(\omega)$ , $G(i\omega_n)$ , $G^{(2)}$ , $X_{AB}$	discretized bath
w2dynamics_interface	ct-qmc hyb / seg exp + worm	~5	$G(\tau)$ , $G^{(2)}$ + worm, $X_{AB}$ , $\rho_{imp}$	small $U/\Delta$ , off diag $\Delta \rightarrow$ sign, $U(\tau)$

## 4. analytic continuation: TRIQS

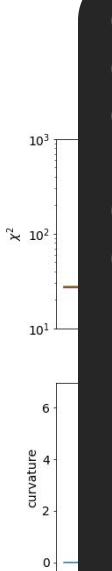
- TRIQS provides several apps to analytically continue Matsubara Green functions:

$$G(i\omega_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{i\omega_n - \omega} A(\omega) \quad \text{or} \quad \mathbf{G} = \mathbf{K}\mathbf{A} \quad (\text{matrix form})$$

- [triqs.github.io/maxent](https://triqs.github.io/maxent)
  - G. J. Kraberger et al. , PRB 96 (2017)
- [triqs.github.io/Nevanlinna](https://triqs.github.io/Nevanlinna)
  - S. Iskakov et al. , CPC (2023)
- [triqs.github.io/omegamaxent\\_interface](https://triqs.github.io/omegamaxent_interface)
  - ΩMaxEnt code, D. Bergeron and A.-M.S. Tremblay, PRE 94 (2016)
- [krivenko.github.io/som](https://krivenko.github.io/som)
  - Stochastic Optimization Method, I. Krivenko et al. , CPC 239 (2019)

# 4. analytic continuation: TRIQS/maxent

- [triqs.github.io/maxent](https://triqs.github.io/maxent)
- different ways to choose  $\alpha$  implemented:
  - line-fit
  - from curvature of  $\log(\chi^2)$  vs  $\log(\alpha)$
  - Bryan
- matrix valued continuation
- self-energy continuation (element-wise)



```
G_iw = GfImFreq(beta=10, indices=[0])
G_iw << SemiCircular(1)-0.5*SemiCircular(0.5)
G_tau = GfImTime(beta=10, indices=[0],
n_points=2501)
G_tau.set_from_fourier(G_iw)
G_tau.data[:, 0, 0] += 1.e-5 *
np.random.randn(len(G_tau.data))

from triqs_maxent import *
tm = TauMaxEnt(cost_function='bryan',
probability='normal')
tm.set_G_tau(G_tau)
tm.set_error(1e-4)
# run maxent
result = tm.run()
result.get_A_out('LineFitAnalyzer')
```

## 4. analytic continuation: solid\_dmft + maxent

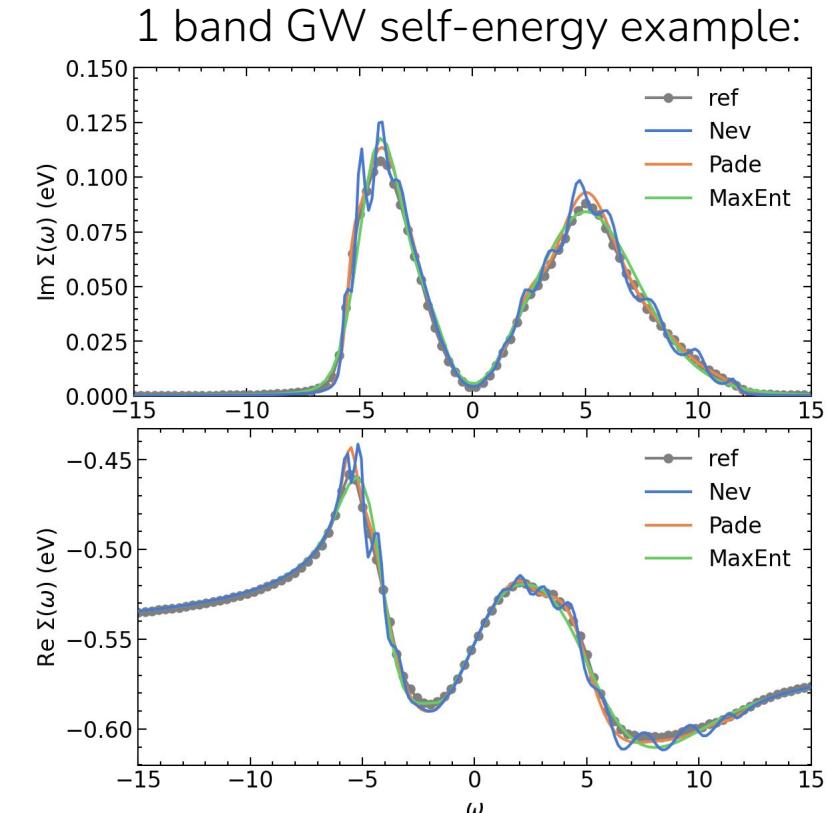
- integration between solid\_dmft and maxent (block structure, DC,  $\mu$ )
- MPI parallelized over blocks
- continuation of:
  - $G_{\text{imp}}$ : postprocessing.maxent\_gf\_imp
  - $G_{\text{latt}}$ : postprocessing.maxent\_gf\_latt
  - $\sum_{\text{imp}}$ : postprocessing.maxent\_sigma
- writes result back to h5
- automatically used by post-processing routines

```
from solid_dmft.postprocessing import maxent_sigma

# use pcb maxent script to continue self energy
Sigma_real_freq =
    maxent_sigma.main(external_path=h5_file,
                      omega_min=-10, omega_max=10,
                      maxent_error=0.03,
                      iteration=None,
                      n_points_maxent=101,
                      n_points_alpha=50,
                      analyzer='LineFitAnalyzer',
                      n_points_interp=2001,
                      n_points_final=1001,
                      continuator_type='inversion_dc')
```

## 4. analytic continuation: TRIQS/nevanlinna & Pade

- Nevanlinna, matrix valued Caratheodory, and Hardy optimizations
- works best for non-continuous spectra
- further extension necessary for noisy data
- [triqs.github.io/Nevanlinna/latest/documentation](https://triqs.github.io/Nevanlinna/latest/documentation)



# 5. FermiSee: WebApp for data visualization

$$A(\omega, \mathbf{k}) = -\frac{1}{\pi} \text{Im} \sum_{\alpha=\alpha'} [\omega + \mu - \epsilon(\mathbf{k}) - \Sigma(\omega)]_{\alpha\alpha'}^{-1}$$



[fermisee.flatironinstitute.org](http://fermisee.flatironinstitute.org)



[github.com/TRIQS/FermiSee](https://github.com/TRIQS/FermiSee)



testers and developers welcome



WANNIER90

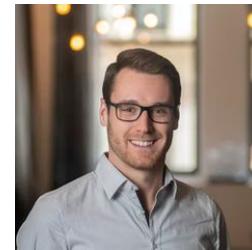
PythTB



S. Beck



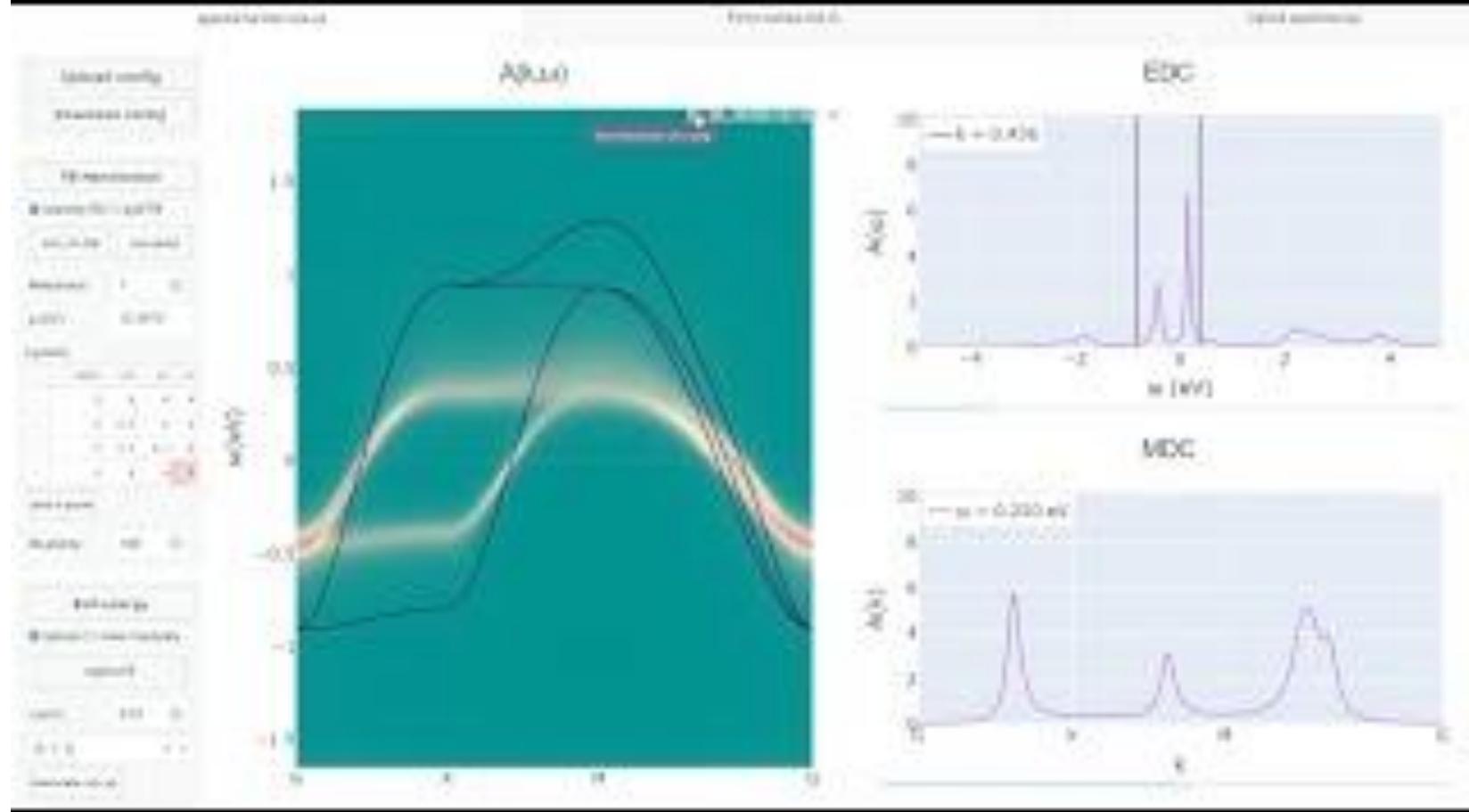
S. Rahim



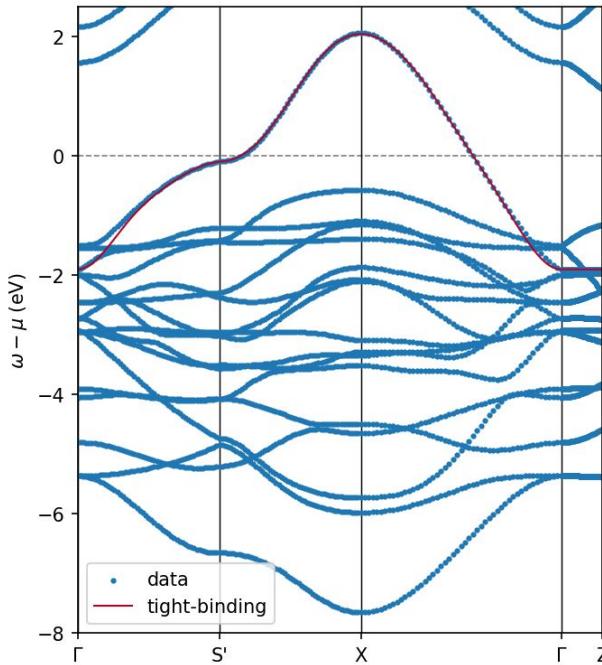
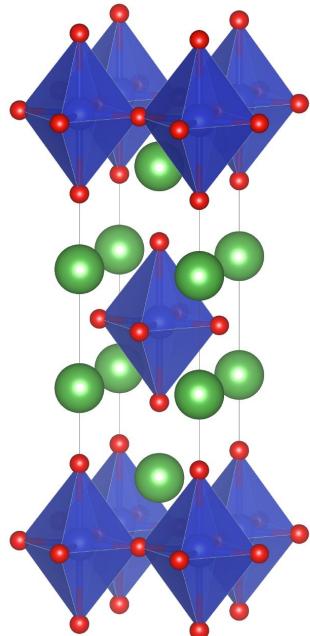
A. Hampel



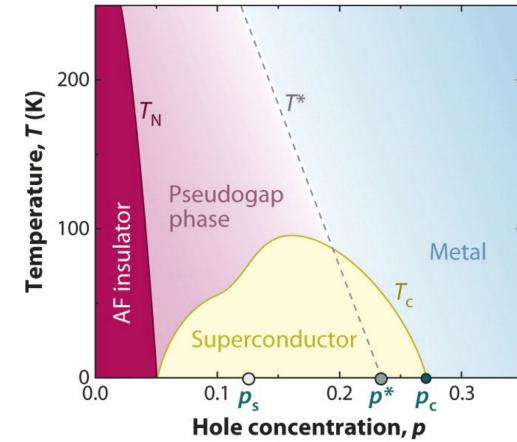
## 5. FermiSee demo



# 6. solid\_dmft tutorial: Mott insulator $\text{La}_2\text{CuO}_4$

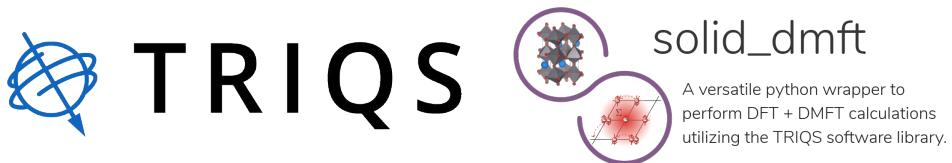


- simple 1-band model,  
2D square-lattice sheets
- 1e<sup>-</sup> per Cu atom, mainly  
 $d_{x^2-y^2}$  character



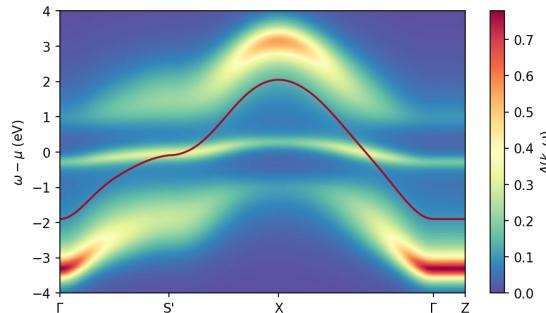
# Summary

-  TRIQS ecosystem to perform ab-initio simulations for correlated electron systems
-  solid\_dmft as flagship implementation for DFT+DMFT and embedding
-  Analytic continuation packages
-  FermiSee: webapp for data visualization



solid\_dmft

A versatile python wrapper to perform DFT + DMFT calculations utilizing the TRIQS software library.



Acknowledgements:



N. Wentzell



S. Beck



O. Parcollet



A. Georges